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FAULT-TOLERANT CONTROL OF A CLASS OF NONLINEAR SYSTEMS

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Abstract: This paper presents a fault-tolerant control scheme for a class of nonlinear systems. A robust nominal controller is designed to ensure system stability and tracking performance before a fault occurs. A monitoring module is used for on-line fault detection and isolation. The fault detection scheme is designed based on some stability criterion of the controlled plant, hence guaranteeing boundedness of system states before fault detection in the presence of a fault. Then using the fault information provided by the monitoring module, the controller is reconfigured after fault detection and isolation, respectively, to compensate the effects of the fault.

1. INTRODUCTION

Fault tolerance can be achieved either passively by the use of feedback control laws that are robust to possible systems faults, or actively through a fault diagnosis (fault detection and isolation (FDI)) and accommodation architecture. However, links between fault diagnosis and fault-tolerant control techniques are still lacking (Patton, 1997). The complexity arises, for instance, from the significant consequences that the fault detection time has on the system stability (Mariton, 1989).

In (Zhang *et al.*, 2001), the authors proposed a unified framework for fault detection, isolation and accommodation. The stability of the closed-loop system is rigorously analyzed for each mode of the controlled plant based on two assumptions: (1) the modeling uncertainty is uniformly bounded; (2) if a fault occurs, its rate of growth to infinity satisfies certain assumption such that the fault is timely detected before some state variable possibly grows unbounded. In this paper, we present a new design procedure that extends the results of (Zhang *et al.*, 2001) by removing these two assumptions.

The presented fault-tolerant control architecture consists of two main modules: an on-line health monitoring (FDI) module and a controller (fault accommodation) module. Before the occurrence of any faults, a nominal controller guarantees the system stability and tracking performance. The fault detection scheme is designed based on a stability criterion of the controlled plant, such that, if a fault occurs the system stability before fault detection is ensured. Once a fault is detected, the nominal controller is reconfigured to maintain some basic stability properties in the presence of the fault. Meanwhile, a bank of fault isolation estimators (FIEs) are activated for the purpose of fault isolation. If the fault is isolated, the controller is reconfigured again to improve the control performance. Due to space limitations, some of the details of the analysis are omitted.

2. PROBLEM FORMULATION

Consider a class of single-input, single-output nonlinear dynamical systems described by:

$$\begin{aligned} \dot{x}_i &= x_{i+1} + \phi_i(x_1,\ldots,x_i) + \eta_i(x,u,t) \\ &\quad + \beta_i(t-T_0) f_i(x_1,\ldots,x_i)\,, \\ &\quad \text{for} \quad 1 \le i \le n-1, \\ \dot{x}_n &= \phi_0(x)u + \phi_n(x) + \eta_n(x,u,t) \\ &\quad + \beta_n(t-T_0) f_n(x)\,, \\ y &= x_1\,, \end{aligned} \tag{1}$$

where $x \triangleq \mathrm{col}\,(x_1,\cdots,x_n)$ is the state vector, $u \in \Re$ is the control input, $y \in \Re$ is the output, ϕ_0

is a nonzero smooth function, and ϕ_i, η_i, β_i and f_i, for $1 \leq i \leq n$, are generic smooth functions. The system

$$\begin{aligned} \dot{x}_i &= x_{i+1} + \phi_i(\bar{x}_i)\,, \quad 1 \leq i \leq n-1 \\ \dot{x}_n &= \phi_0(x)u + \phi_n(x) \\ y &= x_1\,, \end{aligned}$$

where $\bar{x}_i \triangleq \text{col}(x_1, \cdots, x_i)$, represents the known nominal model dynamics. The functions η_i, β_i and f_i represent the modeling uncertainty, the fault time profile and unknown fault function, respectively. The control objective is to force the output $y(t)$ to track a given reference signal $y_r(t)$. It is assumed that $y_r(t)$ and its first n derivatives are known, piecewise continuous, and bounded.

