

## INDUCTION MOTOR MODEL IDENTIFICATION VIA FREQUENCY-DOMAIN FRISCH SCHEME

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**Abstract:** In this paper the frequency domain version of the Frisch identification scheme is applied to identify parameters of the continuous-time model of an induction motor. A formulation of the identification problem in the errors-in-variables framework is given, in particular this formulation allows handling of periodic signals affected by noises with stochastic properties. A new approach, based on Bilinear Matrix Inequalities, is introduced to estimate noise variances of measured signals in the Frisch scheme. Simulations and experimental results are reported to show the properties of the proposed approach. *Copyright © 2002 IFAC*

**Keywords:** Induction motors, frequency-domain, noise characterization, parameters estimation, errors-in-variables, Frisch scheme, Bilinear Matrix Inequalities.

### 1. INTRODUCTION

In electric drives based on Induction Motors (IM) a key point to obtain a high-performance motion control is a good knowledge of the physical parameters of the electric machine. In fact, control of electric machines is quite involved since the IM model is multivariable, non-linear and strongly coupled. Typical control solutions in industrial drives are based on Direct and Indirect Field Orientation (DFO, IFO), often indicated as “vector control” (Leonhard, 1995; Bose, 1997). In nonlinear and adaptive control literature many efforts have been devoted to develop other control algorithms for IM. Although different approaches have been used (see (Ortega *et al.*, 1996; Nicklasson *et al.*, 1997; Peresada and Tonielli, 2000; Marino *et al.*, 2000) for an extensive overview), only partial and quite weak results have been obtained in terms of robustness with respect to parameter uncertainties. Hence, at the state of the art, a good knowledge of the parameters of the electric model is a key point to realize a high performance control of commercial IM drives. In addition, also for diagnosis purpose the electric parameters of a “healthy” induction motor must be identified with high accuracy.

The traditional method for IM parameter estimation based on locked-rotor and no-load tests (Leonhard, 1995) is now supplanted by automatic parameter estimation procedures implemented in modern self-commissioning industrial drives (Vas, 1998). Various identification techniques for IM have been proposed in literature (Pintelon and Shoukens, 2001; Moons and De Moor, 1995; Pappano *et al.*, 1998a; Pappano *et al.*, 1998b; Pappano *et al.*, 1998c). These methods can be divided in two main classes: the “on-line” techniques and the “off-line” techniques. Methods belonging to the first class perform the parameter tuning procedure while the motor is operating normally. Instead, the second-class ones are based on tests performed before starting normal operations and they are more spread and reliable. Typically, off-line identification procedures are performed at standstill in order to be suitable for industrial self-commissioning drives, since it is required that the parameter tuning procedure is performed “in system” (i.e. with the motor already connected with the mechanical load).

In this paper it is presented a new identification method based on the frequency domain version of the Frisch identification scheme (FIS), developed by

the authors in (Beghelli *et al.*, 1997) and theoretically analyzed in (Castaldi and Soverini, 1996). The FIS, which deals with linear errors in variables models (LEIV), is particularly suitable for the estimation of the IM model at standstill. In fact this model is linear and both the input and output measurements are disturbed with noise, hence resulting in a LEIV model. Some theoretical aspects are considered for reconfiguring the FIS scheme in the framework of the classical frequency domain identification methods using periodic excitation signal and parametric stochastic noise model. It is worth observing that proper choice of the excitation signal allow avoiding leakage problems of the Discrete Time Fourier transformation (DFT). Moreover it will be shown that the solution of the FIS problem can be obtained by solving a Bilinear Matrix Inequality (BMI) problem. This formulation allows, also in the practical case, preserving some theoretical positivity conditions concerning the estimated power spectrum of the noiseless input and output signals. These conditions are not fulfilled by the procedures already present in literature. Finally it will be shown, in the case of IM identification, that the solution of the BMI problem is quite easy, thus resulting in an efficient IM identification in the practical case.

