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TRAFFIC CONTROL PROBLEMS FOR NETWORKS VIA INCENTIVE MODEL

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Abstract: This paper deals with traffic rate control problems of networks. The incentive Stackelberg strategy concept was introduced to the networking model that comprises subsidiary systems of users and network. A linear strategy and a nonlinear strategy were proposed to the elastic traffic problem, with the illustrations via examples. The presented method was extended to non-elastic traffic problem.

Keywords: Rate control, Pricing, Elastic traffic, Non-elastic traffic, Incentive Stackelberg strategy

1. INTRODUCTION

The work focus on the system model of charging, routing and flow control, where the system comprises both users with utility functions and a network with capacity constraints. Kelly (1997) showed that the optimization of the system may be decomposed into subsidiary optimization problems, one for each user and one for the network, by using price per unit flow as a Lagrange multiplier that mediates between the subsidiary problems. Low and Varaiya (1993) and Murphy *et al.* (1994) described how such results may be used as the basis for distributed pricing algorithms, and MacKie-Mason and Varian (1994) described a "smart market" based on a per-packet charge when the network is congested.

As mentioned in Kelly's (1997) work, price per unit flow is the mediating variable. And this may cause a particular difficulty for elastic traffic. In an implementation of an ATM available bit rate service, for example, users would be subject to two sources of uncertainty about the service offered. The system optimum can be achieved when users' choice of charges and the network's choice of allocated rates are in equilibrium. For elastic traffic, the equilibrium exists and a system optimum can be achieved. But for most non-elastic traffic, the equilibrium does not exist and the system optimum can not be achieved.

By using the incentive Stackelberg strategy concept, a new way to deal with such kind of routing control problems is proposed. Studies on the game theory and its applications belong to the area of automatic control (Basar and Olsder 1982). In a game theoretic model (Funderberg and Tirole 1992), there are at least two players who control their own inputs to reach their own outcomes from

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the system, respectively. Therefore, game theory (Myerson 1991) provides a systematic framework to treat the dynamic behavior of noncooperative networks. Two major concepts in game theory, Nash and Stackelberg equilibria, have been employed into the study of noncooperative networks (see Bertsekas and Gallager 1992, Douligeris and Mazumdar 1989, Economides and Silvester 1990, Korilis *et al* 1996, Orda *et al* 1993). In these references, the game theoretic models are all based on the classical noncooperate strategy concepts.

In this paper, the traffic rate control problem is discussed by means of the incentive Stackelberg strategy, which was introduced into the game theory by Ho *et al* (1982). To the elastic traffic, a linear strategy and a nonlinear strategy are proposed to reveal the concept of incentive strategies. The method is generalized to non-elastic traffic model. The incentive strategy consists of two parts one of which is the regular price. The other is punishment price which varies on the variance of traffic rate linearly or functionally.

2. SYSTEM MODEL

Consider a network with a set $\mathbf{J}$ of resources, and let C_j be the finite capacity of resource j, for $j \in \mathbf{J}$. A set $\mathbf{S}$ of users use the network with rates $x_s \in \mathbf{S}$, $s = 1, 2, \cdots, S$. For each user s, the utility maximization is as follows.

$USER_s(U_s; \lambda_s)$:

$$\begin{aligned} &\text{maximize } U_s(x_s) - \lambda_s x_s \\ &\text{over } x_s \geq 0 \end{aligned} \tag{1}$$

where $U_s(x_s)$ is the utility function of user s, and it is an increasing, strictly concave and continuously differentiable function of x_s. λ_s is a price charged to user s per unit flow, and also is the component of the vectors of Lagrange multipliers for the following problem of overall system.

