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STABILIZING CONTROL FOR DISCRETE TIME MULTI-INPUT BILINEAR SYSTEMS

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Abstract: This paper presents two stabilizing control methods for discrete time bilinear systems. One is a stabilizing controller for single input systems, and the other is a generalized stabilizing control method for discrete time multi-input bilinear systems. The proposed control methods guarantee the stability for each closed loop systems with single-input case and multi-input case, respectively. This paper introduces a lemma used to extend the stabilizing control method which is useful to represent any bilinear systems as pure differential(difference) matrix equations. The proposed algorithms are verified by a numerical example.

Keywords: Bilinear Systems, Lyapunov methods, Stabilization methods, Discrete-time systems, Multi-input systems

1. INTRODUCTION

Physical systems are inherently nonlinear in nature. Also, many real physical systems in chemical process, nuclear, engineering, biology, ecology, socioeconomic, etc. are described by bilinear models. Detailed reviews of bilinear systems and their control design methods can be found in (Mohler, 1991).

The stabilization problems for the bilinear systems have been widely studied in the past by many researchers, see for examples (Chiou *et al.*, 2000)(Chen *et al.*, 2000)(Chen and Tsao, 2000)(Chen *et al.*, 1991)(Wang and Chiou, 1991). For nonlinear systems, a powerful method in robust stabilization is the so-called Lyapunov approach. The advantage of this approach is that the resulting closed-loop systems are globally asymptotically stable. Many researches are devoted to the continuous time bilinear systems. In (Niculescu *et al.*, 1995) the closed-loop stability of a class of continuous time bilinear systems with time delayed state was studied. In (Chen and Tsao, 2000) a nonlinear controller was designed to exponentially stabilize the open-loop unstable continuous time bilinear systems.

A nonlinear feedback controller for the stabilization of discrete-time multi-input bilinear systems having an external input was discussed in (Wang and Chiou, 1991) by applying the Lyapunov's direct method. In their scheme each entry of a control input vector is separately obtained under the assumption that system matrix is stable. Recently, a novel approach to the design of nonlinear robust stabilizing nonlinear state feedback controllers for a class of singularly perturbed discrete bilinear systems with single input was developed in (Chiou *et al.*, 2000).

The main motivation of this work is to extend the results of (Chiou *et al.*, 2000) to discrete time multi-input bilinear systems. In this paper two control algorithms are presented to solve the stabilizing problem of discrete time bilinear systems. One is a modified stabilizing algorithm based on the results of (Chiou *et al.*, 2000) for discrete time single-input bilinear systems. And generalized stabilizing algorithm for discrete time multi-input bilinear systems is presented. These proposed methods also use the same Lyapunov function. Moreover, this paper presents a lemma used to extend to the multi-input systems which is useful to represent bilinear systems as pure differential(difference) matrix equations.

The organization of this paper is as follows: Section 2 introduces the previous results of (Chiou *et al.*, 2000) and proposes a modified stabilizing algorithm for discrete time single-input bilinear systems. The generalized stabilizing control algorithm to discrete time multi-input bilinear systems is presented in Section 3. In Section 4, a numerical example is given. The conclusion is given in the last section.

2. STABILIZING CONTROL FOR DISCRETE SINGLE-INPUT BILINEAR SYSTEMS

Some necessary notations used in this paper are as follows. Let $\Re^{n\times 1}$ denote the usual n-dimensional vector space and the norm of a vector $x = [x_1 \; \cdots \; x_n]^T$ on $\Re^{n\times 1}$ be denoted by

$$\|x\| = \Big(\sum_{i=1}^{n} x_i^2\Big)^{1/2}$$

If A is an $n\times m$ matrix over $\Re$, then the norm of A is defined by

$$\|A\| = \Big(\sum_{i=1}^{n}\sum_{j=1}^{m} a_{ij}^2\Big)^{1/2}$$

And spectral radius of a matrix A is denoted by $r(A)$.

Consider the discrete time single-input bilinear systems described by difference equation of the form

$$x(k+1) = Ax(k) + Bu(k) + Mx(k)u(k) \tag{1}$$

where $x \in \Re^{n\times 1}$ is a state vector, $u \in \Re$ is a scalar control law, and $A \in \Re^{n\times n}$, $B \in \Re^{n\times 1}$, $M \in \Re^{n\times n}$ are constant matrices. It is assumed that $r(A) < 1$.

