

## INCENTIVE STACKELBERG STRATEGIES FOR FLOW CONFIGURATION OF PARALLEL NETWORKS

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Abstract: This work consider the problem of flow control using incentive strategy in Stackelberg game theory. The network model employed here is that users route their flows from a common source to a common destination node, each of them trying to optimize its individual performance objective. First, the existing Stackelberg routing strategy is briefly introduced. And then, the linear Stackelberg incentive strategies are presented for both single-follower and multi-follower systems, by which the leader (manager) force the followers (the noncooperative users) adopt the team optimal flow configuration as their reply strategies. It is shown that the incentive strategy improves the existing Stackelberg routing strategy. A numerical example is given to illustrate the results of the strategy.

Keywords: Game Theory, Parallel network, Flow configuration, Optimum Routing, Stackelberg Strategy

### 1. INTRODUCTION

The modern telecommunications networks are examples of large-scale complex systems that carry a wide variety of traffic classes, serve many users, and may appear in various structures. System of parallel links represents an appropriate model for seemingly unrelated networking problems. Consider, for example, a network in which resources are pre-allocated to various routing paths that do not interfere. Such case is common in modern networking. In broadband networks, bandwidth is separated among different virtual paths,

resulting effectively in a system of parallel and non-interfering “link” between source/destination pairs. Another example is that of internet working, in which each “link” models a different sub-network.

Traditional centralized control schemes do not scale well as the size of the network increases. Thus, control of modern networks is usually performed in a decentralized fashion. The network, generally, is shared by a set of noncooperative users, each sending its flow in a way that optimizes its individual performance objective. Control of user flows is decentralized, that is, each user is responsible for controlling its own flow. Control decisions are made independently and the users do not make joint control decisions. Each user has its

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individual performance objective to be optimized. The self-optimizing behavior of the users leads to a dynamic behavior of the network, which is the so-called noncooperative networking (see Korilis 1995) that applies to various practical networks. The most common example of noncooperative network is the Internet (see Jacobson 1988).

Game theory (see Basar and Olsder 1982, Myerson 1991) provides the systematic framework to treat the dynamic behavior of noncooperative networks. During the last decade, the area of networking games has been receiving increasing attention due to the realization that single-user models are not sufficient to capture the complex interactions among various network controllers. The study of noncooperative networks can be categorized as network control, network design, and network management problems (Korilis 1995).

Two major concepts in game theory, Nash equilibria and Stackelberg equilibria, have been employed into the study of noncooperative networks. For Nash equilibria, among many others, Orda, Rom and Shimkin (1993) dealt with the optimal capacity allocation problem. Bertsekas and Gallager (1992) gave out a simple and intuitive solution in the single-user case. And then Korilis (1995) generalized this result to the case of noncooperative routing. Meanwhile, Stackelberg equilibria strategies have also been investigated in the context of flow control (Douligeris and Mazumdar 1989), and routing (Economides and Silvester 1990). In these references, however, the leader in the Stackelberg models was a selfish user concerned about its own rather than the system's performance. Korilis (1995, 1996) emphasized the overall network performance and the interests of followers excessively, while did not attempt to optimize the cost of the leader.

The main contribution of this paper is that a novel strategy has been proposed, by introducing the incentive concept of game theory, to ensure the achievement of team equilibria of the entire network system while the benefit of manager is considered. The method presented in this paper improves the existing Stackelberg routing strategy introduced by Korilis (1995, 1996).

## 2. MAIN SYMBOLS AND GENERAL ASSUMPTIONS

Consider the model of one source node and one destination node with several links between them, which is shared by several users who send their data flow on links from source to destination. For convenience, the main symbols used in the paper are introduced.

