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FAST CALCULATION OF THE BESSEL TIME-FREQUENCY DISTRIBUTION

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Abstract: The Bessel distribution, one of the Cohen's class time-frequency distributions, has been reported, in many application areas and particularly on biomedical signal processing, as an efficient spectral estimator. Its usage on practical applications has been deprecated due to its computational burden. This paper presents a Bessel distribution algorithm and the results of its implementation on a variety of commercial off-the-shelf personal computers. The analysis of these results indicates that the Bessel distribution may no longer be put aside for real-time applications. It is demonstrated that the real-time estimation of pulsed Doppler blood flow spectral parameters is achievable.

Keywords: Sequential algorithms, Spectral estimation, Real-time systems,
Time-frequency representation, Biomedical systems

1. INTRODUCTION

Doppler ultrasound is currently used on the non-invasive assessment of blood flow on the cardiovascular human system. Due to the time-varying nature of the blood flow signals, research has been developed towards the establishment of more accurate blood flow estimators rather than those obtained by Fast Fourier transformations, typically encountered on the clinical instrumentation.

Therefore, time-frequency distributions (TFD) have been applied to the analysis of Doppler blood flow signals with a special emphasis on those of the Cohen's class. Among these, the Bessel distribution (BD) has been reported by (Cardoso *et al.*, 1996) (Guo *et al.*, 1994*a*) as being very useful on the time-frequency analysis of Doppler blood flow signals and heart sound (Guo *et al.*, 1994*b*) respectively, but put aside due to its computational burden (Cardoso *et al.*, 1996) (Guo *et al.*, 1994*a*). In fact, the statistical performance of the Bessel spectral estimator of non-stationary signals motivates a deeper study of its implementation.

This paper presents the implementation of the BD distribution for the particular application of Doppler blood flow signals spectral estimation. The Bessel time-frequency distribution, the numerical implementation and some issues related to its application to blood flow evaluation will be presented. Performance is evaluated in terms of execution time. Results with different workloads will be presented showing that real-time estimation is achieved in most cases.

2. BESSEL DISTRIBUTION

The Cohen's class TFD (Cohen, 1989) can be obtained by the solution of the Fourier Transform of the generalised autocorrelation function

$$TDF(t,\omega) = \int_{t}^{+\infty} R'_x(t,\tau)\, e^{-j\omega\tau} \partial\tau \qquad (1)$$

where

$$R'_x(t,\tau) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} R_x(\mu,\tau)\ \Psi(t-\mu,\tau)\ \partial\mu \quad (2)$$

$R_x(\mu,\tau)$ is the instantaneous autocorrelation function of the analysed signal $x(t)$, which given by

$$R_x(t,\tau) = x\Big(t+\frac{\tau}{2}\Big)x^*\Big(t-\frac{\tau}{2}\Big) \qquad (3)$$

and, $\Psi(t-\mu,\tau)$ is the autocorrelation domain kernel, defined as

$$\Psi(t,\tau) = \int_{-\infty}^{+\infty} \Phi(\theta,\tau)\ e^{-j\theta t}\ \partial\theta \qquad (4)$$

where $\Phi(\theta,\tau)$ is the kernel which characterises the particular TFD in use.

For the Bessel distribution, the kernel is defined (Guo *et al.*, 1994*b*) as

$$\Phi(\xi,\tau) = \frac{J_1(2\pi\alpha\xi\tau)}{\pi\alpha\xi\tau} \qquad (5)$$

where J_1 is the first kind Bessel function of order one, ξ is the frequency lag, τ the time lag, and $\alpha > 0$ a scaling factor.

2.1 *Numerical implementation*

For a computer based implementation, the discrete-time BD (Guo *et al.*, 1994*b*) of a complex valued discrete-time signal $x(n)$ is given by

$$BD_f(n,k) = 2\sum_{\tau=-\infty}^{+\infty} W_N(\tau)\ e^{-j2\pi k\tau/N} \left[\sum_{\mu=-2\alpha|\tau|}^{+2\alpha|\tau|} \frac{1}{\pi\alpha|\tau|}\sqrt{1-\Big(\frac{\mu}{2\alpha\tau}\Big)^2}\ x(n+\mu+\tau)\ x^*(n+\mu-\tau)\right] \qquad (6)$$

with

$$\left[\sum_{\mu=-2\alpha|\tau|}^{+2\alpha|\tau|} \frac{1}{\pi\alpha|\tau|}\sqrt{1-\Big(\frac{\mu}{2\alpha\tau}\Big)^2}\ x(n+\mu+\tau)\ x^*(n+\mu-\tau)\right]_{\tau=0} = x(n)x^*(n) \qquad (7)$$

where n and k correspond to the discrete time and frequency variables respectively, $W_N(\tau)$ is a symmetric window with zero values out of the range $[-N/2, N/2]$.

When a real valued signal is employed, one of two actions need to be performed: 1) to increase the sampling frequency above twice the Nyquist rate, or, 2) to use the analytic form of the signal (Guo *et al.*, 1994*a*) (Guo *et al.*, 1994*b*).