$SYSTEM(U, H, A, C)$:

$$\begin{aligned} &\text{maximize } \sum_{s=1}^{S} U_s(x_s) \\ &\text{subject to } Hy = x, Ay \leq C \\ &\text{over } x, y \geq 0 \end{aligned} \tag{2}$$

where H and A are the $0-1$ matrixes, y is the flow pattern. And the Lagrangian form of the problem is

$$\begin{aligned} &L(x, y, z; \lambda, \mu) = \\ &\sum_{s=1}^{S} U_s(x_s) - \lambda^T(x - Hy) + \mu^T(C - Ay - z) \end{aligned} \tag{3}$$

where z is a vector of slack variables. If the network receives a revenue λ_s per unit flow from user s, then the revenue optimization problem for the network is as follows.

$NETWORK(H, A, C; \lambda)$:

$$\begin{aligned} &\text{maximize } \sum_{s=1}^{S} \lambda_s x_s \\ &\text{subject to } Hy = x, Ay \leq C \\ &\text{over } x, y \geq 0 \end{aligned} \tag{4}$$

From Kelly's (1997) work, about these three problems, there exists a price vector $\lambda = (\lambda_1, \lambda_2, \cdots, \lambda_s)$ such that the vector $x = (x_1, x_2, \cdots, x_S)$, formed from the unique solution x_s to $USER_s(U_s; \lambda_s)$ for each $s \in \mathbf{S}$, solves $NETWORK(H, A, C; \lambda)$. The vector x then also solves $SYSTEM(U, H, A, C)$.

Note that $USER_s(U_s; \lambda_s)$ and $SYSTEM(U, H, A, C)$ have the same parts $U_s(x_s)$ in their formulations. So it is easy to find the result in cooperation. But the problem $NETWORK(H, A, C; \lambda)$ is just opposite to the problem $USER_s(U_s; \lambda_s)$, because there are the same parts $\lambda_s x_s$ in their formulations with the opposite symbols. So, they are non-cooperate in general. The conditions allowing the vectors λ and x solve both problems are very strictly.

If the leader wants users be at the rates which is arranged by the network, the leader must have the leadership in the game which is indicated in the following Stackelberg strategy.

$$\xi_s(x_s) = \lambda_s + p_s(x_s) - p_s(x_s^a) \tag{5}$$

where $p_s(x_s)$ is any function of x_s to be determined. x_s^a is a desired point arranged by the network. $p_s(x_s)$ can be taken as a linear function

$$p_s(x_s) = q_s x_s \tag{6}$$

where q_s is some kind of punishment price. It will be determined by the leader.

3. INCENTIVE STRATEGY FOR ELASTIC TRAFFIC PROBLEM

3.1 Linear Incentive Strategy

In this section, the linear function (6) is taken as the Stackelberg incentive strategy to force users to act at the point x_s^a. Replacing λ_s in (1) by ξ_s with linear structure, the problem $USER_s(U_s; \lambda_s)$ becomes

$USER_s(U_s; q_s)$

$$\begin{aligned} &\text{maximize } U_s(x_s) - \lambda_s x_s - q_s(x_s - x_s^a)x_s \\ &\text{over } x_s \geq 0 \end{aligned} \tag{7}$$

To get q_s, calculate the derivative of (7) with respect to x_s, and let it be zero. Therefore,

$$U_s'(x_s) - \lambda_s - 2q_s x_s + q_s x_s^a = 0 \tag{8}$$

Let x_s takes the value at x_s^a. Then

$$q_s = \frac{U_s'(x_s^a) - \lambda_s}{x_s^a} \tag{9}$$

So, the strategy should be

$$\xi_s(x_s) = \lambda_s + \frac{U_s'(x_s^a) - \lambda_s}{x_s^a}(x_s - x_s^a) \tag{10}$$

The following work is to prove that (10) is a incentive Stackelberg strategy, *i.e.*

$$\xi_s(x_s^a) = \lambda_s \tag{11}$$

and

$$\begin{aligned} \arg\max[U_s(x_s) - \lambda_s x_s - q_s(x_s - x_s^a)x_s] \\ = x_s^a \end{aligned} \tag{12}$$

It is easy to see that (11) is held from the structure of $\xi_s(x_s)$ in (10). Eq. (12) means that the following inequality should hold.