The following Lyapunov function candidate is selected to derive the stabilizing control law.

$$V(x(k)) = x^T(k)Px(k) \tag{2}$$

where $P \in \Re^{n\times n}$ in (2) is a unique real symmetric positive-definite matrix satisfying the following discrete Lyapunov equation

$$(1+\gamma)A^TPA - P = -I_n \tag{3}$$

where γ is a positive constant satisfying

$$\sqrt{(1+\gamma)}r(A) < 1$$

Before dealing with the main problems, some important preliminary lemma is reviewed.

Lemma 1. (Furnkama and Shimemura, 1983) Consider the matrices A, B, and C, which have the same dimensions, and let $C = A + B$. For any positive constant γ and positive definite symmetric matrix D, the following relation holds:

$$CDC^T \le (1+\gamma)ADA^T + (1+\gamma^{-1})BDB^T \tag{4}$$

The proof of this lemma follows from (Furnkama and Shimemura, 1983). The following Theorem 2 discussing the stabilization control for discrete time single input bilinear systems described in (1) is essentially an extension of the results of (Chiou *et al.*, 2000) and provides an upper bound of input for global asymptotic stabilizabilty.

Theorem 2. Consider discrete time single input bilinear systems (1). The nonlinear state feedback control law globally asymptotically stabilizes the equilibrium point of (1).

$$u(k) = \frac{-\kappa Cx(k)}{\sqrt{1 + x^T(k)C^TCx(k)}} \tag{5}$$

where $C \in \Re^{1\times n}$ can be arbitrarily designed and κ satisfies the following inequality.

$$\kappa \;<\; \kappa^* = \frac{1}{\sqrt{2}}\min\Big(\Big(\frac{\gamma}{(1+\gamma)^2\|C^TB^TPBC\|}\Big)^{\frac{1}{2}}, \Big(\frac{\gamma^2}{(1+\gamma)^2\|M^TPM\|}\Big)^{\frac{1}{2}}\Big) \tag{6}$$

PROOF. With the control law defined in (5), the closed-loop system (1) can be rewritten as

$$x(k+1) = \Big(A - \frac{\kappa BC}{\sqrt{\varphi}} - \frac{\kappa Mx(k)C}{\sqrt{\varphi}}\Big)x(k) \tag{7}$$

where φ denotes $1 + x^T(k)C^TCx(k)$. Denoting $\Delta V \triangleq \Delta V(x(k))$ and using (7), the Lyapunov forward difference is given by

$$\Delta V = x^T\Big(A - \frac{\kappa}{\sqrt{\varphi}}(BC + MxC)\Big)^T P \Big(A - \frac{\kappa}{\sqrt{\varphi}}(BC + MxC)\Big)x - x^TPx \tag{8}$$

Using Lemma 1 and (3), (8) can be rewritten as

$$\Delta V \le x^T\Big((1+\gamma)A^TPA - P + \frac{\kappa^2}{\varphi}\frac{1+\gamma}{\gamma}(BC+MxC)^TP(BC+MxC)\Big)x$$
$$\le x^T\Big(-I_n + \frac{\kappa^2}{\varphi}\frac{1+\gamma}{\gamma}((1+\gamma)C^TB^TPBC + \frac{1+\gamma}{\gamma}C^Tx^TM^TPMxC)\Big)x(k)$$
$$\le x^T\Big(-I_n + \frac{\kappa^2}{\varphi}\frac{(1+\gamma)^2}{\gamma}C^TB^TPBC + \frac{\kappa^2C^Tx^TxC}{\varphi}\frac{(1+\gamma)^2}{\gamma^2}M^TPM\Big)x \qquad (9)$$

Since the following inequalities are always satisfied for any x and C

$$\frac{C^Tx^TxC}{1+x^TC^TCx} \le 1, \quad \frac{1}{1+x^TC^TCx} \le 1, \qquad (10)$$