The paper is organized as follows. In Section 2 the basic statements of the Frisch identification scheme are given, then a reformulation in terms of BMI problem is presented. In Section 3 simulation and experimental results are reported: particular attention is paid to the selection of the exciting signal and the frequency characterization of the additive noise.

## 2. FREQUENCY DOMAIN DYNAMIC FRISCH SCHEME

Let us consider two scalar periodic signals  $\hat{u}(t)$  and  $\hat{y}(t)$ ,  $t \in \mathcal{R}$ , which are the input and the steady-state output of a linear, lumped, time-invariant, continuous time, stable system described by the transfer function

$$G(s) = \frac{N(s)}{D(s)} = \frac{\sum_{k=0}^m \beta_k s^k}{\sum_{k=0}^n \alpha_k s^k} \quad m \leq n \quad (1)$$

In the case of periodic excitation signals, the DFTs  $\hat{U}_{\text{DFT}}(\omega_k)$  and  $\hat{Y}_{\text{DFT}}(\omega_k)$  of the sampled input and output signal are linked by the relation

$$\hat{Y}_{\text{DFT}}(\omega_k) = G(j\omega_k) \hat{U}_{\text{DFT}}(\omega_k) \quad (2)$$

where  $\omega_k = k\omega_b$ ,  $k = 1 \dots M$  are the angular frequencies of the sinusoids constituting the input and output signals, and  $\omega_b = h \frac{\omega_s}{N}$ ,  $h \in \mathcal{N}$ , with  $\omega_s$  equal to the sampling frequency and  $N > 2Mh$  is the number of samples. It is worth observing that aliasing and leakage problems have been avoided by synchronizing the frequencies  $\omega_b$  and  $\omega_s$  as above reported.

In an errors-in-variables environment the measurements are assumed disturbed by noises fulfilling the following hypothesis.

*Assumption 1.* The measurements  $U_{\text{DFT}}(\omega_k)$  and  $Y_{\text{DFT}}(\omega_k)$  are related to the exact values  $\hat{U}_{\text{DFT}}(\omega_k)$  and  $\hat{Y}_{\text{DFT}}(\omega_k)$  by

$$\begin{aligned} U_{\text{DFT}}(\omega_k) &= \hat{U}_{\text{DFT}}(\omega_k) + \tilde{U}_{\text{DFT}}(\omega_k) \\ Y_{\text{DFT}}(\omega_k) &= \hat{Y}_{\text{DFT}}(\omega_k) + \tilde{Y}_{\text{DFT}}(\omega_k) \end{aligned} \quad (3)$$

By defining

$$\hat{Z}_{\text{DFT}}(\omega_k) = \begin{bmatrix} \hat{U}_{\text{DFT}}(\omega_k) \\ \hat{Y}_{\text{DFT}}(\omega_k) \end{bmatrix} \quad (4)$$

$$\tilde{Z}_{\text{DFT}}(\omega_k) = \begin{bmatrix} \tilde{U}_{\text{DFT}}(\omega_k) \\ \tilde{Y}_{\text{DFT}}(\omega_k) \end{bmatrix} \quad (5)$$

the Frisch scheme framework is defined by *Assumption 2*.

*Assumption 2.* The noise  $\tilde{Z}_{\text{DFT}}(\omega_k)$  is zero-mean, with

$$E \left[ \tilde{Z}_{\text{DFT}}(\omega_k) \tilde{Z}_{\text{DFT}}(\omega_k)^H \right] = \begin{bmatrix} \sigma_u^* & 0 \\ 0 & \sigma_y^* \end{bmatrix} \quad (6)$$

$$\begin{aligned} E \left[ \tilde{Z}_{\text{DFT}}(\omega_k) \tilde{Z}_{\text{DFT}}(\omega_j)^T \right] &= 0 \quad \forall k, j \\ E \left[ \tilde{Z}_{\text{DFT}}(\omega_k) \tilde{Z}_{\text{DFT}}(\omega_j)^H \right] &= 0 \quad \forall k \neq j \end{aligned} \quad (7)$$

with the superscript H indicating the Hermitian transpose.