The modeling uncertainty, represented by η_i in (1), for $i = 1, \cdots, n$, may include external disturbances as well as modeling errors. Throughout the paper the following assumption will be used:

Assumption 1. *The modeling uncertainty η_i is an unknown nonlinear function of x, u, and t, but bounded by some function $\bar{\eta}_i(x,u,t)$, i.e.,*

$$|\eta_i(x,u,t)| \leq \bar{\eta}_i(x,u,t)\,, \qquad (2)$$

$\forall x \in \Re^n$, $\forall u \in \Re^m$, $\forall t \in \Re^+$, *where the bounding function $\bar{\eta}_i(x,u,t) \geq 0$ is known and continuous.*

As to the fault affecting the nominal system modes, from a qualitative viewpoint, the term $\beta_i(t - T_0) f_i(x_1, \cdots, x_i)$, $i = 1, \ldots, n$, represents the deviation in the system dynamics due to the occurrence of a fault. More specifically, the function $\beta_i : \Re \mapsto \Re$ represents the fault time profile in the i-th state equation, with T_0 being the unknown fault occurrence time, and f_i denotes the nonlinear fault function affecting the i-th state equation. In this work, we consider faults with time profiles modeled by:

$$\beta_i(t - T_0) = \begin{cases} 0 & \text{if} \quad t < T_0 \\ 1 - e^{-\alpha_i (t - T_0)} & \text{if} \quad t \geq T_0\,, \end{cases} \qquad (3)$$

where the scalar $\alpha_i > 0$ denotes the unknown fault evolution rate. Small values of α_i characterize slowly developing faults, also known as *incipient faults*. For large values of α_i, the time profile β_i approaches a step function, which models *abrupt faults*.

The fault–tolerant controller proposed in the paper will make explicit use of any information available about the occurred fault $f \triangleq [f_1,\ f_2,\ \cdots,\ f_n]^\top$, i.e., the detection and isolation of the fault provided by the monitoring module. We assume that there are N types of possible faults in the fault class; specifically, $f(x)$ belongs to a finite set of functions given by

$$\mathcal{F} \triangleq \left\{ f^1(x),\ \ \ldots,\ \ f^N(x) \right\}\,. \qquad (4)$$

Each fault function $f^s(x)$, $s = 1, \cdots, N$, is given by

$$f^s \triangleq \left[(\theta_1^s)^\top g_1^s(x_1), (\theta_2^s)^\top g_2^s(x_1, x_2), \cdots, (\theta_n^s)^\top g_n^s(x) \right]^\top$$

where θ_i^s, $i = 1, \cdots, n$, is an unknown parameter vector assumed to belong to a known compact set Θ_i^s (i.e., $\theta_i^s \in \Theta_i^s \subset \Re^{q_i^s}$) and $g_i^s : \Re \mapsto \Re^{q_i^s}$ is a known smooth vector field. This representation characterizes a class of nonlinear faults where the nonlinear vector field g_i^s represents the functional structure of the s-th fault affecting the i-th state equation, while the unknown parameter vector θ_i^s characterizes the "magnitude" of the fault in the i-th state equation. The dimension q_i^s of each parameter vector θ_i^s is determined by both the type of fault and the specific state component considered.

3. FAULT-TOLERANT CONTROL ARCHITECTURE

Now, we present the architecture of the fault-tolerant control scheme. A block diagram representation of the overall design in shown in Fig.1.

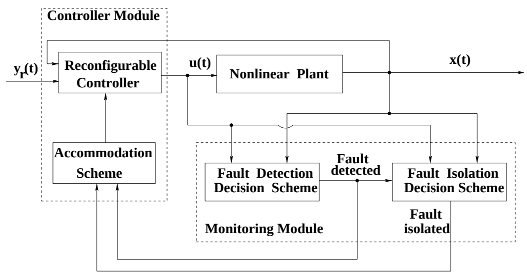

Fig. 1. Architecture of the fault-tolerant control scheme.

The monitoring (fault diagnosis) module shown in the above figure consists of a fault detection and a fault isolation scheme. Under normal operating conditions, the fault detection scheme is the only one operating by monitoring the system for the occurrence of any faults. If a fault occurs, then once it is detected, the fault isolation scheme is activated to determine the particular type of fault that has occurred. Conditions under which a fault can be detected and isolated have been derived analytically in previous work by the authors (Zhang *et al.*, 2000). The fault isolation scheme consists of a bank of N nonlinear adaptive estimators, with each of them corresponding to one of the possible types of nonlinear faults in the fault class $\mathcal{F}$ described by (4). More details of the monitoring module are given later on in the paper.