$i = 0, 1, \dots, I$  - The index of network users

$l = 1, \dots, L$  - The index of communication links

$c_l$  - The allocated capacity of link  $l$

$C = \sum_{l=1}^L c_l$  - The total capacity of the network

$\mathbf{c} = (c_1, \dots, c_L)$  - The capacity configurations

$r^i$  - The average rate for demand of user  $i$

$R = \sum_{i=0}^I r^i$  - The total demand of all users

$R^{-i} = \sum_{j=0, j \neq i}^I r^j$  - The total demand of all users except for user  $i$

$f_l^i$  - The expected flow of user  $i$  sends on link  $l$

$\mathbf{f}_l = (f_l^0, f_l^1, \dots, f_l^I)$  - The flow vector of all users on link  $l$

$f_l = \sum_{i=0}^I f_l^i$  - The total flow of all users on link  $l$

$\mathbf{f}^i = (f_1^i, \dots, f_L^i)$  - The routing strategy vector of user  $i$  on each link

$F^i = \{\mathbf{f}^i \in R^L : 0 \leq f_l^i \leq c_l, \sum_{l=1}^L f_l^i = r^i\}$  - The strategy space for users  $i$

$\mathbf{f} = (\mathbf{f}^0, \dots, \mathbf{f}^I)$  - The routing strategy profile of all users

*Assumption 1.* The following assumptions are made.

1.  $c_1 \geq c_2 \geq \dots \geq c_L$ , 2.  $r^1 \geq r^2 \geq \dots \geq r^I$ ,
3.  $C > R$ , 4.  $\sum_{l=1}^L f_l = R$ .

*Remark 2.* The capacity configuration  $\mathbf{c}$  depends on the actual link capacity and not on the link "labels", so one can rearrange the link "labels" to satisfy the assumption. The second assumption only involves the throughput demands of user  $i$ ,  $1 \leq i \leq I$ . User 0 will be considered as a manager in the Stackelberg strategy problem.  $C > R$  is assumed for the stability of the overall system.

## 3. PROBLEMS

The main problem here is to minimize the total cost  $J(\mathbf{f})$  of the network. Due to its simplicity, the M/M/1 queueing model is concentrated. The time-delay function on link  $l$  is described as

$$T_l(f_l) = \begin{cases} \frac{1}{c_l - f_l}, & f_l < c_l \\ \infty & , f_l \geq c_l \end{cases}. \quad (1)$$

The cost function of user  $i$  sending flow on link  $l$  can be chosen as

$$J_l^i(f_l) = f_l^i T_l(f_l) = \frac{f_l^i}{c_l - f_l}, \quad \text{for } f_l < c_l. \quad (2)$$

In the case  $f_l \geq c_l$ , no matter how much flow user  $i$  send, the delay is infinity as well as the cost function.

The summation of  $J_l^i(f_l)$  on  $l$ , denoted by  $J^i(\mathbf{f})$ , is taken as the total cost function of user  $i$ , which is described as follows.

$$J^i(\mathbf{f}) = \sum_{l=1}^L J_l^i(f_l) = \sum_{l=1}^L \frac{f_l^i}{c_l - f_l}. \quad (3)$$

The total cost of the network is

$$\begin{aligned} J(\mathbf{f}) &= \sum_{i=0}^I J^i(\mathbf{f}) = \sum_{i=0}^I \sum_{l=1}^L \frac{f_l^i}{c_l - f_l} \\ &= \sum_{l=1}^L \frac{1}{c_l - f_l} \sum_{i=0}^I f_l^i = \sum_{l=1}^L \frac{f_l}{c_l - f_l}. \end{aligned} \quad (4)$$

which depends only on the link flow configuration  $(f_1, \dots, f_L)$ . It is evident that  $J(\mathbf{f})$  is a convex function of  $(f_1, \dots, f_L)$ . Therefore, there exists a unique link flow configuration  $(f_1^*, \dots, f_L^*)$  minimizing the total cost. Certainly, the conditions  $f_l^* \geq 0$  and  $\sum_{l=1}^L f_l^* = R$  must be satisfied. This is the team solution of the classical routing optimization problem. There is a necessary and sufficient condition to guarantee the optimization of  $(f_1^*, \dots, f_L^*)$ .

*Lemma 3.*  $(f_1^*, \dots, f_L^*)$ ,  $f_l^* \geq 0$ ,  $l = 1, \dots, L$ ,  $\sum_{l=1}^L f_l^* = R$ , is the network team optimum, if and only if there exist  $\lambda^*$  and  $\mu^* = (\mu_1^*, \dots, \mu_L^*)$ ,  $\mu_l^* \geq 0$  such that

$$\frac{\partial J(\mathbf{f})}{\partial f_l} \Big|_{f_l^*} - \lambda^* - \mu_l^* = 0, \quad \mu_l^* f_l^* = 0, \quad (5)$$

where  $\lambda^*$  and  $\mu^*$  are the Lagrange multipliers.