(9) can be rewritten as

$$\Delta V \le x^T\Big(-I_n + \frac{(1+\gamma)^2\kappa^2}{\gamma}C^TB^TPBC + \frac{(1+\gamma)^2\kappa^2}{\gamma^2}M^TPM\Big)x$$
$$\le x^T\Big(-I_n + \frac{(1+\gamma)^2\kappa^2}{\gamma}\|C^TB^TPBC\|I_n + \frac{(1+\gamma)^2\kappa^2}{\gamma^2}\|M^TPM\|I_n\Big)x$$
$$\le x^T\Big(\frac{2(1+\gamma)^2}{\gamma} \times \max\Big(\|C^TB^TPBC\|, \frac{1}{\gamma}\|M^TPM\|\Big)\kappa^2I_n - I_n\Big)x \qquad (11)$$

Clearly, the right hand side of (11) is negative definite if and only if

$$\max\Big(\|C^TB^TPBC\|, \frac{1}{\gamma}\|M^TPM\|\Big)\kappa^2 < \frac{\gamma}{2(1+\gamma)^2} \qquad (12)$$

This inequality completes the proof of this theorem. ■

Remark 3. The denominator of the second term of right hand side of (6) is not zero, if M in (1) is a zero matrix, then (1) becomes linear systems.

Remark 4. The necessary and sufficient condition for the existence of a unique solution of discrete algebraic Lyapunov equation (3) is that no two eigenvalues have product equal to one (Gajic and Qureshi, 1995), that is

$$\lambda_i\lambda_j \ne 1, \quad i,j = 1,2,\cdots,n \qquad (13)$$

This condition is obviously satisfied if all eigenvalues of $\sqrt{(1+\gamma)}A$ are strictly inside of a unit circle.

Theorem 2 slightly differs in the upper bound κ^* from that of (Chiou *et al.*, 2000) where the upper bound was given by

$$\kappa^* = \min\Big(\Big(\frac{\gamma}{(1+\gamma)^2\|C^TB^TPBC\|}\Big)^{\frac{1}{2}}, \Big(\frac{\gamma^2}{(1+\gamma)^2\|M^TPM\|}\Big)^{\frac{1}{2}}\Big) \qquad (14)$$

3. STABILIZING CONTROL FOR DISCRETE MULTI-INPUT BILINEAR SYSTEMS

A straightforward generalization of the previous result to an extended class of discrete time multi-input bilinear systems is noted in this section. Consider again the discrete time multi-input bilinear systems described by difference equation of the form

$$x(k+1) = Ax(k) + \sum_{i=1}^{n} x_i(k)M_iu(k) + Bu(k) \qquad (15)$$

where $u \in \Re^{m\times 1}$ is an input vector and $B \in \Re^{n\times m}$, $M_i \in \Re^{n\times m}$ are constant matrices and $x_i(k)$ is an i-th element of the state vector. In the following, for convenience, $x_i(k)$ and $\sum_{i=1}^{n} x_i(k)M_i$ are denoted by x_i and $\{xM\}$, respectively. The following lemma is needed to develop the stabilizing control law for discrete time multi-input bilinear systems.

Lemma 5. Consider a vector $x \in \Re^{n\times 1}$ and matrices $M_i \in \Re^{n\times m}$ and $N_i \in \Re^{n\times m}$ with $i = 1,2,\cdots,n$. Then the following relation holds:

$$\sum_{i=1}^{n} x_iM_i = \sum_{i=1}^{n} X_iN_i \qquad (16)$$

with unique N_i as follows:

$$N_i = \begin{bmatrix} n_{i1}^1 & n_{i2}^1 & \cdots & n_{im}^1 \\ n_{i1}^2 & n_{i2}^2 & \cdots & n_{im}^2 \\ & \vdots & & \\ n_{i1}^n & n_{i2}^n & \cdots & n_{im}^n \end{bmatrix} \qquad (17)$$

where n_{ij}^k of N_i is an (i,j)-th element of M_k matrix with $j = 1,\cdots,m$, $k = 1,\cdots,n$. And the i-th row of $X_i \in R^{n\times n}, i = 1,\cdots,n$, is x^T and the other rows of X_i are zero row vectors.