Let us define three important time–instants: T_0 is the fault occurrence time; $T_d > T_0$ is the fault detection time; and $T_{\text{isol}} > T_d$ is the fault isolation time. Then, the structure of the fault-tolerant controller takes on the following general form:

$$u(t) = \begin{cases} u_0(x, y_d, t), & \text{for } t < T_d \\ u_D(x, y_d, t), & \text{for } T_d \le t < T_{\text{isol}} \\ u_I^s(x, y_d, t), & \text{for } t \ge T_{\text{isol}} \end{cases} \quad (5)$$

where $y_d \in \Re^n$ denotes a reference vector to be tracked by the controlled system state vector. The control laws u_0, u_D, u_I^s are nonlinear dynamic functions to be designed according to the following objectives:
(1) Under normal operating conditions (i.e., for $0 \le t < T_0$), a nominal controller u_0 is designed to guarantee the system stability and robust tracking performance in the presence of the modeling uncertainty η.
(2) When a fault occurs at time T_0, the nominal controller u_0 should guarantee the boundedness of all the system signals until the fault is detected, i.e., for $T_0 \le t < T_d$.
(3) After fault detection (i.e., for $T_d \le t < T_{\text{isol}}$) the nominal controller is *reconfigured* to compensate the effect of the (yet unknown) fault; that is, the controller u_D is designed in such a way to exploit the information that a fault occurred to recover some basic control performance.
(4) After fault isolation (i.e., for $t \ge T_{\text{isol}}$) the controller is reconfigured again. The function u_I^s, where $s = 1, \cdots, N$, is designed using the information of the fault type that has been isolated so as to improve the control performances.

In the following sections, the general architecture shown in Fig. 1 will be specified by detailing the *monitoring module* and the reconfigurable controller.

4. NOMINAL CONTROLLER AND FAULT DETECTION SCHEME

In this section, we investigate the design of the nominal controller, the fault detection scheme, and the system stability issue before a fault is detected if a fault occurs (i.e., for $t \in [0, T_d)$). For the sake of compactness of notation, we let $\bar{y}_r^{(i)} \triangleq \text{col}\,(y_r, y_r^{(1)}, \ldots, y_r^{(i)})$.

4.1 *Nominal controller design and system stability before fault occurrence.*

Let us first consider the system behavior in the absence of any faults (i.e., for $t \in [0, T_0)$). The following design procedure is based on the backstepping methodology (Krstic *et al.*, 1995) with a bounding control scheme to account for the modeling uncertainty. A new state vector $z \triangleq \text{col}\,(z_1, \ldots, z_n)$ is defined recursively by the following coordinate transformation:

$$z_i = x_i - \alpha_{i-1}(\bar{x}_{i-1}, \bar{y}_r^{(i-2)}) - y_r^{(i-1)}, \quad (6)$$

where $i = 1, \cdots, n$ and

$$\begin{aligned} \alpha_0 &= 0 \\ \alpha_1 &= -c_1 z_1 - \phi_1 + \rho_1 \\ \alpha_i &= -c_i z_i - z_{i-1} - \phi_i + \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k}(x_{k+1} + \phi_k) \\ &\quad + \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial y_r^{(k-1)}} y_r^{(k)} + \rho_i(\bar{x}_i, \bar{y}_r^{(i-1)}), \\ &\qquad \text{for} \quad 2 \le i \le n, \end{aligned}$$

where, for $1 \le i \le n$, c_i are suitable design constants, and ρ_i are smooth functions to be defined later on. According to the system model (1) and the coordinate transformation described by (6), before the occurrence of any faults, the time-derivative of the new state variable z_1 can be expressed as follows:

$$\begin{aligned} \dot{z}_1 &= x_2 + \phi_1 + \eta_1 - \dot{y}_r = z_2 + \alpha_1 + \phi_1 + \eta_1 \\ &= z_2 - c_1 z_1 + \eta_1 + \rho_1. \end{aligned} \quad (7)$$

Similarly, for $2 \le i \le n-1$, the time-derivative of z_i can be recursively obtained as follows:

$$\begin{aligned} \dot{z}_i &= x_{i+1} + \phi_i + \eta_i - y_r^{(i)} - \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial y_r^{(k-1)}} y_r^{(k)} \\ &\quad - \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k}(x_{k+1} + \phi_k + \eta_k) \\ &= x_{i+1} - \alpha_i + \eta_i - y_r^{(i)} - c_i z_i - z_{i-1} + \rho_i \\ &\quad - \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} \eta_k \\ &= z_{i+1} - c_i z_i - z_{i-1} + \eta_i - \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} \eta_k + \rho_i \end{aligned} \quad (8)$$