The previous model of the frequency characteristics of the noise, jointly with *Assumption 1*, corresponds to time-domain sequences  $\tilde{u}(\cdot)$  and  $\tilde{y}(\cdot)$  which are zero-mean white noises, mutually uncorrelated and uncorrelated to the noiseless signals  $\hat{u}(k)$  and  $\hat{y}(k)$ .

The values  $\sigma_u^*$  and  $\sigma_y^*$  are (up to a known scalar factor depending on the number of samples) the variances of the actual noise sequences  $\tilde{u}(\cdot)$  and  $\tilde{y}(\cdot)$ . In the following it will be assumed that the variances  $\sigma_u^*$ ,  $\sigma_y^*$  are unknown. A situation of this type is referred (Beghelli *et al.*, 1990) as *dynamic Frisch scheme*.

*Remark 1.* In (5) no correlation is assumed between input and output noises. In literature, different methods have been presented to deal with more general cases, where correlated noises are present (Schoukens *et al.*, 1997; Beghelli *et al.*, 1990). By the way, this extension does not seem necessary for IM parameter estimation as enlightened by the experimental results reported in next sections.

From the previous definitions and assumptions, it follows that (note that some notations are introduced too):

$$\begin{aligned} \Phi(\omega_k) &= E \left[ Z_{\text{DFT}}(\omega_k) Z_{\text{DFT}}(\omega_k)^H \right] = \\ &= E \left[ \hat{Z}_{\text{DFT}}(\omega_k) \hat{Z}_{\text{DFT}}(\omega_k)^H \right] + \\ &= E \left[ \tilde{Z}_{\text{DFT}}(\omega_k) \tilde{Z}_{\text{DFT}}(\omega_k)^H \right] = \\ &= \left[ \hat{Z}_{\text{DFT}}(\omega_k) \hat{Z}_{\text{DFT}}(\omega_k)^H \right] + \begin{bmatrix} \sigma_u^* & 0 \\ 0 & \sigma_y^* \end{bmatrix} = (8) \\ &= \begin{bmatrix} \Phi_{\hat{u}\hat{u}}(\omega_k) & \Phi_{\hat{u}\hat{y}}(\omega_k) \\ \Phi_{\hat{y}\hat{u}}(\omega_k) & \Phi_{\hat{y}\hat{y}}(\omega_k) \end{bmatrix} + \begin{bmatrix} \sigma_u^* & 0 \\ 0 & \sigma_y^* \end{bmatrix} = \\ &= \hat{\Phi}(\omega_k) + \check{\Phi}(\omega_k) \end{aligned}$$

Note that, because of *Assumption 2*,  $\Phi_{\hat{u}\hat{y}}(\omega_k) = \Phi_{uy}(\omega_k)$ , i.e. the cross-spectra between the noisy and the noiseless input-output signals are identical.

Matrix  $\Phi(\omega_k)$  is positive definite, while the linear link (2) between the input-output signals leads to a singular nonnegative definite spectral density matrix  $\hat{\Phi}(\omega_k)$  whose entries satisfy the following relation

$$\Phi_{\hat{u}\hat{u}}(\omega_k)\Phi_{\hat{y}\hat{y}}(\omega_k) = \Phi_{uy}(\omega_k)\Phi_{yu}(\omega_k) \quad (9)$$

and

$$G(j\omega_k) = \frac{\Phi_{\hat{y}\hat{y}}(\omega_k)}{\Phi_{\hat{u}\hat{u}}(\omega_k)} = \frac{\Phi_{yu}(\omega_k)}{\Phi_{uy}(\omega_k)}. \quad (10)$$

Under the previous assumptions, the identification problem can thus be stated as follows.