$$\begin{aligned} &U_s(x_s^a) - \lambda_s x_s^a \geq \\ &U_s(x_s) - \lambda_s x_s - q_s(x_s - x_s^a)x_s \end{aligned} \tag{13}$$

Now, denote by x_s^u the optimal rate of user s which maximizes the problem $USER_s(U_s; \lambda_s)$. So

$$\begin{array}{lcl} & U_s'(x_s^u) - \lambda_s = 0 & \\ U_s'(x_s) - \lambda_s > 0 & \text{if} & x_s < x_s^u \\ U_s'(x_s) - \lambda_s < 0 & \text{if} & x_s > x_s^u \end{array} \tag{14}$$

If the optimal rates of users coincide with the arranged rates of the network, i.e. $x_s^u = x_s^a$, the prefer rate of user s is just x_s^a. Therefore,

$$\begin{aligned} U_s(x_s^a) - \lambda_s x_s^a &= U_s(x_s^u) - \lambda_s x_s^u \\ &> U_s(x_s) - \lambda_s x_s \end{aligned} \tag{15}$$

So (13) is satisfied in the case $x_s^u = x_s^a$.

If $x_s^u \neq x_s^a$, two cases need to discuss.

i) $x_s^u > x_s^a$. Denote by $V_s(x_s)$ the entire utility function of user s in problem $USER_s(x_s; q_s)$. Substituting (9) into $V_s(x_s)$, and calculating the first and second derivatives of (7) with respect to x_s, so

$$\begin{aligned} V_s(x_s) &= U_s(x_s) - \lambda_s x_s \\ &- \frac{U_s'(x_s^a) - \lambda_s}{x_s^a}(x_s - x_s^a)x_s \end{aligned} \tag{16}$$

$$\begin{aligned} V_s'(x_s) &= U_s'(x_s) + U_s'(x_s^a) - 2\lambda_s \\ &- \frac{2(U_s'(x_s^a) - \lambda_s)}{x_s^a}x_s \end{aligned} \tag{17}$$

$$V_s''(x_s) = U_s''(x_s) - \frac{2(U_s'(x_s^a) - \lambda_s)}{x_s^a} \tag{18}$$

From (17), one can see $V_s'(x_s^a) = 0$. Since $U_s(x_s)$ is a concave function, so $U_s''(x_s) < 0$ for all x_s. Also from (14), $U_s'(x_s^a) - \lambda_s > 0$. Then one can see that $V_s''(x_s) < 0$. From $V_s'(x_s^a) = 0$, and $V_s''(x_s) < 0$, one can make the conclusion that $V_s(x_s^a) > V_s(x_s)$. i.e. Eq. (13) holds.

In this case, if user s deflected from x_s^a, he would like to tend to x_s^u rather than the other direction. *i.e.* $x_s > x_s^a$.

ii) $x_s^u < x_s^a$. Actually, it could not occur in this case. Since $U_s(x_s)$ is an increasing function, its optimum must be at the right border of its region, if it has a finite region.

3.2 Numerical Example and Geometric Illustration

The example from Kelly's (1997) work is used. Suppose that each source-sink s is served by a single route r, and abbreviate notation by writing $s = r = 1$, rather than $s = \{r\}$; thus $H = I$, the identity matrix. Suppose also that $U_s(x_s) = m_s \log x_s$. Let the finite capacity is $C = 10$. Then $\lambda_s = m_s/10$.

