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STRUCTURAL IDENTIFICATION METHOD OF SIMULTANEOUS EQUATIONS SYSTEMS

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The objective of this paper is to develop the method for selection of the optimal structure of simultaneous equation system from the given model set in the condition of the limited sample of observations. The identification purpose is the model selection with the best predictive possibility. The information criteria for selecting the best model of nonlinear simultaneous equation system from the given models set are offered. The selection properties of these criteria are investigated by Monte-Carlo simulations. *Copyright © 2002 IFAC.*

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1. PROBLEM STATEMENT

The objective of this research is to find a method for selection the best model of nonlinear simultaneous equations system for the small samples of data. This problem is very significant for building and analyzing the models of economic processes in the countries with transition economics, for example in Ukraine, which are characterized by the short period of investigation and the sharp dynamic of economic processes.

The model to be considered is a system of m nonlinear simultaneous equations (Amemiya, 1986)

$$y_{it}=\eta_{it}(\mathbf{y}_t,\mathbf{x}_t,\mathbf{a}_i) + u_{it},\ \ i=1,2...,m,\ t=1,2...,N, \qquad (1)$$

where $\eta_{it}^T=(\eta_{1t}, \eta_{2t},\ldots,\eta_{mt})$ - is the true but an unknown m-vector of models; $\mathbf{y}_t^T=(y_{1t},y_{2t},\ldots,y_{mt})$ - is a m-vector of dependent (endogenous) variables, $\mathbf{x}_t^T=(x_{1t},x_{2t},\ldots,x_{Kt})$ - is a k-vector of independent or controlled input (exogenous) variables, $\mathbf{a}_i^T=(a_{1i},a_{2i},\ldots,a_{pi})$ - is a p_i vector of unknown parameters in a i-th structural equation, and $u_{it}^T=(u_{1t},u_{2t},\ldots,u_{mt})$ - is a m-vector of independent normally distributed random disturbances with zero mean and a variance-covariance matrix Σ_u, N -is the total number of observations. There is usually some prior information about the regions of possible values for variables: $\mathbf{y}\in W_1$ and $\mathbf{x}\in W_2$, where W_1 and W_2 are sets of possible values of the vectors **y** and **x**.

It is necessary to define the optimal for prediction structure of simultaneous equation system and find the estimations of vectors of unknown parameters $\mathbf{a}_i$, i.e. structural coefficients of the system (1) by results of N observations over vectors of endogenous variables **y** and exogenous variables **x**.

2. METHOD OF STRUCTURAL IDENTIFICATION

It is proposed to use the special method for solving this problem:

1. The special case in which the possible models are nested as in polynomial regression models or moving-average models for time series is considered. Let the nested set of models be denoted by

$$\eta_{ij}(\mathbf{y},\mathbf{x},\mathbf{a}_{ij})\in S_j\ , \qquad (j = 1, 2,..., q), \qquad (2)$$

where $\mathbf{a}_{ij}$ - is a vector of parameters in a i-th structural equation of j-th class and $S_1\in S_2\in \ldots \in S_q$, S_j - being the set of all possible models for class j.

For models which are linear in the parameters, model (2) can be rewritten as

$$\eta_{ij}(\mathbf{y},\mathbf{x},\mathbf{a}_{ij})=\mathbf{f}_{ij}^{\mathbf{T}}(\mathbf{y},\mathbf{x})\bullet\mathbf{a}_{ij}, \qquad (3)$$

where $\mathbf{f}_{ij}(\mathbf{y},\mathbf{x})$ is a vector of known functions.

2. The criteria for selecting the best model of nonlinear simultaneous equation system from the given models set are defined. It should be pointed that in this case we consider the system of simultaneous equation and no one criterion from existing set is appropriate.

3. The model for which criterion is minimum is selected for making predictions.

The main problem in that case is the selection of the appropriate criterion. In this paper the following two criteria for the model selection are investigated:

$$Cr = N\ln(\mathrm{tr}R) + 2\sum_{i=1}^{m} p_i\ , \qquad (4)$$

and $$Crn = N\ln(\det R) + 2\sum_{i=1}^{m} p_i \,, \qquad (5)$$

where *tr* - is the trace of matrix, *det* - is the determinant of matrix, *R* - is the matrix of residues for all equations, where the elements of the main diagonal are RSS_i - the residues sum of squares for each equation and the rest elements are covariance between the residues of equations.