Finally, for $i = n$, we have

$$\begin{aligned} \dot{z}_n &= \phi_0 u - \alpha_n - c_n z_n - z_{n-1} + \eta_n - y_r^{(n)} \\ &\quad - \sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_k} \eta_k + \rho_n. \end{aligned}$$

By choosing the nominal control law as

$$u_0(t) = \frac{1}{\phi_0(x)} \left[\alpha_n + y_r^{(n)} \right], \quad (9)$$

we obtain

$$\dot{z}_n = -c_n z_n - z_{n-1} + \eta_n - \sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_k} \eta_k + \rho_n \quad (10)$$

Consider a Lyapunov function of the form

$$V(t) = \frac{1}{2} \sum_{i=1}^{n} z_i^2. \quad (11)$$

After some algebraic manipulation, the time–derivative of V is given by

$$\dot{V}(t) = -\sum_{i=1}^{n} c_i z_i^2 + \sum_{i=1}^{n} z_i \left(\eta_i - \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} \eta_k + \rho_i \right)$$
$$\leq -2cV(t) + \sum_{i=1}^{n} |z_i| \left(\bar{\eta}_i + \sum_{k=1}^{i-1} \left| \frac{\partial \alpha_{i-1}}{\partial x_k} \right| \bar{\eta}_k \right)$$
$$+ \sum_{i=1}^{n} z_i \rho_i$$

where $c \triangleq \min\{c_1, \cdots, c_n\}$.

Now, we consider the design of the bounding control functions ρ_i. In the sequel, the following property of the hyperbolic tangent function is used: for any $\epsilon > 0$ and for any $q \in \Re$,

$$0 \leq |q| - q \tanh\left(\frac{q}{\epsilon}\right) \leq k\epsilon, \qquad (12)$$

where k is a constant that satisfies $k = e^{-(k+1)}$; i.e., $k \simeq 0.2785$. By choosing

$$\zeta_i = \bar{\eta}_i + \sum_{k=1}^{i-1} \left| \frac{\partial \alpha_{i-1}}{\partial x_k} \right| \bar{\eta}_k ,$$
$$\rho_i = -\zeta_i \tanh\left(\frac{z_i \zeta_i}{\epsilon}\right), \qquad (13)$$

and using the property of the hyperbolic tangent function given by (12), the time–derivative of the Lyapunov function satisfies

$$\dot{V}(t) = -2cV(t) + \sum_{i=1}^{n} \left[|z_i|\zeta_i - z_i \zeta_i \tanh\left(\frac{z_i \zeta_i}{\epsilon}\right) \right]$$
$$\leq -2cV(t) + nk\epsilon . \qquad (14)$$

Now, if we let $\kappa \triangleq \frac{nk\epsilon}{2c} > 0$, then by (14) we have

$$0 \leq V(t) \leq \kappa + [V(0) - \kappa] \exp(-2ct) . \qquad (15)$$

Therefore, $z(t)$ and $x(t)$ are uniformly bounded. Furthermore, using (11) and (15), we obtain that given any $\bar{\epsilon} > \sqrt{2\kappa}$, there exists some time T, such that for all $t \geq T$ the output $y = x_1$ satisfies

$$|y(t) - y_r(t)| \leq \bar{\epsilon} .$$

Hence, before fault occurrence, the stability and tracking property of the nominal controller given by (9) is guaranteed.

4.2 *Fault detection scheme and system stability before fault detection.*

Let us now suppose that a fault occurs at some time T_0. Clearly, the occurrence of a fault may affect the system stability. As discussed in Section 3, boundedness of all signals should be maintained until the fault is detected. This property is guaranteed here by the design of a fault detection scheme using some stability criterion of the controlled system. Specifically, based on (15), the following decision scheme for fault detection is proposed:

Fault detection decision scheme: *The decision for the occurrence of a fault (detection) is made when the Lyapunov function $V(t)$ exceeds a corresponding bound $\bar{V}(t)$, i.e., $V(t) > \bar{V}(t)$, where*

$$\bar{V}(t) \triangleq \kappa + [V(0) - \kappa] \exp(-2ct) .$$

More specifically, the fault detection time T_d is defined as

$$T_d \triangleq \inf\{t > T_0 : V(t) > \bar{V}(t)\} . \qquad (16)$$

Intuitively, the nominal controller described by (9) is designed to be robust with respect to modeling uncertainties that satisfy condition (2). Based on the above fault detection scheme, in the presence of a fault, when the overall effects of the unknown fault function and modeling uncertainty grow beyond the stabilizing ability of the nominal controller (i.e., inequality (15) is violated), a fault alarm is generated. Clearly, the system stability before fault detection is ensured by the proposed nominal control scheme.