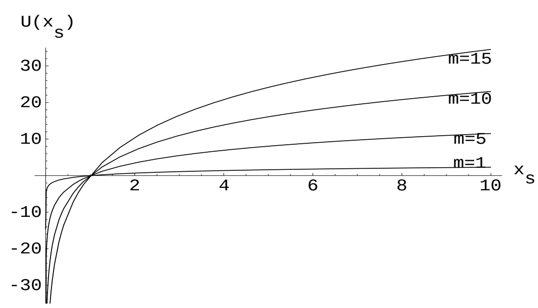

Fig. 1. Curves of the functions $U(x_s) = m_s \log x_s$ for problem (2)

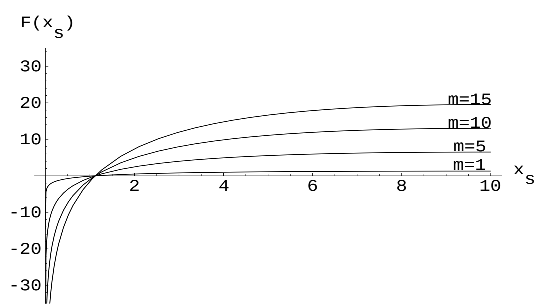

Fig. 2. Curves of the functions $F(x_s) = m_s \log x_s - \lambda_s x_s$ for problem (1)

Figure 1, 2, and 3 give out the curves of the functions with different m_s ($m_s = 1, 5, 10, 15$) for problems $SYSTEM(U, H, A, C)$, $USER_s(U_s; \lambda_s)$ and $NETWORK(H, A, C; \lambda_s)$, respectively. From those curves, one can see that the optimal points for those three problems are the right end of the

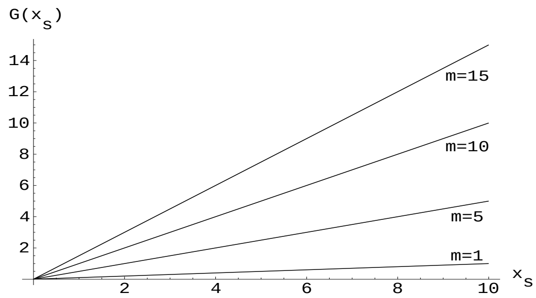

Fig. 3. Curves of the functions $G(x_s) = \lambda_s x_s$ for problem (4)

interval $(0, C]$, no matter how the value of m_s is, i.e. $x_s^u = C = 10$. If the leader won't want the users act at x_s^u but another point such as $x_s^a \neq x_s^u$ for some reason, the incentive strategy should be taken.

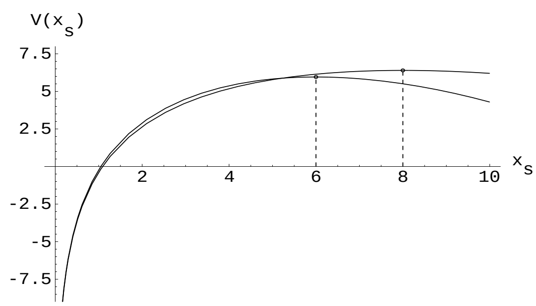

Fig. 4. Curves of the functions $V(x_s) = m_s \log x_s - \lambda_s x_s - q_s(x_s - x_s^a)x_s$ for problem (7)

Figure 4 gives out the result of $USER_s(U_s; q_s)$ with $m_s = 5$ and $\lambda_s = 0.5$ for two different points of x_s^a, respectively. One is $x_s^a = 8$, the other is $x_s^a = 6$. In each case, the maximum is really at x_s^a. For $x_s^a = 6$, a contour illustration is given in Figure 5 from which one can see the optimal point clearly. In Figure 5, the set of curves is the contour of the function for user s in problem $USER_s(U_s; \lambda_s)$ and the line with tangent $q_s = 1/18$ is the incentive strategy. One can see that the maximal value of the function of the user along this line is got at $x_s = x_s^a = 6$ where is the tangent point of the line and the contour curves.