This criteria are the generalization of the Akaike's information criterion $AIC_i=N\ln(RSS_i)+2p_i$ for single equation (Akaike, 1972; Hashimoto, et al., 1981). The optimal model is one for which the criterion is minimum.

It is required to examine the efficiency of these two criteria like the instrument of the optimal model selection. The investigation of these criteria was carried on by the method of statistical examinations (Monte-Carlo simulations).

Let us consider the structural system and its reduced form. The structural model consisted of two equations with two changing parameters **a** and **b** and the rest parameters are given:

$$\begin{cases} y_1 = \eta_1(x_1, y_2) + u_1 = 1 + 0.5x_1 + ax_1^2 + 0.2y_2 + u_1 \\ y_2 = \eta_2(x_2, y_1) + u_2 = 2 + 0.4x_2 + bx_2^2 + 0.7y_1 + u_2 \end{cases} . \qquad (6)$$

The system (6) is identified, because the order condition is satisfied for every equation of the system (Aivazian and Mkhitarian, 1998).

The reduced form of given structural model can be written by:

$$\begin{cases} y_1(a,b) = 1.63 + 0.58x_1 + 1.16ax_1^2 + 0.09x_2 + 0.23bx_2^2 + v_1 \\ y_2(a,b) = 3.14 + 0.4x_1 + 0.81ax_1^2 + 0.46x_2 + 1.16bx_2^2 + v_2 \end{cases} \qquad (7)$$

where v_1, v_2 - are the normally distributed random disturbances with mean 0 and variance 1.

Experimental data were simulated by the following way: exogenous variables x_1 and x_2 took only discrete values in N=12 points with the next designs of the experiments, x_1=[0.5, 0.5, 0.7, 0.7, 0.9, 0.9, 1.1, 1.1, 1.3, 1.3, 1.5, 1.5], x_2=[1.9, 1.8, 1.7, 1.6, 1.5, 1.4, 1.3, 1.2, 1.1, 1.0, 0.9, 0.8]; the random disturbances u_1 and u_2 were simulated by the generator of random numbers built in computer program; the parameter a was changed from 1 to 15, b - from 1 to 20.

The selection of model structure was simulated by the criteria mentioned above for every realization of the experimental data. The experiment was repeated one thousand times. The parameters were estimated by two stage least squares method (2SLS) for every random sample k =1...1000 because it provides the consistent estimates of structural system coefficients (Aivazian and Mkhitarian, 1998; Amemiya, 1986). The selection was made from the following parametric classes of relationships η:

a) linear, i.e. the quadratic terms were excluded from the initial system (6);

b) quadratic (true), i.e. the structure of the initial system was remained the same;

c) cubic, i.e. the exogenous variables in a third power were added to each equation of the initial system.

The mean square error of prediction for the i-th equation of the system is

$$e_{it}^2 = (y_{it} - \eta_{it})^2 . \qquad (8)$$

The general error of prediction for the system of equations is

$$Is = tr\Sigma_e \quad \text{and} \quad Isn = \det \Sigma_e \,, \qquad (9)$$

where

$$\Sigma_u = \begin{pmatrix} \sigma_1^2 & cov_{12} & \cdots & cov_{1m} \\ cov_{21} & \sigma_2^2 & & \vdots \\ \vdots & & \ddots & \vdots \\ cov_{m1} & \cdots & & \sigma_m^2 \end{pmatrix} \qquad (10)$$

$\sigma_i^2 = M\{e_i^2\}$ – variance of prediction error for i-th equation; $cov_{ij} = M\{e_i*e_j\}$ (i,j=1..m) – covariance of prediction errors for i-th and j-th equations.

The expectation of a mean squared error of prediction (EMSEP) is used for evaluation of efficiency of the structural identification method:

$$Rs(a,b) = \sum_{l=1}^{q} vs_l * Is_l(a,b) \,, \qquad (11)$$

where l=1…q - is the number of selected system (1-linear, 2- quadratic, 3- cubic); vs_l - is the probability of the model l selection by criterion Cr (or Crn); $Is_l(a,b)$ (or $Isn_l(a,b)$) - is the loss function for the l-th model.