The first option is easily implemented. The second approach was considered, adopting the procedure proposed by (Marple, 1999) for the computation of the "analytic" form of the signal. A complex-valued N-point discrete-time "analytic" signal is created from a real-valued N-point discrete-time signal $x(n)$ by following the steps:

- compute the N-point discrete-time Fourier Transform $X[m]$
- form the N-point one-sided discrete-time "analytic" signal transform as defined

$$Z[m] = \begin{cases} X[0] & \text{for } m = 0 \\ 2\,X[m] & \text{for } 1 \le m \le N/2-1 \\ X[N/2] & \text{for } m = N/2 \\ 0 & \text{for } N/2+1 \le m \le N-1 \end{cases} \qquad (8)$$

- compute and scale N-point inverse discrete-time Fourier Transform.

Regarding these numerical implementation aspects, some other practical issues should be considered.

2.2 *Application related issues*

In order to analyse a particular area of the vessel, pulse mode Doppler ultrasound is typically used. In this case, directional signals (direct or reverse flow) are obtained upon which estimation of relevant parameters is then performed. Usually two data channels are used providing two real signals containing direct and reverse flow information. If only complex data is supplied by the instrumentation, the signals could be separated (Vaitkus and Cobbold, 1988).

In the case of the signal to analyse being complex and therefore containing relevant information in both its real and imaginary components, the use of an Alias-Free version of the Bessel distribution (Guo *et al.*, 1994*b*) might improve the results. Alias-Free Bessel distribution will not be addressed at this time.

Application of time-frequency distributions to a signal implies general knowledge of the signals behaviour and tuning of the distribution's parameters to that particular signal. Fine-tuning of these parameters is beyond the scope of this study. A wide range of values that include the values determined by other studies ((Leiria, 2000),(Cardoso *et al.*, 1996),(Guo *et al.*, 1994*a*),(Guo *et al.*, 1994*b*)) will be considered.

Overlapping the windowed data time segments might reduce undesirable distortion, and the amount of windowed segment's overlapping depends on the windowed signal under analysis and on the spectral estimator applied. Some results obtained by the use of several time-frequency estimation methods applied to aortic valve and carotid artery pulsed Doppler ultrasound signals can be used to show this behaviour (Leiria, 2000), (Cardoso *et al.*, 1996).

The choice of scaling factor α of the kernel varies with the signal under study. For example, carotid artery blood flow signal (Cardoso, 1998) should use $\alpha = 2$ and femoral artery blood flow signals (Guo *et al.*, 1994*a*) should use $\alpha = 16$.

The application of time-frequency distributions to the real time blood flow characterisation involves the spectral estimation of several data segments on each cardiac cycle and consecutive cardiac cycles need to be estimated. Averaging of a number of cardiac spectrograms is performed, with the number of spectra used typically ranging from 15 to 100 cardiac cycles. For each averaged spectrum the determination of two spectral parameters are required, the centre frequency and bandwidth. The averaging process and the actual parameters calculations are common to the estimation process regardless the estimator applied and will not be included in this study.

In this paper the case study signal is pulsed Doppler blood flow signal from common carotid artery. A file containing 33.61 s. of data was used to assess the implementations' behaviour.

3. PERFORMANCE CRITERIA SELECTION

Performance evaluation aims to ascertain the suitability of each implementation to the goal proposed, preferably in a quantitative form.

In this particular application, real time means that the processing time of a signal acquired during a certain amount of time cannot exceed that amount, so only execution time can be used as a performance criterion.

The acceptable latency is quite high in this application due to the averaging process. The set-up time for the application can be as long as the time needed for the number of cardiac cycles to be averaged to occur.

Throughput does not allow for a direct evaluation. Segment overlapping will vary the workload and the limiting value of acceptable throughput rate of a real time application.

The approach followed in this study takes into account the total elapsed time during the spectral estimation of the whole signal without the need to consider the different workloads brought by the application's arguments values. This work will therefore evaluate if, under the above stated conditions, it is feasible or not to achieve a real-time implementation of a Bessel distribution.

4. IMPLEMENTATION ARCHITECTURES

In order to evaluate the possibility of real-time implementation of the Bessel distribution, COTS (commercial off-the-shelf), quite common computers, equipped with Intel Pentium processors have been used. The operating system used is Debian/GNU Linux version 2.2. The programs were coded in C and compiled with GCC (Gnu Compiler Collection) with optimisation level three. The distinguishing characteristics of each computer are noted in Table 1. To facilitate general conclusions only one processor of *Computer C* was used in the algorithm's implementations at any given time.

Table 1. Computers (A, B and C) main characteristics.

	Proc. model	Clock speed (MHz)	RAM size (MByte)	No. Proc.	Kernel version
A	MMX	166	32	1	2.2.17
B	III	500	64	1	2.2.18pre21
C	II	350	128	2	2.2.15

5. ALGORITHM

The Bessel time frequency distribution computational algorithms are mainly sequential: the analytic form of the data acquired and windowed is calculated, the autocorrelation of the signal with different lags is weighed by the distribution kernel and transformed to the frequency domain by a Fourier transform, representing the power spectral density of the signal under analysis.

