15th Triennial World Congress, Barcelona, Spain

SINGULARITY-FREE STABLE ADAPTIVE CONTROL OF A CLASS OF NONLINEAR DISCRETE-TIME SYSTEMS

Leonid S. Zhiteckij

Int. Centre of Inform. Technologies & Systems Institute of Cybernetics
40, Prospect Akademika Glushkova MSP 03680 Kiev 187 Ukraine
E-mail: zls@d310.icyb.kiev.ua

Abstract: This paper deals with the adaptive control of linearly parameterized discrete-time nonlinear system in the presence of bounded disturbances and unmodeled dynamics. A new adaptive law is presented. No *a priori* knowledge about the bounds on both the plant parameters and unmodeled uncertainty is required to implement the estimation algorithm. The adaptive scheme is free from singularity. This is achieved through the use of some time-varying adaptation gain. It is established that if the plant model nonlinearity is sector-bounded then the BIBO stability of the closed-loop system will be guaranteed. Simulation results are given to demonstrate the performance of the adaptive controller. *Copyright © 2002 IFAC*

Keywords: Adaptive control; convergence analysis; discrete-time systems; estimation algorithm; nonlinear control systems; parametrization; stability analysis.

1. INTRODUCTION

Adaptive control of nonlinear systems has been a subject of very intense research activity during the last decade because of their practical importance. Substantial breakthroughs have been achieved in theoretical frameworks, and new stability results have been obtained for several classes of adaptive nonlinear continuous-time systems represented by some nonlinear models with unknown parameters appearing linearly and nonlinearly; see, e.g., (Annaswamy, *et al*., 1998; Kosmatopoulos and Ioannou, 1999; Krstic, *et al*., 1995; Lozano and Brogliato, 1992; Pomet and Praly, 1992; Sastry and Isidori, 1989; Taylor, *et al*., 1989) and references therein. On the other hand, very few similar works are available in the literature that address the stable adaptive controller design for discrete-time systems with nonlinearities (Agarwal and Seborg, 1987; Chen and Khalil, 1995; Fabri and Kadirkamanathan, 1998; Jagannathan and Lewis, 1996; Kanellakopoulos, 1994; Lin and Yong, 1992; Song and Grizzle, 1993; Spooner, *et al*., 1996; Xie and Guo, 1999; Yeh and Kokotovic, 1995).This is due to the fact that the difficulties pointed out in Kanellakopoulos (1994), Song and Grizzle (1993) and Yeh and Kokotovic (1995) occur in discrete-time case. In particular, Lyapunov-based approach which has usefully been employed in the continuous-time case to establish the global stability, is not directly appropriate in its discrete-time counterpart (Kanellakopoulos, 1994).

Most of paper devoted to the adaptive nonlinear discrete-time control of linear-in-the-parameters (LP) systems guarantee global stability under the assumption that the nonlinearities depending on past outputs have a linear growth rate. Using the Key Technical Lemma (Goodwin and Sin, 1984, Lemma 6.2.1), it is shown by Kanellakopoulos (1994) that a certainty-equivalence controller combined with the standard recursive least-squares (RLS) estimator will ensure the global stability of simple adaptive nonlinear discrete-time LP system if the nonlinearity is sector-bounded. This Lemma is also applied in Yeh and Kokotovic (1995) to prove that all signals in the closed loops of several classes of nonlinear discrete-time LP control systems utilizing some gradient-like and RLS update laws remain bounded if the nonlinearities satisfy certain growth conditions.