PROOF. The proof is easily verified by summing the left hand side of (16). For simplicity we denote

$$\Gamma = \sum_{i=1}^{n} x_iM_i \qquad (18)$$

Then each component Γ_{ij}, $j = 1,\cdots,m$, of Γ is given by

$$\Gamma_{ij} = x_1n_{ij}^1 + x_2n_{ij}^2 + \cdots + x_nn_{ij}^n = \big[x_1 \ x_2 \ \cdots \ x_n\big]\big[n_{ij}^1 \ n_{ij}^2 \cdots \ n_{ij}^n\big]^T \qquad (19)$$

Let Γ_i be a $n \times m$ matrix where (i,j)-th entry of i-th row of Γ_i is Γ_{ij} and the other rows are zero vectors. Using X_i, N_i and from (19), Γ_i can be rewritten as

$$\Gamma_i = X_i N_i = \begin{bmatrix} 0 & \cdots & 0 \\ & \vdots & \\ x_1 & \cdots & x_n \\ & \vdots & \\ 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} n_{i1}^1 & \cdots & n_{im}^1 \\ & \vdots & \\ n_{i1}^2 & \cdots & n_{im}^2 \\ & \vdots & \\ n_{i1}^n & \cdots & n_{im}^n \end{bmatrix} \quad (20)$$

Thus equation (16) holds by summing Γ_i over $[1,\ n]$. ■

Remark 6. This lemma shows that all bilinear systems can be represented by pure difference (differential) matrix equations.

The above lemma leads to the following result.

Corollary 7. The norms of the equation (16) have the following relation.

$$\|\sum_{i=1}^{n} x_i M_i\| \leq \|x\| \sum_{i=1}^{n} \|N_i\| \quad (21)$$

PROOF. The proof is directly verified from Lemma 5. Taking norms on both side of (16), we can obtain

$$\begin{aligned} \|\sum_{i=1}^{n} x_i M_i\| &= \|\sum_{i=1}^{n} X_i N_i\| \\ &\leq \sum_{i=1}^{n} \Big(\|X_i\|\|N_i\|\Big) \end{aligned} \quad (22)$$

Using the fact $\|X_i\| = \|x\|$, we can obtain the inequality of (21). ■

The generalized stabilization control law for discrete time multi-input bilinear systems described in (15) is summarized in the following theorem by using Lemma 5 and Corollary 7.

Theorem 8. Consider discrete time multi-input bilinear systems (15). The following nonlinear state feedback control law globally asymptotically stabilizes the equilibrium point of (15).

$$u(k) = \frac{-\kappa C x(k)}{\sqrt{1 + x^T(k) C^T C x(k)}} \quad (23)$$

where $C \in \Re^{m\times n}$ can be arbitrarily designed and κ satisfies the following inequality.

$$\begin{aligned} \kappa \ < \ \kappa^* = \frac{1}{\sqrt{z+1}} \min\Bigg(&\Big(\frac{\gamma}{(1+\gamma)^2\|C^T B^T PBC\|}\Big)^{\frac{1}{2}}, \\ &\Big(\frac{\gamma^2}{(1+\gamma)^3\|N_1\|^2\|P\|}\Big)^{\frac{1}{2}}, \cdots, \\ &\Big(\frac{\gamma^{n+1}}{(1+\gamma)^{n+1}\|N_n\|^2\|P\|}\Big)^{\frac{1}{2}}\Bigg) \end{aligned} \quad (24)$$

where $1 \leq z \leq n$ is the number of nonzero N_i matrices and all N_i are obtained via Lemma 5.

PROOF. The proof follows the same procedure of the proof of Theorem 2 by substituting the control law defined in (23) into (15), then we can obtain the following closed-loop systems.

$$x(k+1) = \Big(A - B\frac{\kappa C}{\sqrt{\varphi}} - \{xM\}\frac{\kappa C}{\sqrt{\varphi}}\Big)x(k) \quad (25)$$