One can see that the conditions (5) in the above lemma are equivalent to the following:

$$\lambda^* = \frac{\partial J(\mathbf{f})}{\partial f_l} \Big|_{f_l^*}, \quad \text{if } f_l^* > 0, \quad (6)$$

$$\lambda^* \leq \frac{\partial J(\mathbf{f})}{\partial f_l} \Big|_{f_l^*}, \quad \text{if } f_l^* = 0. \quad (7)$$

From (4), the conditions (6) and (7) for M/M/1 queue model can be written as

$$\lambda^* = \frac{c_l}{(c_l - f_l^*)^2}, \quad \text{if } f_l^* > 0, \quad (8)$$

$$\lambda^* \leq \frac{1}{c_l}, \quad \text{if } f_l^* = 0. \quad (9)$$

Let  $\mathbf{f}^*$  be the routing strategy profile of all users according to the optimal flow configuration  $(f_1^*, \dots, f_L^*)$  satisfying (8) and (9). Then, for any strategy profile  $\mathbf{f} \in F$ ,  $J(\mathbf{f}^*) \leq J(\mathbf{f})$ .

Korilis (1995, 1996) presented the following structure of the network optimum  $(f_1^*, \dots, f_L^*)$ .

$$f_l^* = \begin{cases} c_l - \left( \sum_{n=1}^{L^*} c_n - R \right) \frac{\sqrt{c_l}}{\sum_{n=1}^{L^*} \sqrt{c_n}}, & l = 1, \dots, L^* \\ 0, & l = L^* + 1, \dots, L \end{cases}. \quad (10)$$

Note that the flow  $f_l^*$  on link  $l$  is decreasing in the link number  $l$ , i.e.  $f_1^* \geq \dots \geq f_L^*$  (Orda et al 1993). Therefore, there exists some link  $L^*$  such that  $f_l^* > 0$  for  $l \leq L^*$  and  $f_l^* = 0$  for  $l > L^*$ .

*Remark 4.* For given configuration  $(\tilde{f}_1, \dots, \tilde{f}_L)$ , it is evident that there is a routing strategy profile  $\tilde{\mathbf{f}}$  by which the network total cost is  $J(\tilde{\mathbf{f}})$ . Noting the structures of  $\mathbf{f}$ ,  $\mathbf{f}^*$  and  $\mathbf{f}_l$ ,  $f_l$ , it can be shown that there is a number of routing strategy profile  $\mathbf{f}$ , according to  $(\tilde{f}_1, \dots, \tilde{f}_L)$ , which make the network total cost  $J(\mathbf{f}) = J(\tilde{\mathbf{f}})$ . Those strategy profile  $\mathbf{f}$  are collected in the set

$$\tilde{F} = \left\{ \mathbf{f} \in F : \sum_{i=0}^I f_l^i = \sum_{i=0}^I \tilde{f}_l^i \quad \text{for } \forall l \right\}. \quad (11)$$

The case mentioned in Remark 4 is applicable to  $(f_1^*, \dots, f_L^*) \in F^*$ . This case, which was not pointed out by Korilis (1995, 1996), is very important in the discussion of Stackelberg strategies.

#### 4. STACKELBERG MODEL OF THE FLOW CONFIGURATION PROBLEMS

As remarked in Section 3, for the network optimal flow configuration  $(f_1^*, \dots, f_L^*)$ , there exists a set of strategy profiles. How to achieve the network optimum? Which is the best profile in  $\tilde{F}$ ? Korilis (1996) provided the answer to the first question, by means of the Stackelberg routing games.

Consider the simplest case of a Stackelberg routing game, where the network is shared by a single self-optimizing user and a user who aims to optimize the overall network performance by achieving the network optimum, and has knowledge of the behavior of other user.