To prevent the instability of linear discrete-time adaptive control systems subject to bounded disturbances and unmodeled dynamics, dead zone is often incorporated in adaptive law; see, e.g., (Goodwin and Sin, 1984; Ortega and Lozano, 1987). The implementation of standard dead zone approach requires the knowledge of the upper bounds on disturbances and on unmodeled dynamics term. However, such knowledge can hardly be obtained in practice. In order to overcome this difficulty, a time-varying dead-zone adaptive law applied to the adaptive control of linear discrete-time systems with bounded disturbances having unknown bounds has been developed by Feng (1994). Unfortunately, the algorithm of Feng (1994) needs the *a priori* information about a lower bound on the high frequency gain and its sign, and this information is necessary to avoid singularity in adaptive control law. An alternative estimation scheme reported in Zhiteckij (1997) differs from the one used in Feng (1994) in that it does not require any knowledge regarding the high frequency gain and its sign as well. Nevertheless, his control law is free from singularities. Another adaptive algorithm for the singularity-free control of a continuous-time nonlinear system without requiring any *a priori* information on the plant parameters is proposed by Lozano and Brogliato (1992). However, they study the case when the both disturbance and unmodeled dynamics absent. It is not yet clear how their results might be extended to the discrete-time systems with the unmodeled uncertainties whose bounds are unknown. On the other hand, to the best of author's knowledge, there are no adaptive algorithms available in the literature that allow to cope with singularity and to achieve the stability of nonlinear discrete-time system in the presence of bounded disturbance and unmodeled dynamics with unknown bounds.

In this paper, an adaptive nonlinear discrete-time LP control system with parametric and nonparametric uncertainties is designed and analyzed. The proposed approach is based on combining some key ideas employed in Zhiteckij (1997; 1999). It involves the use of a new adaptive law for estimating the plant parameters and the both bounds on unmodeled uncertainty. The basic feature distinguishing this law from existing adaptive schemes is that it requires neither *a priori* information regarding the bounds on unmodeled plant uncertainty nor *a priori* information with respect to the constraints on some bounded region to which the plant parameters belong. To avoid the singularity in control law, a time-varying adaptive gain is incorporated in the estimator. The main effort is focused on establishing the stability properties of resulting closed-loop system.

2. STATEMENT OF THE PROBLEM

The plant to be controlled is a nonlinear single-input single-output (SISO) discrete-time system described by

$$y_t = f(x_{t-1}) + bu_{t-1} + v_t \,, \tag{1}$$

where $y_t, u_t, v_t \in \mathbf{R}$ are the measurable output, control input and unmeasured external disturbance, respectively (integer t denotes the discrete time). $f(\bullet):\mathbf{R}^N \to \mathbf{R}$ represents a smooth nonlinear function depending on the vector $x_{t-1}^{\mathrm{T}} = [y_{t-1}, \cdots, y_{t-N}]$ of N past outputs. b is a constant but unknown nonzero scalar. It is assumed that $f(\bullet)$ is unknown but this function can be approximated within $\mathbf{R}^N$ by suitable $\hat{f}(x,\theta)$ defined as follows:

$$\hat{f}(x,\theta) = \theta^{\mathrm{T}}\phi(x)\,, \tag{2}$$

where $\theta \in \mathbf{R}^d$ represents some unknown parameter vector and $\phi(x):\mathbf{R}^N \to \mathbf{R}^d$ is a vector function whose components are the group of d basis functions chosen by the designer.

Using (2), equation (1) may be rewritten in the form

$$y_t = \theta^{\mathrm{T}}\phi(x_{t-1}) + bu_{t-1} + v_t + \Delta_t \,, \tag{3}$$

where

$$\Delta_t = f(x_{t-1}) - \hat{f}(x_{t-1}, \theta) \tag{4}$$

denotes the approximation error arising due to the mismatch between true plant (1) and its LP model that exploits (2). In (3), Δ_t plays the role of an unmodeled dynamics.

The following assumptions regarding the plant model in equations (3), (4) will be made.

(A1) $\phi(x)$ satisfies the sector condition

$$\| \phi(x) \|_2 \le k_1 \|x\|_2 + k_0 \tag{5}$$

for some nonnegative k_0, k_1, where $\|\bullet\|_2$ denotes the Euclidean vector norm.

