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TIME-DOMAIN FILTERING AND CONTROL WITH EMPIRICAL DATA

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Abstract: The transmission matrix, introduced by Friedland in 1957, can be used to characterize a linear, time invariant system having an emprically-determined impulse response. The Wiener-Kalman filter can be determined by Cholesky factorization of a covariance matrix formed from the transmission matrix. An analogous result is given for linear, quadratic control. The method is illustrated by several examples.

Keywords: Impulse response, Kalman filtering, Wiener filtering, Control algorithms, Dynamic matrix control

1. INTRODUCTION

State-space methods for control and estimation have been enormously successful for many years. But these methods rely upon the availability of state-space models (differential or difference equations) to characterize the dynamics of the processes of interest. It often happens that the requisite model is not readily available, but it is possible to determine the characteristics of the process of interest empirically, by applying a test input and recording the response.

When a state-space model is not available, one approach is to use a model-identification technique to fit a state-space model to the empirical data. The literature abounds with such techniques and many practical algorithms are readily available for application.

There are situations, however, in which the available identification techniques are either cumbersome or fail to produce satisfactory results. Moreover, one might ask why it should be necessary to pay the price of using a sophisticated identification method in order to use a state-space method for the design of a control system or a filter. Isn't it possible to use the empirical data directly, without first having to establish a state-space model?

A method for dealing with linear systems directly in terms of their empirically measured impulse response data, without the necessity of first having to establish state-space models, has been available for over forty years (Friedland, 1957). Owing perhaps to computational limitations of the rudimentary digital computers of that era, however, this method was rarely exploited outside the field of process control where it is known as “Dynamic Matrix Control” (The terminology is attributed to Cutler and Ramaker (Ogunnaike, 1983)). State-space methods seemed obviously more appropriate in most applications. The enormous increase in computing power (speed and memory) since that era makes it timely to revisit the methods and results of that era.

2. REPRESENTATION OF LINEAR SYSTEMS BY TRANSMISSION MATRICES

A linear discrete-time system H can be represented by the superposition summation

$$y(n) = \sum_k h(n,k)u(k) \tag{1}$$

where $u(n)$ and $y(n)$ represent the value at the nth sampling instant of the input and output signals, respectively, and $h(n,k)$ is the "unit (impulse) response" of H.

Assuming that signals start a $n = 0$ allows an alternative representation (Friedland, 1957) of the input-output relation for H [1] :

$$\mathsf{y} = \mathsf{H}\mathsf{u} \tag{2}$$

where signals are represented as vectors, e.g.,

$$\mathsf{x} = [x(0), x(1), \ldots, x(n), \ldots]'$$

and linear systems are represented by "transition matrices", e.g.

$$\mathsf{H} = \begin{bmatrix} h(0,0) & h(0,1) & h(0,2) & \cdots \\ h(1,0) & h(1,1) & h(1,2) & \cdots \\ h(2,0) & h(2,1) & h(2,2) & \cdots \\ \vdots & \vdots & \vdots & \end{bmatrix}$$

In a causal system, $h(n,k) = 0$ for $k < n$, the transition matrix for a causal system is *lower triangular*, e.g.

$$\mathsf{H} = \begin{bmatrix} h(0,0) & 0 & 0 & \cdots \\ h(1,0) & h(1,1) & 0 & \cdots \\ h(2,0) & h(2,1) & h(2,2) & \cdots \\ \vdots & \vdots & \vdots & \end{bmatrix}$$

Finally, if H is time-invariant, $h(n,k) = h(n-k)$ and hence each column of H is the same as the previous column, but pushed down one element, e.g.

$$\mathsf{H} = \begin{bmatrix} h(0) & 0 & 0 & \cdots \\ h(1) & h(0) & 0 & \cdots \\ h(2) & h(1) & h(0) & \cdots \\ \vdots & \vdots & \vdots & \end{bmatrix} \tag{3}$$

For a general, a so-called *infinite impulse response – IIR* system, the transmission matrix is of infinite dimension. In a *finite impulse response — FIR* the impulse response terminates after a fixed number N of terms.