A preliminary study of Bessel algorithm behaviour using a similar approach to (Madeira *et al.*, 1999) was performed. The kernel of the distribution is function of the α parameter (specific to the application case) and of the window size. To avoid repeated calculus for each segment estimation, at the expense of memory space, the kernel is pre-processed at the initialisation phase and kept in memory. The window lengths were considered to have a power of two size. A Fast Fourier Transform algorithm was used in the computation of the analytic form of the signal and in the last algorithm's phase, this is, the computation of the signals power spectral density.

A single application, heavily dependent on dynamic memory allocation, was tested on the different computers and the execution times were noted. The application arguments are: the type of windowing function to be used – Boxcar, Bartlett, Hanning or Hamming; the size of the window, in data elements – values used were 64, 128, 256 and 512; the time step between consecutive windows, also in data elements, conveying segment overlap; the usage or not of the analytic form of the signal and the kernel α parameter – values used were 0.5, 1, 2, 5, 10, 15 and 20.

The type of window used did not influence the application execution times. The size of the window produces a noticeable impact on execution time but real time is always achieved. With the highest tested segment size (512), different values of segment overlapping were tested. The execution times are proportional to the workload increase but real-time is not always achieved, revealing a strong dependence on execution speed of the computer used. *Computer C* is capable of maintaining real-time execution up to 75% segment overlapping, while *Computer B* holds real-time up to 50% segment overlapping. The usage of the analytic form of the signal, although producing a higher impact on higher segment lengths application runs, can be achieved in real-time in all but one case: *Computer A* with segment length equal to 512.

The α parameter of the kernel is the one that influences application behaviour the most. Figure 1 presents the execution times of the Bessel distribution algorithm, in seconds, obtained by the execution on Computer A, varying with segment lengths 64, 128, 256 and 512 points and the kernel α parameter with values 0.5, 1, 2, 5, 10, 15 and 20. A line was added to the graph, representing the real time limit, in this case at 33.61 seconds. It can be observed that real time is achieved in less than half the cases tested.

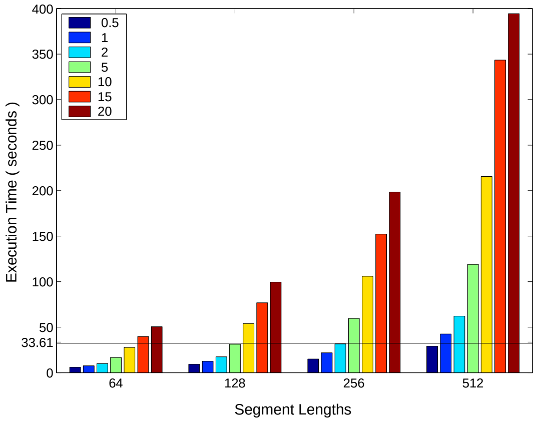

Fig. 1. Execution times of Bessel Distribution on Computer A varying segment length and kernel α parameter. Horizontal line, at 33.61s, indicates the real time limit.

A joint representation of the results obtained with computers B and C is presented in Figure 2, with lines and bars, respectively. A line indicating the real time limit was added to the graph.

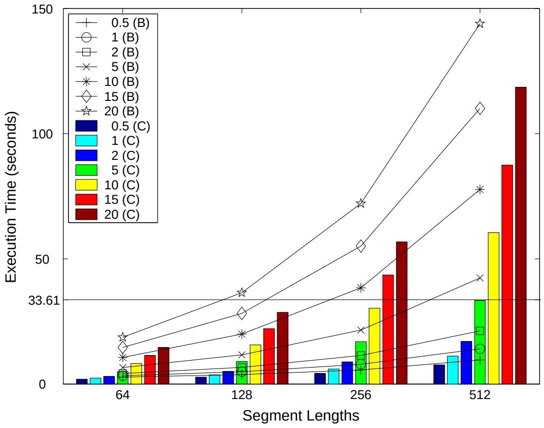

Fig. 2. Execution times of Bessel Distribution on Computers B and C varying segment length and kernel α parameter. Horizontal line, at 33.61s, indicates the real time limit.

At the lowest segment length both computers are able to execute the Bessel Distribution in real time. As the segment length increases, Computer C presents a better performance than Computer B although B is faster. It can be concluded that the memory management is very important when values of $\alpha > 2$ are desired.

It can be concluded that the algorithm can be used for the real-time estimation of blood flow signal spectrum. The actual parameters to be used depend on the signal under study. The application behaviour can be enhanced by the usage of static memory allocation and the usage of a shared memory space to hold the kernel matrix.

6. FINAL REMARKS

Several studies demonstrated the superior capability of Bessel time-frequency distribution to estimate non-stationary signals with application in various areas. In this paper the particular case of blood flow spectral parameters estimation is addressed. The numerical implementation of the Bessel algorithm and some particular application related issues are presented. The algorithm is tested on three off-the-shelg computers using an exaggerated set of possible parameters thus demonstrating the suitability of Bessel algorithm to be used for real-time estimation.

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