(A2) The disturbance v_t is bounded by an unknown ε,

$$\| v_t \|_\infty \le \varepsilon \,, \tag{6}$$

where $\| z_t \|_\infty$ denotes the l_∞-norm of any function $z_t : \mathbf{Z}^+ \to \mathbf{R}$ defined as $\|z_t\|_\infty = \sup_{0 \le t < \infty} |z_t|$.

(A3) The modeling error Δ_t given by (4) is bounded in the l_∞-norm by a known function g_t multiplied by an unknown but bounded variable δ_t, i.e.,

$$\|\Delta_t\|_\infty \le \delta_t g_t \,, \tag{7a}$$

where

$$\|g_t\|_\infty < \infty \,, \quad \|\delta_t\|_\infty \le D < \infty \tag{7b}$$

with some unknown positive constant D.

Remark 1. Note that assumption (A1) implying that the nonlinearity $\phi(x_t)$ has a growth rate not faster than linear, i.e.,

(C1) $\quad \|\phi(x_t)\| = O(\|x_t\|)$ as $\|x_t\| \to \infty$

is essentially used later to derive the stability result. Of course, condition (C1) is restrictive too, and one would like to relax this condition replacing it by

(C2) $\quad \|\phi(x_t)\| = O(\|x_t\|^{\beta})$ as $\|x_t\| \to \infty$

with some $\beta > 1$. Recently, the work of Xie and Guo (1999) who dealt with the standard RLS adaptive laws applied to nonlinear LP discrete-time systems in the presence stochastic disturbances $\{\nu_t\}$ shed some light on such a possibility. They theoretically argued that in the multiparameter case $(d > 1)$ subjected to (C2), the global stabilization of the system of form (3) with $b = 1$, $\Delta_t \equiv 0$ and a Gaussian white noise ν_t is impossible, in general, if $\beta > 1$. However, the question of how far can one go from (C1) to (C2) in the case of non-stochastic bounded ν_t when b is unknown remains open up to now.

Remark 2. Assumption (A3) is first made by Yao and Tomizuka (1997) in their Assumption 1 to study a class of nonlinear continuous-time adaptive systems with bounded unmodeled uncertainties. ■

Let y^* ($y^* \equiv \text{const}$) be a given set-point for output y_t. The problem is to design an adaptive controller such that, in spite of the modeling error Δ_t, the resulting closed-loop control system is bounded input - bounded output (BIBO) stable, and the output error

$$e_t = y^* - y_t \tag{8}$$

is close as possible to its minimum value.

3. ADAPTIVE CONTROL ALGORITHM

Using the standard certainty equivalence principle, one can conclude from equation (4) together with (6) - (8) that if θ and b are known (nonadaptive case), a controller whose output u_t is determined as

$$u_t = b^{-1}[y^* - \theta^{\mathrm{T}} \phi(x_t)] \tag{9}$$

ensures

$$\| e_t \|_{\infty} \le \varepsilon + g_t D. \tag{10}$$

(Note that the upper bound on $\| e_t \|_{\infty}$ specified by (10) is its minimum.) Based on equation (9), a natural choice of the adaptive control law for the case when θ and b are indeed unknown is

$$u_t = b_t^{-1}[y^* - \theta_t^{\mathrm{T}} \phi(x_t)], \tag{11}$$

where b_t and θ_t are the current estimates of b and θ, respectively. It is clear that the condition

$$b_t \ne 0 \quad \forall t. \tag{12}$$

must be satisfied in order to avoid any *singularity* in such an adaptive control scheme.