Since our concern in this paper is with empirical data, we assume that we are dealing with FIR systems. Moreover, implicit in the assumption that the impulse response can be determined empirically is the fact that the system is time invariant and thus has a transmission matrix of the form given by (3).

3. WIENER-KALMAN FILTERING

In terms of transmission matrices, the Kalman filtering problem can be stated as follows:

Consider a system H with transmission matrix H. Suppose that the input u is a white noise sequence with variance σ_u^2. To the output $\mathsf{y} = \mathsf{H}\mathsf{u}$ is added another white noise sequence v with variance σ_v^2 The covariance matrix variance of the sum $\mathsf{z} = \mathsf{y} + \mathsf{v}$ is

$$\mathbf{P}_z = \sigma_u^2 \mathsf{H}\mathsf{H}' + \sigma_v^2 \mathsf{I} \tag{4}$$

The goal is to obtain an estimate $\hat{y}$ of y by means of a *causal* linear filter processing z, i.e.,

$$\hat{\mathsf{y}} = \mathsf{K}\mathsf{z} = \mathsf{K}(\mathsf{H}\mathsf{u} + \mathsf{v})$$

such that the "residual"

$$\mathsf{r} = \mathsf{y} - \hat{\mathsf{y}} = (\mathsf{I} - \mathsf{K})\mathsf{H}\mathsf{u} - \mathsf{K}\mathsf{v} \tag{5}$$

has the minimum covariance matrix.

A simple calculation gives the covariance matrix of the residual:

$$\mathbf{P}_r = \sigma_u^2 (\mathsf{I} - \mathsf{K})\mathsf{H}\mathsf{H}'(\mathsf{I} - \mathsf{K})' + \sigma_v^2 \mathsf{K}\mathsf{K}' \tag{6}$$

There is obviously no loss in generality in assuming that $\sigma_u^2 = 1$ and replacing σ_v^2 by

$$\rho = \frac{\sigma_v^2}{\sigma_u^2}$$

the "noise-to-signal" ratio.

Cursory examination of (6) reveals that for $\rho = 0$ the minimization is achieved with $\mathsf{K} = \mathsf{I}$, i.e., simply accepting the noisy output as the best estimate of the noise-free output, because there is no observation noise.

If not for the requirement that K be causal, finding the transmission matrix K that minimizes $\mathbf{P}_r$ is a straigtforward calculus problem.

The causality requirement, however, turns the problem into the discrete-time version of the famous Wiener-Hopf integral equation which was elegantly solved in the frequency domain by Bode and Shannon (1950) using the "spectral shaping method" in which the observed signal is first transformed to white noise and then the resulting white noise is filtered to obtain the desired estimate.

Friedland (1958) showed that the transmission matrix analog of Bode-Shannon spectral shaping is *Cholesky factorization* of the analogous spectral density matrix P_z given above. In particular, let the Cholesky factorization of $\mathbf{P}_z$ be given by

$$\mathbf{P}_z = \mathsf{C}\mathsf{C}' \tag{7}$$

where C is lower triangular and C' is upper triangular. Since $\mathsf{H}\mathsf{H}' \geq 0$, the presence of $\sigma_u^2\mathsf{I}$ in (4) ensures that $\mathbf{P}_z$ is positive definite, hence C^{-1} exists. It is shown in Friedland (1958) that the transmission matrix of the desired filter is given by

$$\mathsf{K} = [\mathsf{H}\mathsf{H}'(\mathsf{C}')^{-1}]_R \mathsf{C}^{-1} \tag{8}$$

[1] The sans-serif typeface is used to designate signals and transmission matrices in this formulation

where the symbol

$$[\mathsf{M}]_R \text{ denotes the “realizable part of”} \qquad (9)$$

the matrix M, obtained from M by deleting all elements above its principal diagonal.