##### 4.1 Stackelberg Routing Strategy

Assume that the network optimal flow configuration  $(f_1^*, \dots, f_L^*)$  has been obtained as in Section 3. The purpose is to realize it under some strategies of the users. If the users are in the cooperative situation, it is easy to achieve the optimum as action in team. In practical systems as well as in the Stackelberg models, however, the situation among the users (or between the leader and the followers) is noncooperative. The role of manager

is very important to control the entire network. His strategies will affect the overall system performance as well as each user. The manager should make such decision that the total flow sent on link  $l$  by both leader and follower is equal to the optimal configuration  $f_l^*$ ,  $1 \leq l \leq L$ . Korilis (1995, 1996) dealt with such problem on basis of routing strategy, i.e. the manager attempts to optimize the system performance through the control of its portion of the flow. In such case, when the leader takes his flow strategy on link  $l$  as  $f_l^{0*}$ , the follower's flow on link  $l$  which minimizes his own cost function must coincide with  $f_l^* - f_l^{0*}$ , denoted as  $f_l^{1*}$ . That is just the Stackelberg idea.

By using routing strategy, Korilis (1995, 1996) presented a Stackelberg strategy in the form as

$$f_l^{0*} = \begin{cases} c_l \frac{\sum_{n=1}^{L^1} f_n^* - r^1}{\sum_{n=1}^{L^1} c_n}, & l = 1, \dots, L^1, \\ f_l^*, & l = L^1 + 1, \dots, L \end{cases}, \quad (12)$$

where  $L^1$  is some link that is determined by  $r^1$  as follows. The flow  $f_l^1$  the follower sends on link  $l$  is decreasing in the link number  $l$ . Therefore, there exist some link  $L^1$  such that  $f_l^1 > 0$  for  $l \leq L^1$  and  $f_l^1 = 0$  for  $l > L^1$ , that is, the follower sends his flow precisely over the links in  $\{1, \dots, L^1\}$ . It is also evident that  $L^1 \leq L^*$ .

As the rational reaction of the follower to the strategy (12), the follower's sends his flow over the network system as

$$f_l^{1*} = \begin{cases} f_l^* - c_l \frac{\sum_{n=1}^{L^1} f_n^* - r^1}{\sum_{n=1}^{L^1} c_n}, & l = 1, \dots, L^1, \\ 0, & l = L^1 + 1, \dots, L \end{cases}. \quad (13)$$

The Stackelberg strategy pair  $(f_l^{0*}, f_l^{1*})$  makes the overall system performance optimal. There are, however, some issues which should be considered:

- (1) The leader has to send flows to links whose average time delays are higher, so as to "tame" the follower.
- (2) Under  $(f_l^{0*}, f_l^{1*})$ , the value of the leader's cost function is much larger. For the purpose of achieving  $f_l^*$ , the leader has to sacrifice its own throughput demand.
- (3) When the flow that the leader wants to send on the system is very small, he will not be able to control the system. Especially, if the manager is not the user of the system, i.e. he has no flow to send, how will he manage the overall system?

#### 4.2 Incentive Stackelberg Strategy

In this subsection, a new Stackelberg strategy for flow configuration will be proposed. At first, the

model should be improved. The cost function of the overall network is still the same as before. So the overall optimum is still  $(f_1^*, \dots, f_L^*)$ . As a manager, the leader in the network considers not only the overall network performance, but also the benefit of the users including himself. So the manager can choose any  $f$  from  $F^*$  as the proper routing strategy profile of the network. Suppose that the manager select  $f^\alpha \in F^*$ . Then, there are

$$\begin{aligned} \mathbf{f}^\alpha &= (\mathbf{f}^{0\alpha}, \dots, \mathbf{f}^{I\alpha}), & \mathbf{f}^{i\alpha} &= (f_1^{i\alpha}, \dots, f_L^{i\alpha}), \\ \mathbf{f}_l^\alpha &= (f_l^{0\alpha}, \dots, f_l^{I\alpha}), & f_l^\alpha &= \sum_{i=0}^I f_l^{i\alpha} = f_l^*. \end{aligned}$$

To be able to control the followers and "tame" them to  $f_l^{i\alpha}$ , the manager must possess the ability to affect the followers in some way. There are two ways to do so. One is to expand the manager's strategy as a function of the followers' strategies, by which the manager can adjust his strategy depending on the followers' actions. The other way is to put a price on the followers' cost functions. Both these two ways were introduced as the incentive mechanism.