Based on the design of the nominal controller and the fault detection scheme presented in Section 4.1 and Section 4.2, respectively, the stability property of the controlled plant before fault detection can be summarized as follows:

Theorem 1. *Before the detection of any faults (i.e., for $t \in [0, T_d)$), the nominal control law (9) with bounding control design (13) and fault detection scheme (16) guarantee that:*

(1) all the signals are uniformly bounded, i.e., $z(t)$ and $x(t)$ are bounded for all $t \in [0, T_d)$;

(2) given any $\bar{\epsilon} > \sqrt{2\kappa}$, there exists $T(\bar{\epsilon})$ such that $|y(t) - y_r(t)| \leq \bar{\epsilon}$, for all $t > T(\bar{\epsilon})$.

5. FAULT ACCOMMODATION DESIGN

5.1 *Basic controller reconfiguration: accommodation before fault isolation.*

After a fault occurs, the control performance may degrade rapidly, since the nominal controller (9) is no longer tuned to stabilize the system in the presence of a fault. Therefore, after fault detection, the nominal controller has to be reconfigured to help retain some of the basic stability properties of the system.

Before the fault is isolated, no information about the fault function is available. On-line approximators such as neural network models can be used to estimate the unknown fault function $\beta_i f_i$.

By denoting $\hat{f}_i(\bar{x}_i, \hat{\theta}_i)$ as the neural network approximation model with adjustable weights $\hat{\theta}_i$, the system model (1) can be rewritten as follows:

$$\begin{aligned} \dot{x}_i &= x_{i+1} + \phi_i(\bar{x}_i) + \hat{f}_i(\bar{x}_i, \theta_i^*) + \beta_i \delta_i(\bar{x}_i) + \eta_i \\ &\quad + (\beta_i - 1)\hat{f}_i(\bar{x}_i, \theta_i^*), \quad 1 \le i \le n-1, \\ \dot{x}_n &= \phi_0(x)u + \phi_n(x) + \hat{f}_n(x, \theta_n^*) + \beta_n \delta_n(x) \\ &\quad + \eta_n + (\beta_n - 1)\hat{f}_n(x, \theta_n^*), \end{aligned} \tag{17}$$

where $\delta_i(\bar{x}_i) \triangleq f_i(\bar{x}_i) - \hat{f}_i(\bar{x}_i, \theta_i^*)$ is the network approximation error for the i-th network, and θ_i^* is the optimal weight vector, given by

$$\theta_i^* \triangleq \arg \inf_{\hat{\theta}_i \in \Re^{p_i}} \left\{ \sup_{\bar{x}_i \in \Re^i} |f_i(\bar{x}_i) - \hat{f}_i(\bar{x}_i, \hat{\theta}_i)| \right\}.$$

For each network we make the following assumption on the network approximation error:

Assumption 2. *For each $i = 1, \cdots, n$,*

$$|\delta_i(\bar{x}_i)| \le \psi_{i\delta}\, s_{i\delta}(\bar{x}_i), \tag{18}$$

where $\psi_{i\delta} \ge 0$ are unknown parameters and $s_{i\delta} : \Re^i \mapsto \Re^+$ are known smooth bounding functions.

The details of the design of the fault-tolerant controller $u_D(t)$ defined in (5) has been considered in previous work (Zhang *et al.*, 2001) (available on-line). Due to space limitations, no more description is given here.

The fault-tolerant controller $u_D(t)$ guarantees the system stability and asymptotic tracking performance to a neighborhood of zero after the fault is detected. Since no further information about the fault is available at this stage, the neural networks are used to approximate the unknown fault model and Assumption 2 provides a bounding function on the network approximation error to facilitate the design of the fault-tolerant control law $u_D(t)$. However, in some applications, this critical assumption (18) may result in conservative bounds and/or possibly limiting requirements, which motivates the advanced controller reconfiguration procedure described in the next section.