In this proof the same Lyapunov function defined in (2) is used and it can be written by substituting (25) and (15) into (2)

$$\begin{aligned} V(x(k+1)) = x^T(k)&(A - \tau\kappa BC - \tau\kappa\{xM\}C)^T P \\ &\times(A - \tau\kappa BC - \tau\kappa\{xM\}C)x(k) \end{aligned} \quad (26)$$

where

$$\tau = \frac{1}{\sqrt{1 + x^T(k) C^T C x(k)}}$$

Using Lemma 1, the Lyapunov forward difference is given by

$$\begin{aligned} \Delta V = x^T&\Big(A - \tau\kappa BC - \tau\kappa\{xM\}C\Big)^T P \\ &\times\Big(A - \tau\kappa BC - \tau\kappa\{xM\}C\Big)x - x^T P x \\ \leq x^T&\Big((1+\gamma)A^T P A - P \\ &+\frac{(1+\gamma)\tau^2\kappa^2}{\gamma}\Big((1+\gamma)C^T B^T PBC \\ &+(1+\gamma^{-1})C^T\{xM\}^T P\{xM\}C\Big)\Big)x \end{aligned} \quad (27)$$

Using Lemma 1, 5, the term $C^T\{xM\}^T P\{xM\}C$ of (27) can be rewritten as.

$$\begin{aligned} C^T &\Big(\sum_{i=1}^{n} x_i M_i\Big)^T P\Big(\sum_{i=1}^{n} x_i M_i\Big)C \\ &= C^T\Big(\sum_{i=1}^{n} X_i N_i\Big)^T P\Big(\sum_{i=1}^{n} X_i N_i\Big)C \\ &\leq \sum_{i=1}^{n-1} \frac{(1+\gamma)^i}{\gamma^{i-1}} C^T N_i^T X_i^T P X_i N_i C \\ &\quad+\frac{(1+\gamma)^{n-1}}{\gamma^{n-1}} C^T N_n^T X_n^T P X_n N_n C \end{aligned} \quad (28)$$

Thus the inequality (27) becomes

$$\begin{aligned} \Delta V \leq x^T&\Big((1+\gamma)A^T P A - P \\ &+\beta^2\gamma\kappa^2\tau^2 C^T B^T PBC \\ &+\beta^2\kappa^2\tau^2\Big(\sum_{i=1}^{n-1}\beta^i\gamma C^T N_i^T X_i^T P X_i N_i C \\ &+\beta^{n-1} C^T N_n^T X_n^T P X_n N_n C\Big)\Big)x \end{aligned} \quad (29)$$

where

$$\beta^i = \frac{(1+\gamma)^i}{\gamma^i}$$

Since $0 < \tau \leq 1$, the inequality (29) can be rewritten by using Corollary 7

$$\Delta V \leq x^T\Big(- I_n + \beta^2\gamma\kappa^2\|C^TB^TPBC\|I_n + \beta^2\kappa^2\frac{1}{1+\|x^T\|\|C^T\|\|C\|\|x\|} \times\Big(\sum_{i=1}^{n-1}\beta^i\gamma\zeta_i + \beta^{n-1}\zeta_n\Big)I_n\Big)x \qquad (30)$$

where

$$\zeta_i = \|C^T\|\|N_i^T\|\|X_i^T\|\|P\|\|X_i\|\|N_i\|\|C\|$$

Since the following relation is always satisfied for any x and C

$$\frac{\|C^T\|\|x^T(k)\|\|x(k)\|\|C\|}{1+\|x^T(k)\|\|C^T\|\|C\|\|x(k)\|} \leq 1 \qquad (31)$$

(30) can be rewritten (30) as follows:

$$\begin{aligned}\Delta V &\leq x^T\Big(- I_n + \beta^2\gamma\kappa^2\|C^TB^TPBC\|I_n \\ &\quad +\beta^2\kappa^2\Big(\sum_{i=1}^{n-1}\beta^i\gamma\|N_i^T\|\|P\|\|N_i\| \\ &\quad +\beta^{n-1}\|N_n^T\|\|P\|\|N_n\|\Big)I_n\Big)x \\ &\leq x^T\Big(- I_n + (n+1)\times\max\Big(\\ &\quad \frac{(1+\gamma)^2}{\gamma}\|C^TB^TPBC\|, \\ &\quad \frac{(1+\gamma)^3}{\gamma^2}\|N_1\|^2\|P\|, \\ &\quad \frac{(1+\gamma)^4}{\gamma^3}\|N_2\|^2\|P\|,\cdots, \\ &\quad \frac{(1+\gamma)^{n+1}}{\gamma^{n+1}}\|N_n\|^2\|P\|\Big)I_n\kappa^2\Big)x \end{aligned} \qquad (32)$$