*Problem 1.* Let the noisy measurements be generated in accordance with the dynamic Frisch scheme *Assumption 1* and *2* and let the noisy spectrum  $\Phi(\omega_k)$  be given. Determine the transfer function  $G(s)$  and the values (variances)  $\sigma_{\hat{u}}^*$  and  $\sigma_{\hat{y}}^*$ .

The problem of identifying the transfer function  $G(s)$  can be solved in the following way (Beghelli *et al.*, 1997):

- (1) for a given frequency,  $\omega_k$ , determine all the nonnegative definite matrices of structure  $\tilde{\Phi} = \text{diag}(\sigma_{\hat{u}}, \sigma_{\hat{y}})$ , i.e. all the couples  $(\sigma_{\hat{u}}, \sigma_{\hat{y}})$ , such that matrix  $\hat{\Phi}(\omega_k)$  is singular nonnegative definite

$$\begin{cases} \hat{\Phi}(\omega_k) = \Phi(\omega_k) - \tilde{\Phi} \geq 0 \\ \det(\hat{\Phi}(\omega_k)) = 0 \end{cases} \quad (11)$$

- (2) analyze how the solution set obtained at the previous step varies with  $\omega_k$ .

With reference to the first step, in (Beghelli *et al.*, 1997), it has been shown that, for a given frequency  $\omega_k$  the solution set of relation (11) describes in the first quadrant of the plane  $(\sigma_{\hat{u}}, \sigma_{\hat{y}})$  the segment of hyperbole with equation

$$(\Phi_{uu}(\omega_k) - \sigma_{\hat{u}})(\Phi_{yy}(\omega_k) - \sigma_{\hat{y}}) = |\Phi_{uy}(\omega_k)|^2 \quad (12)$$

with the constraints  $(0 \leq \sigma_{\hat{u}} \leq \sigma_{\hat{u}}^{max}(\omega_k))$ ,  $(0 \leq \sigma_{\hat{y}} \leq \sigma_{\hat{y}}^{max}(\omega_k))$  where

$$\begin{aligned} \sigma_{\hat{u}}^{max}(\omega_k) &= \Phi_{uu}(\omega_k) - \frac{\Phi_{yu}(\omega_k)\Phi_{uy}(\omega_k)}{\Phi_{yy}(\omega_k)} \\ \sigma_{\hat{y}}^{max}(\omega_k) &= \Phi_{yy}(\omega_k) - \frac{\Phi_{uy}(\omega_k)\Phi_{yu}(\omega_k)}{\Phi_{uu}(\omega_k)} \end{aligned} \quad (13)$$

Every point  $(\sigma_{\hat{u}}, \sigma_{\hat{y}})$  on the curve is associated to the complex value  $G_{(\sigma_{\hat{u}}, \sigma_{\hat{y}})}(j\omega_k)$ .

When the spectrum  $\Phi(\omega_k)$  has been generated by an errors-in-variables model with noises fulfilling *Assumption 2*, the curves of type (12) for all values of  $\omega_k$  necessarily have at least one common point  $(\sigma_{\hat{u}}^*, \sigma_{\hat{y}}^*)$  whose coordinates are the true values (variances) of the noises. In correspondence of this point, by means of (10) it is possible to obtain the true transfer function  $G(j\omega_k)$ .

The search of a solution for the identification problem may, thus, start from determination of this point on the plane  $(\sigma_{\hat{u}}, \sigma_{\hat{y}})$ . Once the noise variances have been estimated, the values of the transfer function  $G(j\omega_k)$  at different frequencies can be computed by means of relation (10).