5.2 *Advanced controller reconfiguration: accommodation after fault isolation.*

After a fault is detected, the fault isolation scheme is activated. The fault isolation module consists of a bank of N nonlinear adaptive estimators, where N is the number of possible faults in the fault class $\mathcal{F}$. Specifically, the following nonlinear adaptive estimators are used as isolation estimators:

$$\begin{aligned} \dot{\hat{x}}^s &= -\Lambda(\hat{x}^s - x) + \phi(x,u) + \hat{f}^s(x,u,\hat{\theta}^s) \\ \hat{f}^s &= \left[(\hat{\theta}_1^s)^\top g_1^s(x,u), \cdots, (\hat{\theta}_n^s)^\top g_n^s(x,u)\right]^\top \\ \dot{\hat{\theta}}_i^s &= \mathcal{P}_{\Theta_i^s}\{\Gamma_i^s g_i^s(x,u)\epsilon_i^s\}, \end{aligned} \tag{19}$$

where, for $s = 1, \cdots, N$ and $i = 1, \cdots, n$, $\hat{\theta}_i^s$ is the estimate of the fault parameter vector in the i-th state equation with the projection operator $\mathcal{P}$ restricting $\hat{\theta}_i^s$ to the corresponding known set Θ_i^s, $\epsilon_i^s(t) \triangleq x_i(t) - \hat{x}_i^s(t)$ is the i-th component of the state estimation error vector, and Λ and $\Gamma_i^s > 0$ are design matrices.

Fault isolation decision scheme: *If, for each $r \in \{1, \cdots, N\} \backslash \{s\}$, there exist some time $t^r > T_d$ and some $i \in \{1, \cdots, n\}$, such that at least one component of the state estimation error vector exceeds its threshold $\mu_i^r(t^r)$, i.e., $|\epsilon_i^r(t^r)| > \mu_i^r(t^r)$, then the occurrence of fault s is concluded. The fault isolation time is defined as*

$$T_{\text{isol}}^s \triangleq \max\{t^r, r \in \{1, \cdots, N\} \backslash \{s\}\}. \tag{20}$$

Clearly, a basic role in the above fault isolation scheme is played by the adaptive thresholds $\mu_i^s(t)$. The design of threshold $\mu_i^s(t)$ and some properties of the fault isolation scheme, such as fault isolability condition and fault isolation time, have been rigorously investigated in (Zhang *et al.*, 2000). Due to space limitations, no details are given here.

Let us now assume that the isolation procedure provides the information that fault s has been isolated at some time T_{isol}^s. Then, using the fault model described by (4), the system model (1) can be rewritten as follows:

$$\begin{aligned} \dot{x}_i &= x_{i+1} + (\beta_i(t - T_0) - 1)(\theta_i^s)^\top g_i^s(\bar{x}_i) \\ &\quad + \phi_i + (\theta_i^s)^\top g_i^s(\bar{x}_i) + \eta_i, \quad 1 \le i \le n-1 \\ \dot{x}_n &= \phi_0(x)u + (\beta_n(t - T_0) - 1)(\theta_n^s)^\top g_n^s(x) \\ &\quad + \phi_n + (\theta_n^s)^\top g_n^s(x) + \eta_n \\ y &= x_1, \end{aligned} \tag{21}$$

where, as described in Section 2, the vector field g_i^s characterizes the functional structure of the fault in the i-th state equation, and θ_i^s denote the magnitude of the fault in that state equation.

Comparing (21) with the system model (17) before fault isolation, the network approximation error $\delta_i(\bar{x}_i)$ no longer exists. Consequently, the critical Assumption 2 can be removed for the design of fault-tolerant controller $u_I^s(t)$ used after fault isolation. Again, the details for designing the stable adaptive control law $u_I^s(t)$ are omitted here due to space limitations and the reader is referred to (Zhang *et al.*, 2001) for more details.

6. SIMULATION RESULTS

In this section, we consider a simulation example of a single-link robot whose motion dynamics (Kim *et al.*, 1997) is given by

$$\begin{aligned} \ddot{q} &= \frac{1}{M}\left[u(t) - \frac{1}{2}mgl\sin(q)\right] + \eta(q, \dot{q}, t) \\ &\quad + \beta(t - T_0)f(q, \dot{q}) \\ y &= q, \end{aligned}$$

where q is the link position angle, u is the input torque, M is the moment of inertia, g is the acceleration due to gravity, m and l are the mass and length of the link, η, f and β represent the modeling uncertainty and fault function, and the fault time profile, respectively. Moreover, the desired output is $y_r = \sin 2t$, for $t \geq 0$. Letting $x_1 = q$ and $x_2 = \dot{q}$, the state-space representation of the system is