The implementation of the algorithm described above at the time it was developed (before the digital computer era) was impractical for large values of N owing the the amount of calculation required. Nowadays, however, the calculation is all but trivial, even for N of several hundred. A Matlab m-file that implements the algorithm is given in the Appendix.

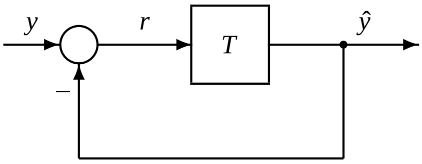

Fig. 1. *The least-squares filter in a feedback configuration.*

It is noted that the result is the transmission matrix of the closed-loop filter. If a feedback implementation as shown in Figure 1, in which the residual $r = y - \hat{y}$ is explicitly determined, is desired, the forward loop transmission matrix is given by

$$\mathsf{T} = \mathsf{K}(\mathsf{I} - \mathsf{K})^{-1} \qquad (10)$$

4. FEEDBACK CONTROL

In the Wiener filtering problem the main goal is to determine the transmission matrix of the filter. Kalman's state space formulation provides a means, but not the only means to that goal. In the feedback control problem, however, the goal is *not* to determine the transmission matrix of the closed-loop system (the analog of the Wiener filter) but rather to determine the compensator D in the closed-loop system that includes the actual process (not simply a mathematical model of the process, as in the case of the Kalman filter) as shown in Figure 2. Determination of the compensator by solution of a quadratic optimization problem is but one of many methods that can be employed.

In principle one could specify the desired closed-loop transmission matrix K and determine the desired forward loop transmission matrix T using (10), finally, when H is non-singular (i.e., $h(0) \neq 0$ implying no process delay) solving for D

$$\mathsf{D} = \mathsf{H}^{-1}\mathsf{T}$$

If H is singular, the “Smith predictor” (Smith, 1958) or another technique for dealing with delays could be used.

In the state-space formulation the linear, quadratic (LQ) control problem and the least-squares estimation problem are “dual”. The transmission matrix formulation of the optimum control problem is similar to the filtering problem, but is not its exact dual. The most obvious difference is that the goal of the LQ control problem is to return the *state* x of the plant to zero rapidly, whereas the state is nowhere present in the transmission matrix formulation and the goal is rather for the closed-loop system to “track” an input y_d (from a specified class).

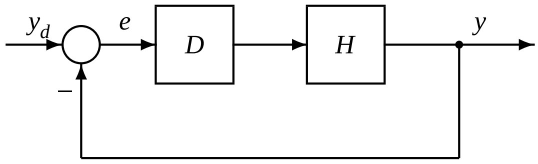

Fig. 2. *Feedback control system*

The classical control problem is to minimize the sum of quadratic forms in the system error

$$e(n) = y_d(n) - y(n)$$

and the control input $u(n)$. Thus a performance criterion

$$V = \sum_{k=0}^{N} e^2(k) + q^2u^2(k) \qquad (11)$$

is defined.

In terms of transmission matrices

$$\mathsf{e} = \mathsf{y}_d - \mathsf{H}\mathsf{u} \qquad (12)$$

so (10) can be written

$$V = (\mathsf{y}_d' - \mathsf{u}'\mathsf{H}')(\mathsf{y}_d - \mathsf{H}\mathsf{u}) + q^2\mathsf{u}'\mathsf{u} \qquad (13)$$

Suppose a linear control law is used. Then

$$\mathsf{u} = \mathsf{G}\mathsf{y}_d \qquad (14)$$

and $\mathsf{y} = \mathsf{H}\mathsf{u} = \mathsf{H}\mathsf{G}\mathsf{y}_d$ and hence the closed-loop transmission matrix is

$$\mathsf{K} = \mathsf{H}\mathsf{G} \qquad (15)$$

Also (13) becomes

$$V = \mathsf{y}_d'\mathbf{M}\mathsf{y}_d \qquad (16)$$

where

$$\mathbf{M} = (\mathsf{I} - \mathsf{G}'\mathsf{H}')(\mathsf{I} - \mathsf{H}\mathsf{G}) + q^2\mathsf{G}'\mathsf{G} \qquad (17)$$

From (17) it is apparent that as the control weighting q^2 approaches zero $\mathsf{K} = \mathsf{H}\mathsf{G}$ approaches the identity matrix, i.e., the best closed-loop system simply reproduces the desired input.