Thus the right hand side of (32) is negative if and only if

$$\begin{aligned}&-1+(n+1)\times\max\Big(\frac{(1+\gamma)^2}{\gamma}\|C^TB^TPBC\|, \\ &\frac{(1+\gamma)^3}{\gamma^2}\|N_1\|^2\|P\|, \frac{(1+\gamma)^4}{\gamma^3}\|N_2\|^2\|P\|, \\ &\cdots, \frac{(1+\gamma)^{n+1}}{\gamma^{n+1}}\|N_n\|^2\|P\|\Big)\kappa^2 < 1\end{aligned} \qquad (33)$$

Since N_i matrix obtained via Lemma 5 may be a zero matrix, any zero matrices N_i cannot be considered as a candidate for minimum function of (24) to prevent a denominator from being zero. Therefore inequality (24) is satisfied. ■

4. A NUMERICAL EXAMPLE

In this section, the proposed control method is applied to discrete time multi-input bilinear systems described by

$$x(k+1) = \begin{bmatrix}0 & 1\\ -0.5 & -1\end{bmatrix}x(k) + \begin{bmatrix}-1 & 1\\ -2 & 0\end{bmatrix}x_1(k)u(k) + \begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}x_2(k)u(k) + \begin{bmatrix}-0.5 & 1\\ 2 & 1.5\end{bmatrix}u(k)$$

with the initial condition $x^T(0) = \begin{bmatrix}1.0 & 1.5\end{bmatrix}$. C of (23) is chosen by $\begin{bmatrix}0 & 1|2 & 0\end{bmatrix}$. Since C can be arbitrarily designed, κ^* is obtained as 0.0149 according to (24) and (3) and γ satisfying $\sqrt{(1+\gamma)}r(A) < 1$. The simulation results are presented in Figure 1 and 2 which show the trajectories of the state and inputs, respectively. These figures show that state asymptotically converges to the equilibrium point.

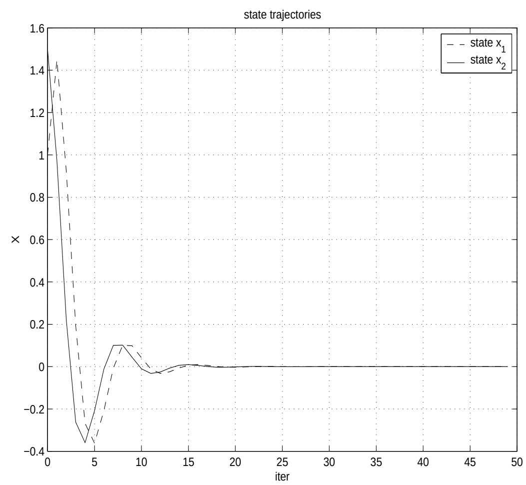

Fig. 1. State trajectories

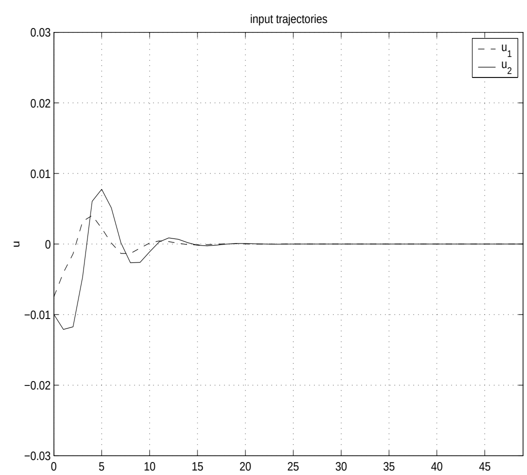

Fig. 2. Input trajectories

5. CONCLUSION

In this paper, two stabilizing control schemes for discrete time bilinear systems by using the Lyapunov method are presented. For the single input systems, the stabilizing control law is presented in Theorem 2 and a generalized stabilizing control method for the multi-input bilinear systems is described by Theorem 8. Using the lemma in Section 3, all bilinear systems can be represented by pure difference(differential) matrix equations. The simulation results show that the proposed control methods globally stabilize the discrete time multi-input bilinear systems.

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