Utilizing the approach of Zhiteckij (1997; 1999), the adaptive algorithm for estimating the parameter vector

$$\theta_0^{\mathrm{T}}(t) = [\theta_t^{\mathrm{T}}, b_t] \tag{13}$$

is derived by recursive solving the inequalities

$$\left| y_t - \hat{\theta}^{\mathrm{T}} \phi(x_{t-1}) - \hat{b} u_{t-1} \right| \le \hat{\varepsilon} + \hat{D} g_t, \quad t \in \mathbf{Z}^+ \tag{14}$$

with respect to the unknown extended vector $\hat{\tilde{\theta}}_0^{\mathrm{T}} = [\hat{\theta}_0^{\mathrm{T}}, \hat{D}, \hat{\varepsilon}]$. These inequalities are compatible because they are satisfied for $\hat{\tilde{\theta}}_0 = \tilde{\theta}_0$, where $\tilde{\theta}_0^{\mathrm{T}} = [\theta_0^{\mathrm{T}}, D, \varepsilon]$. (This fact follows directly from equation (3) together with (6) and (7).)

Introduce the dead zone function proposed by Zhiteckij (1997; 1999)

$$f_0(e, D_0) = \begin{cases} D_0 - |e| & \text{if } \ |e| \ge D_0 \ (D_0 > 0) \\ 0 \ \text{ otherwise} \end{cases} \tag{15}$$

and define the parameter identification error

$$\bar{e}_t = y_t - \theta_{t-1}^{\mathrm{T}} \phi(x_{t-1}) - b_{t-1} u_{t-1}. \tag{16}$$

Let

$$w_t^{\mathrm{T}} = [\chi_{t-1}^{\mathrm{T}} \operatorname{sign} \bar{e}_t, g_t, 1] \tag{17}$$

with

$$\chi_{t-1}^{\mathrm{T}} = [\phi^{\mathrm{T}}(x_{t-1}), u_{t-1}]. \tag{18}$$

It can be shown that the estimate vector

$$\tilde{\theta}_0^{\mathrm{T}}(t) = [\theta_0^{\mathrm{T}}(t), D_t, \varepsilon_t] \tag{19}$$

obtaining as the recursive solution of (14) becomes

$$\tilde{\theta}_0(t) = \tilde{\theta}_0(t-1) + \gamma_t \frac{f_0(\bar{e}_t, D_0(t))}{\|w_t\|_2^2} w_t, \tag{20}$$

where

$$D_0(t) = \varepsilon_{t-1} + D_{t-1} g_t \tag{21}$$

and γ_t is a tuning factor specified later. Equation (20) together with (17), (19) yield straightforward the estimation algorithms for updating the parameter vector

$$\theta_0(t) = \theta_0(t-1) + \gamma_t \frac{f_0(\bar{e}_t, D_0(t))}{1+\|\chi_{t-1}\|_2^2 + g_t^2} \chi_{t-1}^{\mathrm{T}} \operatorname{sign} \bar{e}_t \tag{22}$$

defined by (13), and the estimates

$$D_t = D_{t-1} + \gamma_t \frac{f_0(\bar{e}_t, D_0(t))}{1+\|\chi_{t-1}\|_2^2 + g_t^2} g_t \tag{23}$$

$$\varepsilon_t = \varepsilon_{t-1} + \gamma_t \frac{f_0(\bar{e}_t, D_0(t))}{1+\|\chi_{t-1}\|_2^2 + g_t^2} \tag{24}$$

utilized for calculating the current size $D_0(t)$ of dead zone (15).

The tuning factor γ_t is chosen from the range

$$0 < \gamma_{\min} \le \gamma_t \le \gamma_{\max} < 2 \tag{25}$$

so that the components of $\theta_0(t)$ in (13) including b_t does not go to zero. Such a choice is always possible due to the fact that, in equation (22), $\theta_0(t)$ is the linear function in γ_t. It allows to meet requirement (12), and thus to overcome the singularities which might arise in control law (11). Note that similar tool has already been used by Ortega and Lozano (1987) in their linear adaptive control system with bounded disturbances whose bounds are known; see also (Goodwin and Sin, 1984).

Remark 3. In contrast to Zhiteckij (1997; 1999) no projection is needed to implement adaptive law (22).