Figures 1,2 show the dependencies of EMSEP on the parameters a and b for the structural system, obtaining by simulation experiments. Figure 1 shows this dependency by the criterion Cr, and figure 2 - by the criterion Crn. Figures 3,4 show the dependencies of EMSEP for the reduced system by the criteria Cr and Crn respectively.

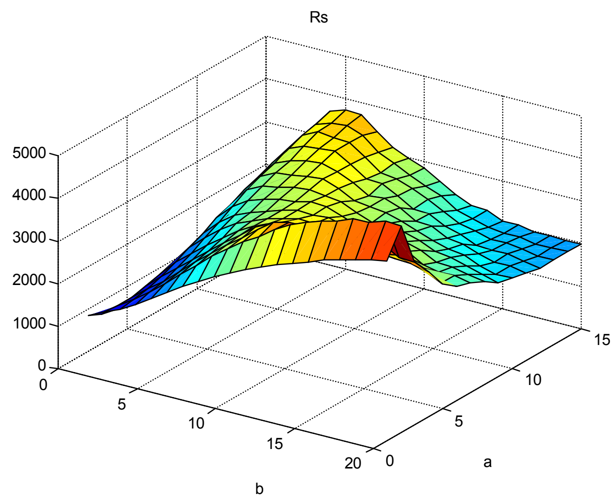

Fig.1 - EMSEP for the structural system by the criterion Cr

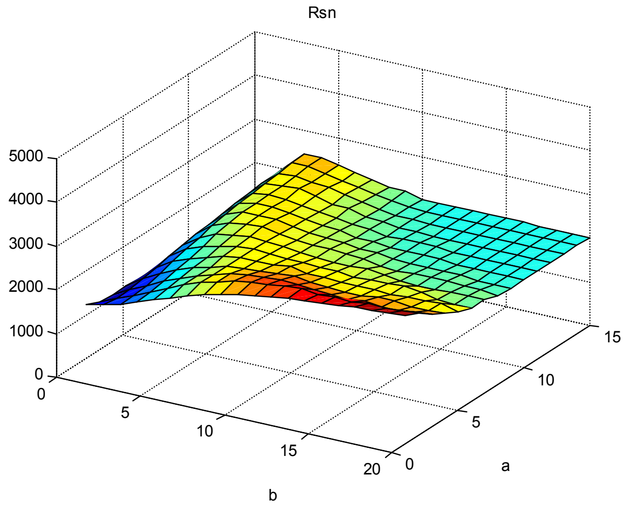

Fig.2 - EMSEP for the structural system by the criterion Crn

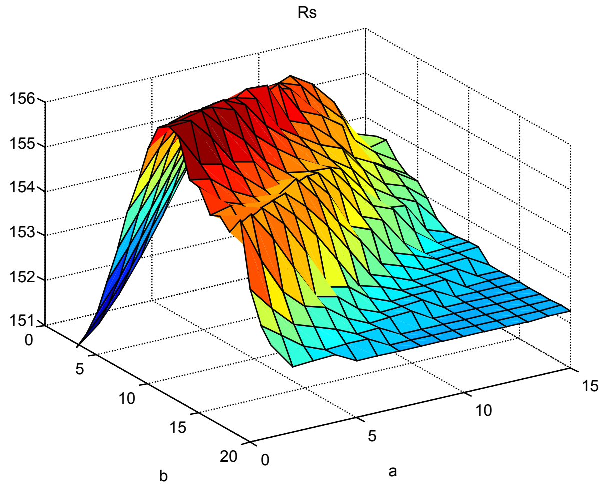

Fig.3 - EMSEP for the reduced system by the criterion Cr

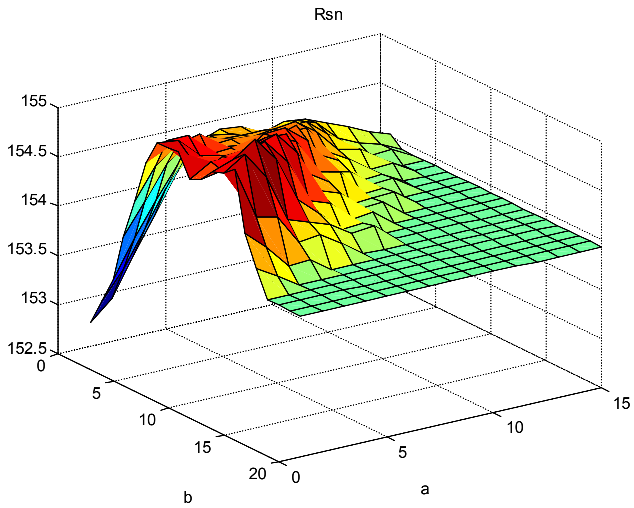

Fig.4 - EMSEP for the reduced system by the criterion Crn

Figures 5,6 show the dependencies of probability of the true model selection on the parameters a and b by the criteria Cr and Crn for structural and reduced systems respectively.