To induct the followers act as the manager expects and punish them when they deflect from the point desired by the manager, the incentive strategy can be chosen as

$$f_l^0 = f_l^{0\alpha} + \sum_{i=1}^I g_i(f_l^i, f_l^{i\alpha}), \quad l = 1, \dots, L. \quad (14)$$

To elucidate both the intuition behind the structure of the incentive strategy and the methodology to derive it, the simplest case of two users (the leader and the follower) incentive Stackelberg routing game is dealt with here.

The simplest incentive strategy structure can be the linear form in  $g(\cdot, \cdot)$ , such as

$$f_l^0 = f_l^{0\alpha} + Q_l(f_l^1 - f_l^{1\alpha}), \quad l = 1, \dots, L, \quad (15)$$

where  $(f_l^{0\alpha}, f_l^{1\alpha})$  is the expected solution of the manager. It is evident that  $f_l^0 = f_l^{0\alpha}$  when  $f_l^1 = f_l^{1\alpha}$ , if (15) is available.

Consider the follower's cost function

$$J^1(\mathbf{f}) = \sum_{l=1}^L \frac{f_l^1}{c_l - f_l} = \sum_{l=1}^L \frac{f_l^1}{c_l - f_l^0 - f_l^1}. \quad (16)$$

Substituting (15) into (16), there will be

$$J^1(\mathbf{f}) = \sum_{l=1}^L \frac{f_l^1}{c_l - f_l^{0\alpha} - Q_l(f_l^1 - f_l^{1\alpha}) - f_l^1}. \quad (17)$$

If the follower chooses  $f_l^{1\alpha}$ , then

$$J^1(\mathbf{f}^{*\alpha}) = \sum_{l=1}^L \frac{f_l^{1\alpha}}{c_l - f_l^{0\alpha} - f_l^{1\alpha}} = \sum_{l=1}^L \frac{f_l^{1\alpha}}{c_l - f_l^*}, \quad (18)$$

where  $f^{*\alpha}$  indicates that  $f_l^{i\alpha}$ ,  $i = 0, 1$ ,  $l = 1, \dots, L$ , have been taken as the final flow configurations.

Now, it needs to be shown that there is a proper  $Q$  such that

$$J^1(\mathbf{f}) > J^1(\mathbf{f}^{*\alpha}), \quad (19)$$

for any  $f_l^1 \neq f_l^{1\alpha}$ .

There would be two cases for the follower to chose his flow configuration on link  $l$ :  $f_l^1 > f_l^{1\alpha}$  or  $f_l^1 < f_l^{1\alpha}$ , if he were not to obey  $f_l^1 = f_l^{1\alpha}$ .

*Theorem 5.* There exist some  $Q_l$  for (15) being a incentive Stackelberg strategy to make (19) hold, where  $Q_l$  possesses the following structure.

$$Q_l = \begin{cases} -1 & , \quad \text{if } f_l^{1\alpha} \leq f_l^1 < r^1 \\ -\frac{c_l}{f_l^{1\alpha}} & , \quad \text{if } 0 < f_l^1 < f_l^{1\alpha} \end{cases} \quad (20)$$

The proof is omitted.

Because of the fundamental assumption of Stackelberg game that the follower is a rational reaction player, there is no possibility for the follower to chose  $f_l^1 = 0$ .

#### 4.3 Multi-Follower Incentive Stackelberg Routing Game

The incentive Stackelberg strategy for flow configuration presented for one-leader one-follower system in Section 4.2 will be expanded to one-leader multi-follower system under the assumption of Nash equilibrium for the non-cooperative self-optimizing users who play the follower role in the routing game. Suppose also that the leader select  $f^\alpha \in F^*$ .