### 2.1 Determination of the common point: a BMI problem

Now by defining

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \sigma_{\hat{u}} \\ \sigma_{\hat{y}} \\ \sigma_{\hat{u}}\sigma_{\hat{y}} \end{bmatrix} \quad (14)$$

the determination of the common point solution can be obtained solving the following Bilinear Matrix Inequality (BMI) optimization problem (Vandenberghe and Venkataramanan, 1997)

$$\begin{cases} \Phi(\omega_k) - \begin{bmatrix} x_1 & 0 \\ 0 & x_2 \end{bmatrix} \geq 0 \\ x_1 x_2 \geq x_3 - \epsilon \\ -x_1 x_2 \geq -x_3 - \epsilon \\ x_1 \geq 0 \\ x_2 \geq 0 \\ \epsilon \geq 0 \\ \min[\ell(x_1, x_2, x_3, \epsilon)] \\ \ell(x_1, x_2, x_3, \epsilon) = \sum_k [x_3 - \Phi_{uu}(\omega_k)x_2 \\ - \Phi_{yy}(\omega_k)x_1 + \Phi_{uu}(\omega_k)\Phi_{yy}(\omega_k) \\ - |\Phi_{uy}(\omega_k)|^2] + \epsilon \end{cases} \quad (15)$$

Note that  $\ell(x_1, x_2, x_3, \epsilon)$  is the linear function to optimize under BMI constraints. This is a non-convex problem and few algorithms for solving the general problem are already present in literature (branch and bound, global optimization algorithms). On the other hand, this case is quite easy to solve, thanks to the presence of the nonlinear term  $x_1 x_2$  only. For this reason, as it will be shown in the following, an easy optimization procedure can be used in order to efficiently find the solution of the BMI problem.

## 3. SIMULATION AND EXPERIMENTAL RESULTS

The identification scheme reported in previous section has been applied to the IM model, performing an off-line procedure with the motor at standstill.

Under the hypothesis of linear magnetic circuits and balanced operating condition, the equivalent two-phase model of a squirrel-cage Induction Motor, represented in the  $(a-b)$  stator reference frame, is (Leonhard, 1995; Bose, 1997; Peresada and Tonielli, 2000):

$$\frac{d\psi_a}{dt} = -\alpha\psi_a - \omega\psi_b + \alpha L_m i_a$$

$$\begin{aligned}
\frac{d\psi_b}{dt} &= -\alpha\psi_b + \omega\psi_a + \alpha L_m i_b \\
\frac{di_a}{dt} &= -\gamma i_a + \alpha\beta\psi_a + \beta\omega\psi_b + \frac{1}{\sigma}V_a \\
\frac{di_b}{dt} &= -\gamma i_b + \alpha\beta\psi_b - \beta\omega\psi_a + \frac{1}{\sigma}V_b \\
\frac{d\omega}{dt} &= \frac{1}{J} \left( (\psi_a i_b - \psi_b i_a) - T_L \right) \quad (16)
\end{aligned}$$

where  $(V_a, V_b)$ ,  $(i_a, i_b)$ ,  $(\psi_a, \psi_b)$ ,  $\omega$  are stator voltages, stator currents, rotor fluxes and rotor speed and  $T_L$  is the load torque.

Positive constants in model (16), related to IM electrical parameters, are defined as:  $\sigma = L_s \left(1 - \frac{L_m^2}{L_s L_r}\right)$ ,  $\beta = \frac{L_m}{\sigma L_r}$ ,  $\alpha = \frac{R_r}{L_r}$ ,  $\gamma = \left(\frac{R_s}{\sigma} + \alpha L_m \beta\right)$ ,  $= \frac{3L_m}{2L_r}$ , where  $R_s, R_r, L_s, L_r$  are stator/rotor resistances and inductances respectively, while  $L_m$  is the mutual inductance between stator and rotor windings.