Compare (17) to (6) and notice that they are of similar form. The minimization with respect to the unknown matrix tranmission matrix G can be accomplished by the same method as was used to obtain (8).

For the control problem there are two cases to consider:

Case 1: Non-singular H. If the system has no "pure delay" ($h(0) \neq 0$) then H is nonsingular and the solution G to the optimization problem defined by (16) is given by

$$\mathsf{G} = \mathsf{L}^{-1}[(\mathsf{H}\mathsf{L}^{-1})']_R$$

where L is the causal Cholesky factor of $\mathsf{H}'\mathsf{H} + q^2\mathsf{I}$, i.e,

$$\mathsf{L}'\mathsf{L} = \mathsf{H}'\mathsf{H} + q^2\mathsf{I}$$

and, hence the "optimum" closed-loop transmission matrix is given by

$$\mathsf{K} = \mathsf{H}\mathsf{G} = \mathsf{H}\mathsf{L}^{-1}[(\mathsf{H}\mathsf{L}^{-1})']_{\mathsf{R}} \tag{18}$$

and the forward loop transmission

$$\mathsf{T} = \mathsf{H}\mathsf{D} = \mathsf{K}(\mathsf{I} - \mathsf{K})^{-1}$$

Thus the compensator transmission matrix is

$$\mathsf{D} = \mathsf{H}^{-1}\mathsf{K}(\mathsf{I} - \mathsf{K})^{-1} \tag{19}$$

Case 2: General H . If H is singular, however, the above solution method is not valid for two reasons: first, the compensator cannot be realized using (19). And, more subtly, $\mathsf{H}\mathsf{L}$ is a causal (lower triangular) matrix; so $\mathsf{H}\mathsf{L}'$ is an upper triangular matrix. Its realizable part is the simply the matrix consisting of the diagonal elements of $\mathsf{H}\mathsf{L}'$, which are all zero if $h(0)$ is zero. So (19) gives the product of an infinite matrix with a zero matrix. One work-around is to put small but non-zero number in the place of $h(0)$. Another work-around is to change the quadratic form to be minimized and thereby change the matrix to be minimized. In particular, suppose the matrix to be minimized is

$$\mathbf{Q} = \mathsf{H}'\mathbf{M}\mathsf{H} \tag{20}$$

$$= (\mathsf{I} - \mathsf{P})'\mathsf{H}'\mathsf{H}(\mathsf{I} - \mathsf{P}) + q^2\mathsf{P}'\mathsf{P} \tag{21}$$

where

$$\mathsf{P} = \mathsf{G}\mathsf{H} \tag{22}$$

Note that $\mathsf{G}\mathsf{H} \neq \mathsf{H}\mathsf{G}$ hence $\mathsf{P} \neq \mathsf{T}$, but P can be used to obtain D as shown below. The solution to the optimization problem defined by (21) is

$$\mathsf{G} = \mathsf{L}^{-1}[\mathsf{L}^{-1}\mathsf{H}'\mathsf{H}]_R \tag{23}$$

5. EXAMPLES

The forgoing theory is intended for use with a system having an empirically measured impulse of perhaps comprising several hundred terms. For illustrative purposes, however, we consider an example of a system having an FIR with only a few non-zero terms, namely

$$\mathsf{h} = [0\ 3\ 2\ 1\ 0\ 0\ 0\ 0\ 0 \cdots]$$

(The extra zeros in H permit closer convergence to the steady-state solution as the following result shows.)