4. CONVERGENCE AND STABILITY ANALYSIS

The preliminary basic result establishing the convergence of the adaptive estimator constructed above is given in the following lemma.

Lemma. The adaptive law defined by equations (22)-(25) together with (15)-(18) and (21) possesses the properties that:

a) the sequences $\{D_t\}$ and $\{\varepsilon_t\}$ are bounded and converge to some finite $D_\infty < \infty$ and $\varepsilon_\infty < \infty$, respectively, as t tends to the infinity;

b) $\dfrac{f_0(e_t, D_0(t))}{(1+\|\chi_{t-1}\|_2^2 + g_t^2)^{1/2}} \in l_2$

provided that the plant of form (3) satisfies the assumptions (A1) and (A3) and $\left\|\tilde{\theta}_0(0)\right\|_2 < \infty$.

Proof. The proof follows closely the steps used in proving the similar result in Feng (1994). Defining a Lyapunov-like function

$$V_t = \left\| \tilde{\theta}_0 - \tilde{\theta}_0(t) \right\|_2^2, \tag{26}$$

one has

$$V_t - V_{t-1} \le -\gamma_t (2-\gamma_t) \frac{f_0^2(\bar{e}_t, D_0(t))}{1+\|\chi_{t-1}\|_2^2 + g_t^2}$$

$$\le -\gamma_{\min}(2-\gamma_{\max}) \frac{f_0^2(\bar{e}_t, D_0(t))}{1+\|\chi_{t-1}\|_2^2 + g_t^2} \text{ [using (25)].} \tag{27}$$

The first inequality of (27) has been derived after a number of successive steps involving equation (3), inequalities (6), (7), (14) and the definitions of χ_{t-1}, $\tilde{\theta}_0$ and $\tilde{\theta}_0(t)$. Since $\gamma_{\min} > 0$ and $\gamma_{\max} < 2$ (see (25)), it follows from the second inequality of (27) that $\{V_t\}$ is nonincreasing:

$$V_t \le V_{t-1}. \tag{28}$$

On the other hand, V_ts are all nonnegative. This together with (28) gives that $\{\tilde{\theta}_0(t)\}$ is bounded sequence. (Here definition (26) and the condition that $\left\|\tilde{\theta}_0(0)\right\|_2 < \infty$ was used.) From definition (19), one can see that $\{D_t\}$ and $\{\varepsilon_t\}$ are both bounded. Further, from equations (23) and (24) of D_t and ε_t, respectively, and definition (15) of dead zone $f_0(\bullet,\bullet)$ it follows that these sequences are both nondecreasing. From the boundedness of D_ts and ε_ts, it allows to conclude that there exist some $D_\infty < \infty$ and $\varepsilon_\infty < \infty$ such that $\lim_{t\to\infty} D_t = D_\infty$, $\lim_{t\to\infty} \varepsilon_t = \varepsilon_\infty$. This proves part a) of the lemma.

To show the validity of part b), it is necessary to sum both sides of the second inequality of (27) from $t = 1$ to some N. Then one gets

$$V_t \le V_0 - \gamma_{\min}(2-\gamma_{\max}) \sum_{t=1}^{N} \frac{f_0^2(\bar{e}_t, D_0(t))}{1+\|\chi_{t-1}\|_2^2 + g_t^2}. \tag{29}$$

Since V_t is nonnegative and $\gamma_{\min}(2-\gamma_{\max}) > 0$, inequality (29) gives that if $N \to \infty$ then the series

$$\sum_{t-1}^{\infty} f_0^2(\bar{e}_t, D_0(t)) \Big/ (1+\|\chi_{t-1}\|_2^2 + g_t^2)$$

will be convergent. Hence,

$$\lim_{t\to\infty} f_0^2(\bar{e}_t, D_0(t))/(1+\|\chi_{t-1}\|_2^2 + g_t^2) = 0.$$

Using the fact that equations (11) and (16) together with (8) cause $e_t = -\bar{e}_t \ \forall t$, it results in

$$\lim_{t\to\infty} f_0^2(e_t, D_0(t))/(1+\|\chi_{t-1}\|_2^2 + g_t^2) = 0, \tag{30}$$

which implies part b). This completes the proof of the lemma. ∎

With the convergence properties of the proposed adaptive law given in parts a), b) of the lemma, using similar arguments as those in (Feng, 1994; Kanellakopoulos, 1994; Ortega and Lozano, 1987), the following stability result can be shown to be valid.