As the manager of the network system, the leader-user shall proposes proper strategy to force the follower-users adopting  $f_l^{i\alpha}$ ,  $i = 1, \dots, I$ ,  $l = 1, \dots, L$ , as their strategies in the flow configuration game. The linear incentive Stackelberg strategy will be employed here.

$$f_l^0 = f_l^{0\alpha} + \mathbf{Q}_l^T (\mathbf{f}_l - \mathbf{f}_l^\alpha), \quad l = 1, \dots, I, \quad (21)$$

where  $\mathbf{Q}_l = (Q_l^1, Q_l^2, \dots, Q_l^I)^T$  is parameter vector which should be determined such that (21) can force the follower-users acting as the manager expects.

In the case of one follower-user,  $\mathbf{Q}_l$  is a scale parameter and can be determined by making (19)

hold. In the multi-follower case, however, the complexity of formulations and calculations increases rapidly making the problem difficult. One reason is that the followers are non-cooperative and self-optimizing users. To overcome the difficulty and simplify the calculations, the Nash equilibrium should be introduced among them. Contrary with eq. (19), there should be

$$J^i(\mathbf{f}^0(\mathbf{f}^i), \mathbf{f}^i, \mathbf{f}^{-i\alpha}) > J^i(\mathbf{f}^{*\alpha}), \quad \text{for } \forall i, \quad (22)$$

where  $\mathbf{f}^{-i\alpha} = (f^{1\alpha}, \dots, f^{(i-1)\alpha}, f^{(i+1)\alpha}, \dots, f^{I\alpha})$  and  $\mathbf{f}^0(\mathbf{f}^i)$  indicates that  $\mathbf{f}^0$  is a function of  $\mathbf{f}^i$  as see in eq. (21) with  $\mathbf{Q}_l$  determined.

The following theorem states the result on this question.

*Theorem 6.* There exists some  $\mathbf{Q}_l$  for (21) being an incentive Stackelberg strategy to make (22) hold, where  $\mathbf{Q}_l = (Q_l^1, \dots, Q_l^I)^T$  possesses the following structure.

$$Q_l^i = \begin{cases} -1 & , \quad \text{if } f_l^{i\alpha} \leq f_l^i < r^i \\ -\frac{c_l}{f_l^{i\alpha}} & , \quad \text{if } 0 < f_l^i < f_l^{i\alpha} \end{cases} \quad (23)$$

## 5. NUMERICAL EXAMPLE

Consider a numerical example of a system of parallel links with capacity configuration  $\mathbf{c} = (12, 7, 5, 3, 2, 1)$ , shared by  $I = 100$  identical self-optimizing users, with total demand  $r$ , and the manager. This example was employed to illustrate the Stackelberg routing strategy by Korilis (1995, 1996) in which the performance of the network was under Stackelberg and Nesh scenaria. The example there showed that the network performance was always better under the Stackelberg scenario. The cost of the manager was, however, always higher under the Stackelberg scenario. The reason is that the Stackelberg routing strategy is only the open-loop strategy (see Basar *et al* 1982). Figure 1 shows that the cost of leader under the

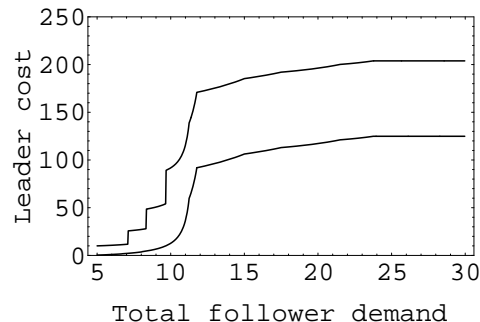


Fig. 1. Leader cost as a function of total follower demand, the low curve under the incentive Stackelberg strategy, the up curve under the Stackelberg routing strategy

proposed incentive Stackelberg strategy is much

less than that under the Stackelberg routing strategy. Figure 2 shows that the performance of the

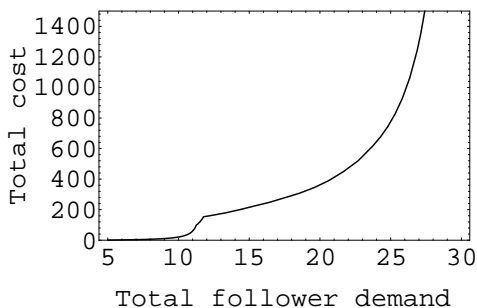


Fig. 2. Total cost as a function of total follower demand

network will achieve the network optimum that is the same as in Korilis's work.