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = \frac{1}{M}(u(t) - \frac{1}{2}mgl\sin(x_1)) + \beta(t - T_0)f(x) + \eta$$

The class of faults is assumed to be as follows:
(1) A fault which results in a reduction in the mass of the link. Then the fault function takes on the form of $f^1 = \theta^1 g^1(x)$, where $g^1(x) = -\frac{1}{2M}mgl\sin(x_1)$ and $\theta^1 \in [-1, 0]$ represents the percentage of change in the mass.
(2) A fault that occurs due to a tangle of complex factors; the fault is assumed to be a nonlinear change (in the robotic system dynamics) described by $f^2 = \theta^2 g^2(x)$, where $g^2(x) = x_1 x_2$ and $\theta^2 \in [-1, 0]$.

Without loss of generality, in the following we consider a fault of type 1 with $\theta^1 = -0.75$ and fault time profile $\beta = 1 - e^{-0.2(t-10)}$, i.e., the fault evolution rate is $\alpha = 0.2$ and the fault occurrence time is $T_0 = 10$ s. Using the methodology described in Section 4.2 and Section 5.2, the monitoring module consists of a detection scheme and a bank of two fault isolation estimators (FIEs). The filter pole is chosen as $-\lambda = -1$.

The nominal controller is given by (9). For the basic controller reconfiguration (for the sake of simplicity), the bound on the network approximation error $\delta(x)$ described in Assumption 3 is taken as $|\delta(x)| \leq \psi_\delta$, where ψ_δ is an unknown constant (i.e., s_δ is selected as a unit constant function). The design parameters are selected as follows: the controller gains $c_1 = c_2 = 0.3$, $\epsilon = 0.2$.

As we can see from Fig. 2, after the fault occurs at $t = 10$ s, it is detected at approximately $t = 12$ s. At that time, the nominal controller is reconfigured, and two isolation estimators are activated to isolate the fault. The decision of occurrence of fault 1 can be made at $t = 15.5$ s. Then the second controller reconfiguration is triggered. The actual output with fault diagnosis and accommodation (FDA) and the desired output are shown in Fig. 2. For comparison purposes, the system output without FDA is also included.

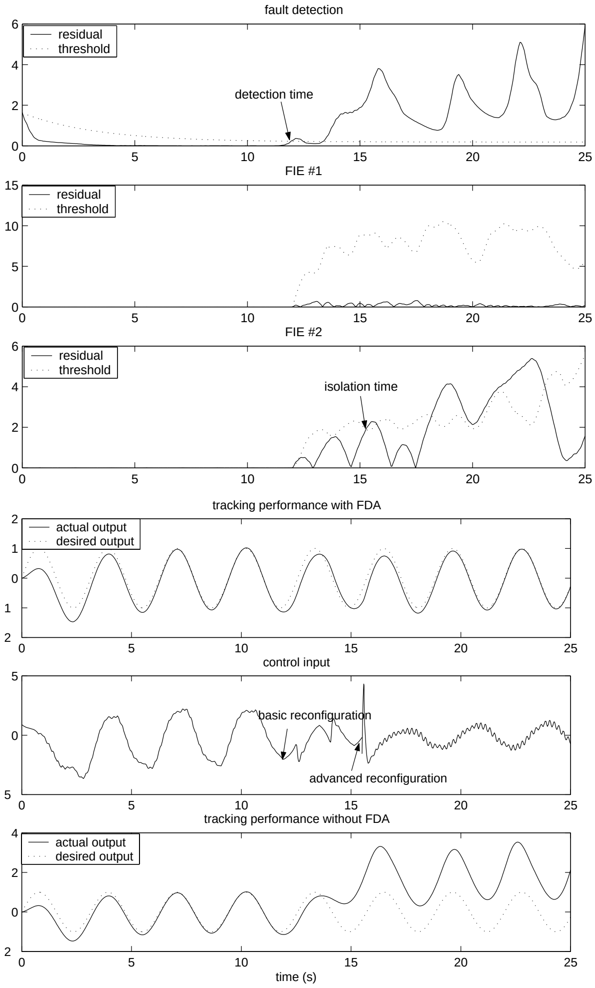

Fig. 2. Fault detection and isolation, tracking performance, and control input.

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