From (16), with zero initial states and null load torque, whereas only one phase of the IM is excited, it follows that no torque is produced, the standstill condition is preserved and the two phases ‘‘a’’ and ‘‘b’’ are fully decoupled. Therefore, if  $V_b(t) = 0$ , only the two equations related to ‘‘a’’ variables can be considered, since all the variables of the *b*-phase are assumed equal to zero. Hence, the IM at standstill is modelled as a 2-nd order LTI system and it is represented by the following transfer function between the stator voltage  $V_a(t)$  and the stator current  $i_a(t)$ :

$$G(s) = \frac{i_a(s)}{V_a(s)} = \frac{\frac{1}{\sigma}(s + \alpha)}{s^2 + s(\gamma + \alpha) + \alpha(\gamma - \alpha L_m \beta)} \quad (17)$$

The identification procedure for the IM electrical parameters is based on the measures of stator currents  $y(kT_s)$  and stator voltages  $u(kT_s)$ , with  $k = 0 \dots N - 1$ , where  $N$  is the number of samples and  $T_s$  is the sample time.

The frequency domain identification of the IM relies on the discrete Fourier transform of the input and output signals  $U_{DFT}(\omega_k), Y_{DFT}(\omega_k)$ , obtained from the sampled I/O variables. It is well known that a proper choice of the excitation signal is necessary to avoid spectral leakage and alias errors in the computation of the discrete Fourier transform. In particular, the sample frequency should be sufficiently high to avoid aliasing. Moreover, a periodic signal should be considered and an integer number of signal periods should be used as temporal window: in this way, the DFT coincides with the Fourier transform of the signal in the sampled frequencies (Kay, 1988; Stoica and Moses, 1997).

To obtain a correct estimation of the IM model, it is necessary to excite the motor in a broad frequency range. Two different voltage signals have been separately imposed to the IM. With the first one, the IM is excited in the low-frequency range, applying a multisine in the band  $[0.122\text{Hz}, 5.737\text{Hz}]$  with step  $f_b =$

0.244Hz. With the second one, the IM is excited in the high-frequency range, applying a multisine in the band  $[5.859\text{Hz}, 95.703\text{Hz}]$  with step  $f_b = 3.906\text{Hz}$ . The estimated transfer function  $\hat{G}(j\omega_k)$  has been obtained combining the data deriving from the two different excitation signals.

The initial phases of the sine functions have been imposed as  $\phi_0^{(i)} = -\frac{i(i-1)\pi}{M}$  to optimize the crest factor (Pintelon and Shoukens, 2001). Moreover, the current and voltage signals have been sampled only after the initial transient for the identification procedure, to consider only the steady-state behavior of the IM. The amplitude of the multisine signal has been set in order to avoid the excitation of the IM in the magnetic saturation region, ensuring the validity of the linear model (17).

In the error-in-variables framework presented in the previous paragraphs, the measured input-output signals (and hence the DFTs) are the sum of a deterministic term (the ‘‘actual’’ signals) and a stochastic one (the noise). According to assumptions reported in the problem statement, the noises are supposed uncorrelated and white. Unfortunately, as it is well known (Pintelon and Shoukens, 2001), the spectral matrix of *independent random finite sequences*, calculated by means of DFT, is variable with frequency and dependent on the particular realization. An additive diagonal noise power spectrum  $diag(\sigma_u, \sigma_y)$ , independent by the frequency as required in problem statement, can be obtained if the mean value over an infinite amount of realizations is considered. For this purpose, the adopted solution consists in performing the average of DFTs relative to different realizations obtained by considering a certain number of shifted time windows. In this way, a reduction of the noise variance in the frequency domain is achieved. Note that no distortion is introduced on the noiseless signal, since the time windows are multiple of the adopted multisine period.

The BMI optimization problem for the noise identification with the Frisch scheme has been solved utilizing the function for constrained nonlinear optimization problem *fmincon* of the Matlab Optimization Toolbox. The results reported in the sequel have been achieved executing twice the noise-level identification algorithm, since the first result has been used to remove some outliers in the frequency range. In fact, the noise spectrum of experimental data, even after averaging, is not constant. This fact, combined with ‘‘positive definition constraints’’ of the BMI problem leads to an underestimation of the noise level. This phenomenon has been partially compensated removing the frequencies (8% of the total number) where the determinant of spectral matrix  $\hat{\Phi}(\omega_k)$  after first ‘‘cleaning process’’ is closer to zero (outliers).