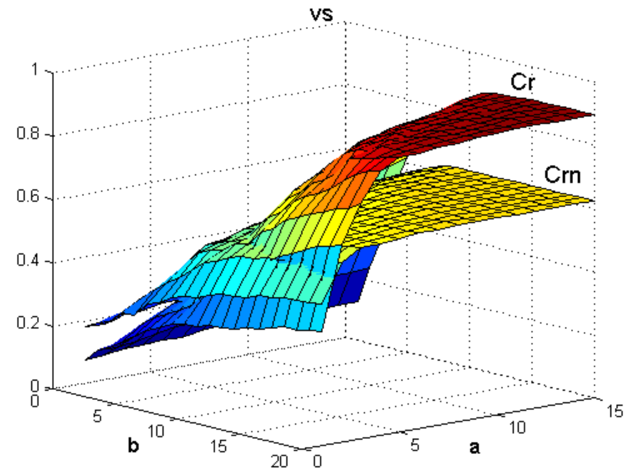

Fig.5 - Probability of the true structural system selection by the criteria Cr and Crn

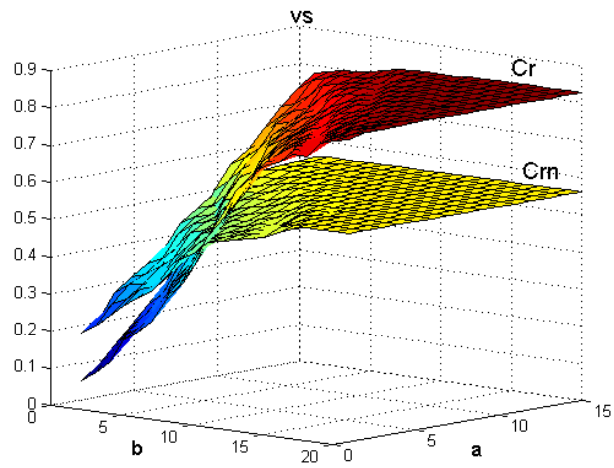

Fig.6 - Probability of the true reduced system selection by the criteria Cr and Crn

It can be seen from figures 1,2 that maximum value of EMSEP for the structural form is smaller by the criteria Crn. From the point of view of the speed of convergence to the true model the criterion Crn is more preferable. The same result can be seen for reduced system by analyzing figures 3,4.

Also the results of experiments (displayed in figures 5,6) demonstrating that the criterion Crn is better than the criterion Cr under small values of the parameters a and b and Cr is more efficient than Crn under big values of this parameters for both the structural and reduced forms of the model.

The next selection properties of proposed method can be formulated:

1. Method allows to select the optimal model for prediction from the given models set.
2. Method can be used for the selection both the structural systems of simultaneous equation and their reduced forms.
3. For the purposes of the minimization of the prediction error and the speed of convergence to the true model the criterion Crn is more preferable than Cr;
4.The criteria efficiency depends on the region of the true model parameters variation.
5. Proposed criteria behave similarly to AIC for single equation.

3.CONCLUSIONS

It is necessary to carry out the further theoretical investigation about the predicting possibility of the structural and the reduced forms of simultaneous equation system. Moreover it would be valuable to derive the analytical expressions for the general error of prediction of the system for verification of the simulation experiments results.

The proposed method and criteria with selection properties pointed above have been used for selecting the optimal model structure of the Sevastopol region economics and good results are received in the conditions of the limited sample of observations (Gorobets, 1999).

The results of this research allow one to have wide applicability of the new method in the many fields of modeling, for example, the macroeconomic modeling for changing society, the analysis of transformations in the economics and the government forecasting and planning.

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