Furthermore, making some change in this example, the network behavior could be shown clearly as the network capacity increases. Three capacity levels are considered in this example:  $\mathbf{c} = (18, 10.5, 7.5, 4.5, 3, 1.5)$ ,  $\mathbf{c} = (24, 14, 10, 6, 4, 2)$ ,  $\mathbf{c} = (36, 21, 15, 9, 6, 3)$ .

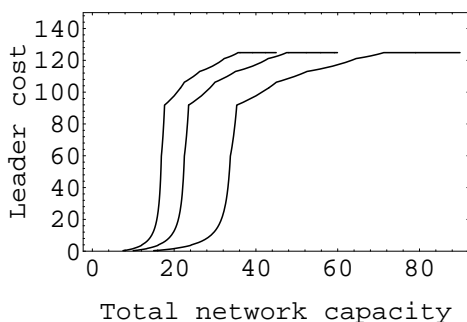


Fig. 3. Behavior of leader cost as capacity increased

From Figure 3, one can see that, the limit of the leader's cost function will be a constant number, for instance, 124.844 here, when the total follower demand is closed to the total capacity. The more the capacity is, the later the leader cost achieves the limit. Therefore, the leader's cost can be kept under the limit.

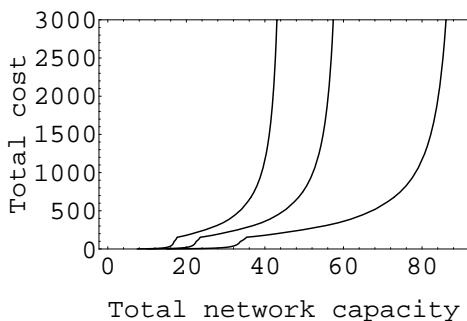


Fig. 4. Behavior of total network cost as capacity increased

Figure 4 gives more information about the performance of the network than Figure 2. It was already seen from eq. (4) that the cost would be

infinite when the total follower's demand was up to the network capacity.

The result of this example indicates that the leader of the network will be able to manage the entire system by using the incentive Stackelberg strategy at a low cost level.

## 6. CONCLUSION

The flow configuration problems of a network of parallel links were considered by means of the concept of incentive strategy. A new method was proposed to solve the problem on achieving team optimal solution in the entire noncooperative network. The proposed strategy overcomes some of the problems of existing Stackelberg strategy under which the leader cannot get the benefit from the game. Essentially, in the past work, the strategy is a routing strategy which cannot overcome the problems of existing strategy perfectly. When the leader does not have any flow to send into the network, he cannot construct a routing strategy. In such case, another type of strategy such as price (or cost) routing strategy should be employed, which will be investigated in future work.

## REFERENCES

- Basar, T. and G.J. Olsder (1982), *Dynamic Non-cooperative Game Theory*, Academic Press, NY.
- Bertsekas, D. and R. Gallager (1992), *Data Networks*, Englewood Cliffs, NJ: Prentice-Hall.
- Douligeris, C. and R. Mazumdar (1989), Multilevel flow control of queues, in Proc. of Johns Hopkins Conference on Inf. Sci., Baltimore, MD.
- Economides, A.A. and J.A. Silvester (1990), Priority load sharing: An approach using Stackelberg games, in Proc. of 28th Annual Allerton conference on Communi. Contr. and Computing.
- Jacobson, van (1988), Congestion avoidance and control, in Proc. of ACM SIGCOMM, Computer Communi. Review, vol.18, no.4, pp.316-329.
- Korilis, Y.A. (1995), *Architecting Noncooperative Networks*, Ph.D. dissert., Columbia University.
- Korilis, Y.A., A.A. Lazar, and A. Orda (1996), The role of the manager in a noncooperative network, in Proc. of IEEE INFOCOM'96.
- Myerson, R.B. (1991), *Game Theory: Analysis of Conflict*, Cambridge, Massachusetts: Harvard University Press.
- Orda, A., R. Rom, and N. Shimkin (1993), Competitive routing in multiuser communication networks, IEEE/ACM Trans. on Networking, vol.1, pp.510-521.