Once estimated the additive noise, the estimated covariance matrix has been determined and the ‘‘physical’’ parameters of the IM transfer function can be identified considering the nonlinear weighted least

Table 1. IM parameters

$R_s$	6.6 $\Omega$	$\sigma$	0.041
$R_r$	5.5 $\Omega$	$\beta$	23.3
$L_s$	475mH	$\alpha$	11.6
$L_r$	475mH	$\gamma$	283
$L_m$	454mH		

square problem with cost function

$$\sum_k \|\hat{G}(j\omega_k) - \bar{G}(j\omega_k)\|^T W \|\hat{G}(j\omega_k) - \bar{G}(j\omega_k)\|$$

where  $W$  is a weight matrix for the magnitude errors at different frequencies,  $\hat{G}(j\omega)$  is the identified transfer function, given by the ratio (1) and estimated by means of relation (10), and  $\bar{G}(j\omega)$  depends on the estimated physical parameters as (17). During tests, matrix  $W$  has been chosen equal to the identity matrix.

The motor used in simulation and experimental tests is a 1.1kW 50Hz IM. Parameters identified using traditional methods based on no-load and locked-rotor tests are reported in Table 1. With these parameters the resulting poles of  $G(s)$  in (17) are -288 and -6.46, the zero is -11.6 and the static gain is 0.151.

During experimental tests, stator currents have been measured using closed-loop Hall sensors, while stator voltages have been measured with a Y-connected resistive divider. The stator voltages have been imposed by a standard three-phases inverter with a 10KHz symmetrical-PWM control. Simple techniques based on phase current sign (Jeong and Park, 1991) have been used to compensate for the effects of the dead-time, set to 1.5  $\mu$ s. The proposed estimation scheme have been implemented on a control board equipped with a floating-point DSP, TMS320C32. The adopted sampling time has been set to 200  $\mu$ s for the voltage signal generation, while for the identification procedures the input/output signals have been sampled with  $T_s = 64ms$  in the low-frequency test and  $T_s = 4ms$  in the high-frequency test. For both tests, the number of samples used for the DFTs was  $N=128$ , while 1024 samples have been acquired to perform the DFT averages, as described before. Hence, the frequency resolution is equal to 0.767rad/s and 12.27rad/s respectively. The off-line identification algorithm has been implemented in Matlab. The computational burden of the BMI optimization problem is affordable with a standard PC: with a Pentium III 350MHz the noise estimation is performed in about 2 sec.

At first, the performances of the proposed method have been analyzed by means of a simulated experiment. The detailed results for the two feeding multisines are summed up in Table 2, where the imposed and estimated standard deviations of the superimposed random input and output measurement noise are also reported, for the two different experiments at low and high frequency. Note that in ideal conditions (flat spectra of the noise) the estimation algorithm works perfectly, while the estimation errors are only due to the non-flatness of the noise spectra related to finite data

Table 2. Simulation results

	Low-freq.	High-freq.
$V_{a_{RMS}}$	6.92V	6.92V
$\sigma_{\hat{u}}$	1.22V	1.09V
$\sigma_{\hat{u}}$	0.85V	0.76V
$\sigma_{\hat{y}}$	0.30A	0.24A
$\sigma_{\hat{y}}$	0.21A	0.17A

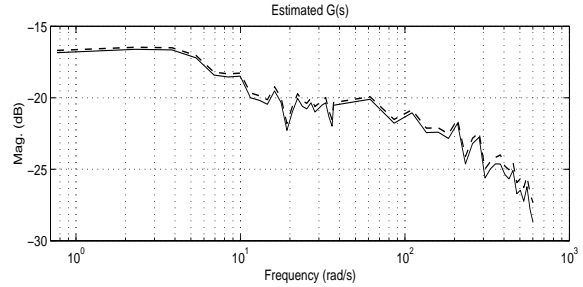


Fig. 1. Bode plots of  $\hat{G}(s)$  prior (*dashed*) and after (*solid*) the application of Frisch scheme (simulation data).