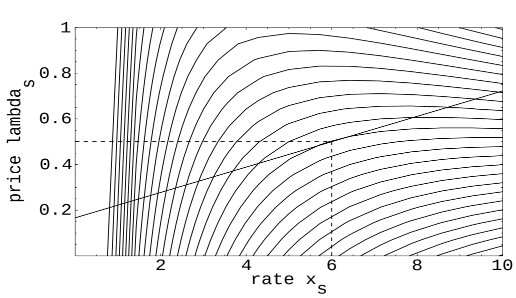

Fig. 5. Contour illustration of the optimal problem

3.3 Non-linear Incentive Strategy

Choose the function as

$$p_s(x_s) = \begin{cases} \dfrac{\lambda_s(x_s^a - x_s)}{x_s} & \text{if } x_s < x_s^a \\ 0 & \text{if } x_s = x_s^a \\ \dfrac{U_s(x_s) - U_s(x_s^a)}{x_s} & \text{if } x_s > x_s^a \end{cases} \quad (19)$$

It is easy to see that (5) becomes $\xi_s(x_s) = \lambda_s$, if $x_s = x_s^a$. It is just the first condition on an incentive strategy in Stackelberg game theory (Basar and Olsder 1982). To meet the second condition, substitute (5) into (1) with the structure (19). The problem $USER_s(U_s; \lambda_s)$ becomes

$USER_s(U_s; p_s(\cdot))$:

$$\begin{gathered} \text{maximize } W_s(x_s) \\ \text{over } x_s \geq 0 \end{gathered} \quad (20)$$

where $W_s(x_s) = U_s(x_s) - \lambda_s x_s - p_s(x_s)x_s$.

Consider it in both cases of $x_s < x_s^a$ and $x_s > x_s^a$, respectively.

i) $x_s < x_s^a$. In this case, $p_s(x_s) = \lambda_s(x_s^a - x_s)/x_s$. Then

$$W_s(x_s) = U_s(x_s) - \lambda_s x_s^a \quad (21)$$

So, one can see $W_s(x_s) < W_s(x_s^a)$ for $U_s(x_s) < U_s(x_s^a)$.

ii) $x_s > x_s^a$. In this case, $p_s(x_s) = (U_s(x_s) - U_s(x_s^a))/x_s$. Then

$$W_s(x_s) = U_s(x_s^a) - \lambda_s x_s \quad (22)$$

So, one can see $W_s(x_s) < W_s(x_s^a)$ for $\lambda_s x_s > \lambda_s x_s^a$.

In both cases, it is shown that $W_s(x_s^a) > W_s(x_s)$ which indicates the satisfaction of the second condition for incentive strategy.

The same example is used again here to illustrate how the non-linear incentive strategy works on forcing users to act at x_s^a. Let again $U_s(x_s) = m_s \log x_s$. According to (19), $\xi_s(x_s) = \lambda_s + \lambda_s(x_s^a - x_s)/x_s$ when $x_s < x_s^a$, $\xi_s(x_s) = \lambda_s + m_s(\log x_s - \log x_s^a)/x_s$ when $x_s > x_s^a$, and $\xi_s(x_s) = \lambda_s$ when $x_s = x_s^a$. Figure 6 gives out the result in the contour curves, where $m_s = 5$, $\lambda_s = 0.5$ and $x_s^a = 6$. The folding curve is the non-linear incentive strategy $\xi_s(x_s)$. one can see that, along the curve, the maximal point of $m_s \log x_s - \lambda_s x_s$ is at $x_s = 6$ and $\lambda_s = 0.5$.

Compare with Figure 5, one can see that non-linear incentive strategy is stronger than the linear one. It means that if the user deflected from x_s^a, he would be punished much more under the non-linear incentive strategy than under the linear

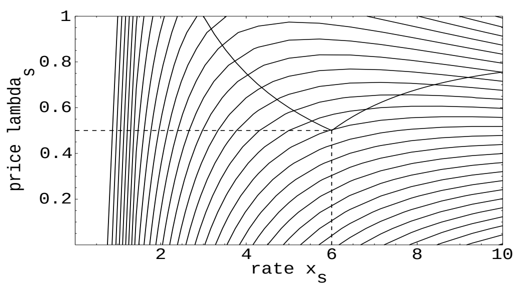

Fig. 6. Contour illustration of the optimal problem with non-linear incentive strategy

incentive strategy. But the linear strategy is easier to be realized than the nonlinear one.