Theorem. Under the assumption (A1)-(A3), and arbitrary initial $\widetilde{\theta}_0(0)$ such that $b_0 \neq 0, D_0 \geq 0$, and $\varepsilon_0 \geq 0$, the closed-loop control system consisting of the plant of form (3) and the adaptive controller described in equations (11), (22)-(24) together with (16), (21) and (15), is (global) stable in the sense that the variables y_t and u_t remain bounded for all t provided that $y^* < \infty$. Moreover,

$$\limsup_{t\to\infty} |e_t| \leq \varepsilon_\infty + D_\infty g_t . \tag{31}$$

Proof. It can be shown that equation (3) together with (6) and (7) yield the inequality

$$|u_{t-1}| \leq \max_{\tau\leq t} |y_\tau| + C_1 \max_{\tau\leq t} \|\phi(x_{t-1})\|_2 + C_2 \tag{32}$$

with some constants $C_1, C_2 > 0$. Applying (32) together with condition (5) and using the definition of x_{t-1} and utilizing the fact that y^* is bounded, due to definition (18) of χ_{t-1}, obtain

$$\|\chi_{t-1}\|_2 \leq \|\varphi(x_{t-1})\|_2 + |u_{t-1}|$$
$$\leq C_3 \max_{\tau\leq t} |e_\tau| + C_4 , \tag{33}$$

where $C_3, C_4 < \infty$ represent corresponding positive constants.

With the convergence properties of $\{D_t\}$ and $\{\varepsilon_t\}$ given in item a) of Lemma and with the fact that (7b) implies the boundedness of g_t one gets from (21) that $D_0(t)$ is upper bounded. This and second inequality of (33) gives that there exist finite constants C_5, C_6 such that

$$\|\chi_{t-1}\|_2 \leq C_5 \max_{\tau\leq t} |f_0(e_\tau, D_0(\tau))| + C_6 , \tag{34}$$

where definition (15) of $f_0(\bullet,\bullet)$ was also used.

Since $\|g_t\|_\infty < \infty$, it is seen from (30) and (34) that the conditions of the Key Technical Lemma (Goodwin and Sin, 1984, Lemma 6.2.1) are satisfied. By this Lemma, $\|\chi_{t-1}\|_2$ is bounded and

$$f_0(e_t, D_0(t)) \to 0 \text{ as } t \to \infty . \tag{35}$$

Since $\|\chi_{t-1}\|_2$ is bounded, from the definition (18) of χ_{t-1}, the boundedness of y_t and u_t follows.

Taking into account (15), (21) and (35) and the fact that $\varepsilon_t \to \varepsilon_\infty$ and $D_t \to D_\infty$, one can finally conclude that (31) holds. This proves the theorem.

5. SIMULATION EXAMPLE

To illustrate some features of the proposed adaptive control algorithm, the closed-loop system containing a plant described by

$$y_t = \theta \frac{y_{t-1}^3}{1 + y_{t-1}^2} + b u_{t-1} + u_t + \Delta_t$$

with $\theta = -0.5$, $b = 4.0$ and the adaptive controller with control law (11) and estimator (22)-(24) was simulated. The nonlinearity $\phi(x_{t-1}) = y_{t-1}^3 / (1 + y_{t-1}^2)$ was chosen to satisfy requirement (5). v_t was generated as 10-digit PRBS of amplitude 0.5. The upper bound on Δ_t given by (7) was chosen as follows: δ_t was a pseudorandom variable taken from the range [0, 1.0] and $g_t = |G \sin \omega t|$ with $G = 1.0$, $\omega = 0.6$. The initial estimates were: $\theta_0 = 1.5$, $b_0 = 0.5$, $D_0 = 0.0$, $\varepsilon_0 = 0.0$.