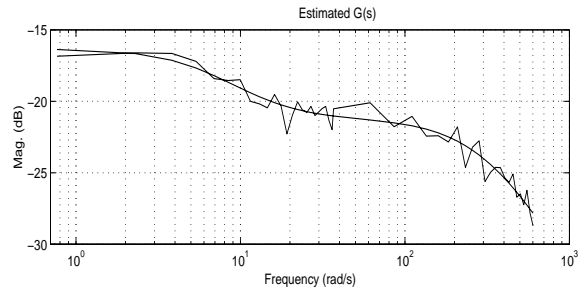


Fig. 2. Bode plots of  $\hat{G}(s)$  and  $\bar{G}(s)$  (simulation data).

Table 3. Estimated IM parameters and standard deviations (simulation data).

	Real value	with Frisch sch.	without Frisch sch.
$\gamma$	283	292(23)	315(22)
$\alpha$	11.57	11.62(2.49)	11.68(2.51)
$L_m\beta$	10.56	11.18(2.10)	11.75 (2.22)
$\sigma$	0.0411	0.0394 (0.0020)	0.0361 (0.0016)

sequences. Anyway, satisfactory estimation of noise variance is obtained and positive definiteness of the covariance matrix is assured. Fig. 1 shows the Bode plot of the transfer function  $G(s)$  prior and after the application of the Frisch scheme. In fig. 2 the sampled and optimum transfer function are shown.

A set of 10 simulations has been performed in order to evaluate the performance of the identification procedure for IM parameters using the Frisch scheme. Table 3 reports the estimated IM parameters obtained using the Frisch scheme. Also the IM parameters estimated without the implementation of the Frisch scheme are reported as comparison. Note that in this case a biased estimation is obtained, due to uncorrect estimation of the IM transfer function.

In the second test, experimental results are presented. The procedure for noise estimation and parameter identification has been repeated 10 times and the mean values of the estimated parameters are presented. The results relative to one of the experimental tests are re-

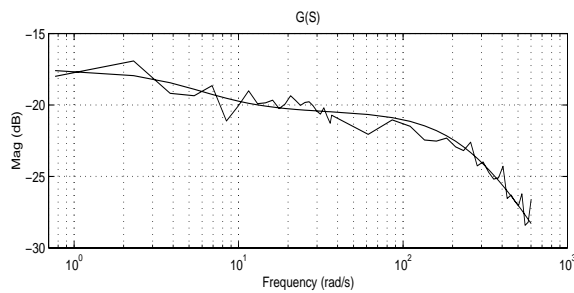


Fig. 3. Bode plots of  $\hat{G}(s)$  and  $\bar{G}(s)$  (experimental data).

Table 4. Estimated IM parameters (experimental data).

	$\gamma$	$\alpha$	$L_m\beta$	$\sigma$
value	271	8.2	10.0	0.039
std	20	1.8	3.2	0.005

ported in fig. 3, where the Bode plots of the estimated  $\hat{G}$  and optimum  $\bar{G}$  (after parameter identification) are shown. The estimated noise standard deviations are 0.71V (0.22A) for the input (output) signal in the low frequency test and 0.57V (0.117 A) in the high frequency test. Finally, table 4 sums up the results of all the experimental tests, reporting the estimated values of the physical IM parameters and the standard deviations.

#### 4. CONCLUSIONS

A new identification procedure based on the frequential Frisch scheme has been proposed. The FIS has been reconfigured to deal with periodic signals jointly with parametric stochastic noise models. It has been shown that the solution of the FIS problem can be obtained by means of an optimization problem with BMI constraints. The effectiveness of the method for identifying IM model by means of an off-line procedure has been illustrated by experimental results.

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