4. INCENTIVE STRATEGY FOR NON-ELASTIC TRAFFIC PROBLEMS

In previous sections, the users' utilities $U_s(x_s)$ have an increasing character. By the assumption, one can easily determine the optimum of problem $USER_s(U_s;\lambda_s)$ and/or $SYSTEM(U,H,A,C)$.

However, in a practical network, the delay can be faced very often. The more the traffic rate is closed to the capacity of resources, the higher the delay will be. Therefore, the utility function can not be always increasing. In this section, the utility function $U_s(x_s)$ is concave but no longer be increasing. In such conditions, the parameter λ_s in problem $USER_s(U_s;\lambda_s)$ can not be the Lagrange multiplies of problem $SYSTEM(U,H,A,C)$, because it is possible that the optimum of problem $SYSTEM(U,H,A,C)$ can be got in the inner of the set $\mathbf{X}$.

$$\mathbf{X}=\{(x,y)\mid Hy=x, Ay\le C, x,y\ge 0\} \quad (23)$$

As known, the lagrange multipies of such problem should be zero and can not be taken as the price for per unit flow.

So $\lambda_s > 0$ must be considered now as a regular price determined by the network. Assume that x_s^* is the optimal rate for problem $SYSTEM(U,H,A,C)$, i.e. $U_s^{'}(x_s^*)=0$. And assume also that x_s^u is the optimal rate for problem $USER_s(U_s;\lambda_s)$, i.e. $U_s^{'}(x_s^u)-\lambda_s=0$. It is obvious that $x_s^*\neq x_s^u$ if $\lambda_s\neq 0$. The problem here is to find an incentive strategy to force users to act at the point x_s^* rather than x_s^u. Use the linear function here again, under which problem $USER_s(U_s;\lambda_s)$ becomes

$USER_s(U_s;p_s)$:

$$\text{maximize } U_s(x_s)-\lambda_s x_s - q_s(x_s-x_s^*)x_s \quad (24)$$

$$\text{over } x_x\ge 0$$

By the following steps, one can determine what q_s should be.

i) Calculate the derivative of (24) with respect to x_s, and let it be zero, *i.e.*

$$U_s^{'}(x_s)-\lambda_s-2q_s x_s+q_s x_s^*=0 \quad (25)$$

ii) Let x_s take the value at x_s^*. So

$$q_s=-\frac{\lambda_s}{x_s^*} \quad (26)$$

It can be easily shown that it is an incentive Stackelberg strategy under the condition

$$\lambda_s< -\frac{U_s^{''}(x_s^*)x_s^*}{2}. \quad (27)$$

Eq. (27) means that the regular price should be determined in a reasonable range.

To illustrate the result, let $U_s(x_s)=m_{s1}\log x_s + m_{s2}\log(C-x_s)$. one can see that $U_s(x_s)$ is nolonger increasing always in $(0,C]$. The example is just the extension of that in (?).

Figure 7 and Figure 8 give out the non-increasing concave utility functions $U_s(x_s)$ and the functions $U_s(x_s)-\lambda_s x_s$ of users with different m_{s1} and m_{s2}. The three curves in each of Figure 7 and Figure 8 are related to three cases (a) $m_{s1}>m_{s2}$, (b) $m_{s1}=m_{s2}$, (c) $m_{s1}<m_{s2}$, as (m_{s1},m_{s2}) takes values at $(7,3),(5,5),(3,7)$, respectively. From Figure 7, one can see that $\arg\max U_s(x_s)$ is in the open set $(0,C)$, *i.e.* x_s^*, in the three cases, are $7,5,3$, respectively, while $U_s^{''}(x_s^*)$, $x_s^*=7,5,3$, are $-10/21,-2/5,-10/21$, respectively. And λ_s can be chosen to be less than $5/7$, 1, $5/3$ in these three cases.