Figures 1-3 show the outcome of the 100-step long simulation with $y^* = 1.0$ if $0 \leq t \leq 40$ and $y^* = 5.0$ if $40 \leq t \leq 100$. It is seen that the ultimate behavior of the closed-loop system is satisfactory.

6. CONCLUSION

This paper is an extension and generalization of previous works (Zhiteckij, 1997; 1999) to a class of nonlinear discrete-time LP control systems. A new adaptive scheme has been developed and analyzed. It does not need *a priori* knowledge regarding the bounds on both the plant parameters and the external disturbance and unmodeled dynamics.

A disadvantage is that restrictive condition (C1) is required to guarantee the BIBO stability of resulting closed-loop adaptive system. However, no one knows yet how (C1) might be removed to ensure the stabilization of (3). It seems that a new tool should further be devised to emerge from (C1). To this end, another stability concept has to be advanced.

ACKNOWLEDGMENTS

The author would like to thank Professor Petros A. Ioannou who gave him a lot of helpful suggestions and also anonymous reviewers for their valuable comments.

This work was supported in part by NATO Grant PST.CLG. 976958.

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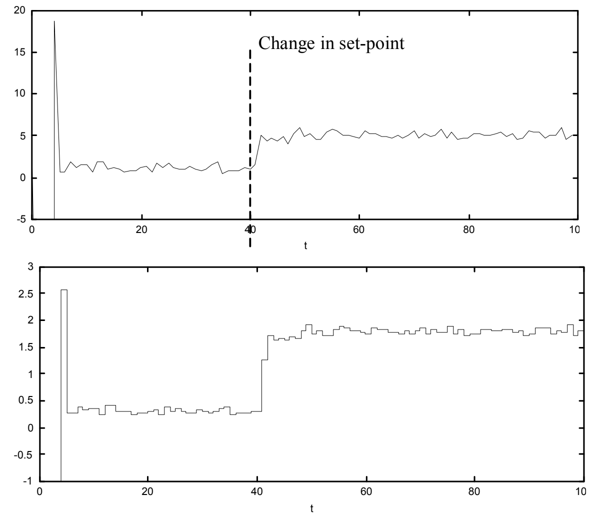

Fig. 1. Plant output y_t (upper plot) and control input u_t (lower plot).

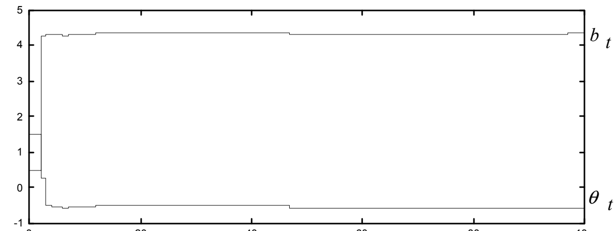

Fig.2. Parameter estimates b_t and θ_t .

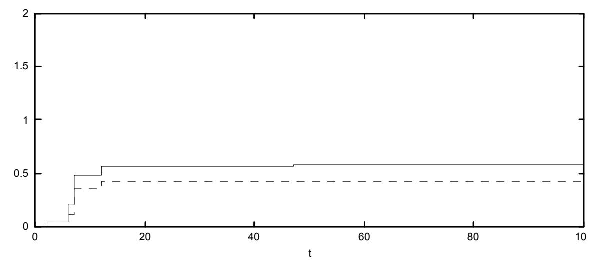

Fig. 3. Estimates ε_t (__) and $D_t(\cdots)$.