Using the appended m-file, with $\rho = 1$, the transmission matrix of the filter is found to be

```
K =
0       0       0       0       0       0       0
0  0.9000       0       0       0       0       0
0  0.0577  0.9038       0       0       0       0
0 -0.0055  0.0573  0.9039       0       0       0
0 -0.0133 -0.0065  0.0573  0.9041       0       0
0  0.0095 -0.0126 -0.0064  0.0571  0.9042       0
0 -0.0018  0.0094 -0.0126 -0.0064  0.0570  0.9042
```

A state-space model

$$\mathbf{x}_{n+1} = \mathbf{\Phi}\mathbf{x}_n + \mathbf{\Gamma}\mathbf{u}_n$$

$$\mathbf{y}_n = \mathbf{C}\mathbf{x}_n$$

for this system has

$$\mathbf{\Phi} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{\Gamma} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$$

The Kalman filter and the linear, quadratic control can be designed by state-space methods. In particular, with $\sigma_u^2 = \sigma_v^2$ the steady state Kalman filter, found with the aid of Matlab, has the transfer function

$$K(z) = \frac{0.9042z^2 + 0.5953z + 0.2874}{z^2 + 0.5953z + 0.2874}$$

to which corresponds the impulse response

```
k=
[0.9042,0.0570,-0.0064,-0.0126,0.0093,-0.0019,...]
```

It is observed that this impulse response is identical to the last row of the transmission matrix K as determined above by the method of this paper.

In view of the issue raised above in determining the compensator when there is a delay between the input and the output, we assume that the observed impulse response is given by

$$\mathsf{h} = [3\ 2\ 1\ 0\ 0\ 0\ 0\ 0 \cdots]$$

making $h(0) \neq 0$.

For a control weighting of $q^2 = 1$ the closed-loop transfer function calculated by the formula of Case 1 is found to be

```
K =
```

```
 Columns 1 through 6
   0.6429         0         0         0         0         0
   0.1236    0.7875         0         0         0         0
  -0.0390    0.1007    0.8295         0         0         0
   0.0180   -0.0464    0.0907    0.8425         0         0
  -0.0095    0.0245   -0.0479    0.0873    0.8467         0
   0.0052   -0.0136    0.0265   -0.0483    0.0862    0.8480
  -0.0029    0.0076   -0.0149    0.0271   -0.0484    0.0858
   0.0017   -0.0043    0.0084   -0.0153    0.0273   -0.0484
  -0.0009    0.0024   -0.0047    0.0087   -0.0154    0.0274
   0.0005   -0.0014    0.0027   -0.0049    0.0087   -0.0155
  -0.0003    0.0008   -0.0015    0.0028   -0.0049    0.0088
  -0.0001    0.0002   -0.0004    0.0008   -0.0014    0.0025

  Columns 7 through 12
        0         0         0         0         0         0
        0         0         0         0         0         0
        0         0         0         0         0         0
        0         0         0         0         0         0
        0         0         0         0         0         0
        0         0         0         0         0         0
   0.8484         0         0         0         0         0
   0.0857    0.8485         0         0         0         0
  -0.0484    0.0857    0.8486         0         0         0
   0.0274   -0.0485    0.0857    0.8486         0         0
  -0.0155    0.0274   -0.0485    0.0856    0.8486         0
  -0.0044    0.0077   -0.0136    0.0241   -0.0426    1.3625
```