From Figure 8, however, one can see that $x_s^u=\arg\max[U_s(x_s)-\lambda_s x_s]$ are 5.78046, 3.81966 and 2.15477. It indicates that users prefer x_s^u (for instance, 5.78046 in the case $m_{s1}=7$) rather than x_s^* $(=7)$. It can also be seen in Figure 9 where the line $\xi_s(x_s)=0.5$ is just tangent to contour of the function $U_s(x_s)-\lambda_s x_s$ at $x_s=5.78046$. Note that $\xi_s(x_s)=0.5$ means that the regular price is $\lambda_s=0.5$ and the punishment price is null. Along line $\xi_s(x_s)=0.5$, the maximum point is just $x_s=5.78046$. It means that users would rather take $x_s^u=5.78046$ rather than $x_s^*=7$, if there were no punishment price in the strategy.

Figure 10 gives out the geometric illustration of the incentive strategy. The line of the incentive function is just tangent to the contour of the utility function $U_s(x_s)-\lambda_s x_s$ at $x_s=7$. Along the line, the maximum point is just $x_s=7$. So, under the incentive strategy (indicated by the line), users have to choose the rate $x_s^*=7$. If they still choose the rate $x_s^u=5.78046$ they prefer, their outcomes will be less than at $x_s^*=7$.

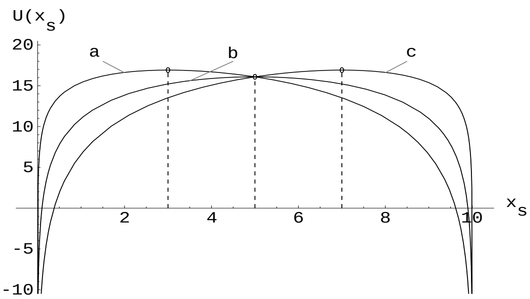

Fig. 7. Non-increasing concave utility functions $U(x_s) = m_{s1} \log x_s + m_{s2} \log(C - x_s)$ and their optimum

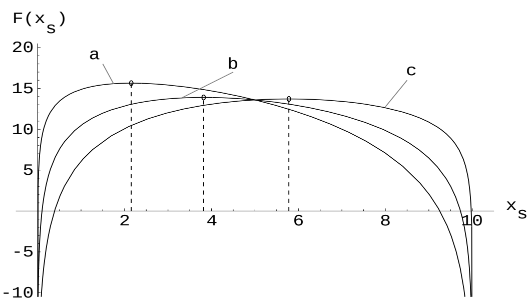

Fig. 8. Users' non-increasing concave functions $F(x_s) = m_{s1} \log x_s + m_{s2} \log(C - x_s) - \lambda_s x_s$ and their optimum

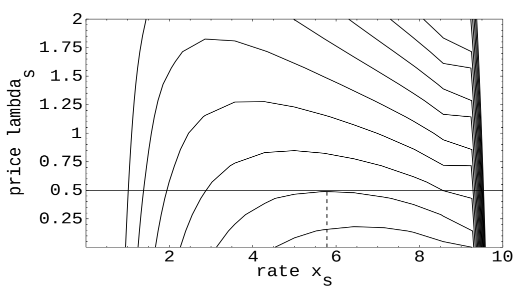

Fig. 9. Contours for problem (1)

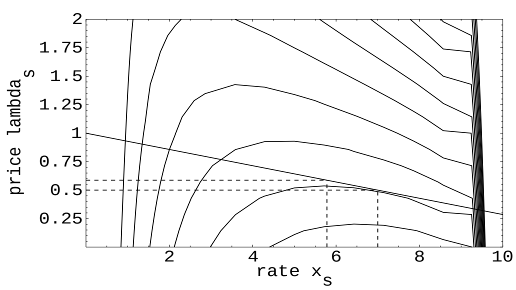

Fig. 10. Contours for problem (1) with non-increasing concave utility function and the linear incentive strategy

5. CONCLUSIONS

In this paper, the traffic rate control problem for a kind of network systems is discussed. The networking models based on elastic and non-elastic traffic are considered and the valid incentive Stackelberg strategies are proposed. It is a quite new way that the networking traffic control problem is dealt with by using the tool of game theory.

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