The transmission matrix of the corresponding compensator, using (19) is found to be

```
D =
  Columns 1 through 6
   0.6000         0         0         0         0         0
   0.1430    1.2353         0         0         0         0
   0.0119    0.1027    1.6216         0         0         0
   0.0003    0.0028    0.0450    1.7836         0         0
   0.0000    0.0000    0.0004    0.0161    1.8407         0
   0.0000    0.0000    0.0000    0.0000    0.0053    1.8595
   0.0000    0.0000    0.0000    0.0000    0.0000    0.0017
   0.0000    0.0000    0.0000    0.0000    0.0000    0.0000
   0.0000    0.0000    0.0000    0.0000    0.0000    0.0000
  -0.0000    0.0000   -0.0000   -0.0000    0.0000    0.0000
   0.0000    0.0000    0.0000   -0.0000    0.0000    0.0000
   0.0000   -0.0000   -0.0000   -0.0000    0.0000   -0.0000

  Columns 7 through 12
        0         0         0         0         0         0
        0         0         0         0         0         0
        0         0         0         0         0         0
        0         0         0         0         0         0
        0         0         0         0         0         0
        0         0         0         0         0         0
   1.8656         0         0         0         0         0
   0.0006    1.8676         0         0         0         0
   0.0000    0.0002    1.8682         0         0         0
   0.0000    0.0000    0.0001    1.8684         0         0
   0.0000    0.0000    0.0000    0.0000    1.8685         0
  -0.0000   -0.0000   -0.0000   -0.0000   -0.9870   -1.2529
```

From the middle rows of D it is seen that the compensator for this design is essentially nothing more than a constant gain of 1.868.

For the design method of Case 2, the closed-loop transmission matrix, calculated by first determining P and then determining $\mathsf{K} = \mathsf{HPH}^{-1}$, and using the same control weighting, is determined to be

```
K =
  Columns 1 through 6
   0.7253         0         0         0         0         0
   0.0947    0.8546         0         0         0         0
  -0.0228    0.0690    0.8900         0         0         0
   0.0086   -0.0289    0.0585    0.9007         0         0
  -0.0041    0.0146   -0.0299    0.0550    0.9041         0
   0.0022   -0.0079    0.0163   -0.0301    0.0539    0.9052
  -0.0012    0.0044   -0.0091    0.0168   -0.0302    0.0535
   0.0007   -0.0025    0.0051   -0.0095    0.0170   -0.0302
  -0.0004    0.0014   -0.0029    0.0054   -0.0096    0.0170
   0.0002   -0.0008    0.0016   -0.0030    0.0054   -0.0096
  -0.0016    0.0027   -0.0043    0.0067   -0.0106    0.0167
  -0.0003    0.0005   -0.0008    0.0013   -0.0021    0.0035

  Columns 7 through 12
        0         0         0         0         0         0
        0         0         0         0         0         0
        0         0         0         0         0         0
        0         0         0         0         0         0
        0         0         0         0         0         0
        0         0         0         0         0         0
   0.9055         0         0         0         0         0
   0.0534    0.9057         0         0         0         0
  -0.0302    0.0534    0.9057         0         0         0
   0.0171   -0.0302    0.0533    0.9057         0         0
  -0.0265    0.0424   -0.0682    0.1103    0.8202         0
  -0.0056    0.0091   -0.0150    0.0248   -0.0412    1.3625
```

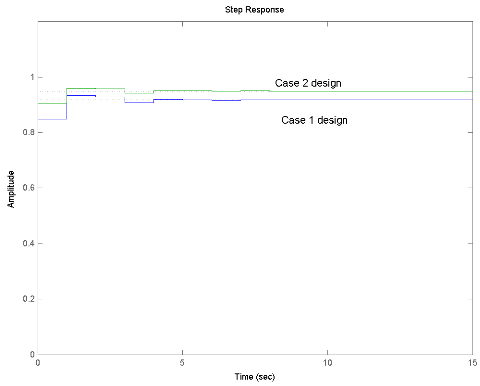

Fig. 3. *Step responses of example designs.*

and the transmission matrix of the corresponding compensator is

```
D =
  Columns 1 through 6
   0.8800         0         0         0         0         0
   0.2041    1.9595         0         0         0         0
   0.1077    0.1304    2.6958         0         0         0
  -0.0532    0.0926   -0.0117    3.0244         0         0
   0.0387   -0.0578    0.0886   -0.0896    3.1433         0
  -0.0263    0.0395   -0.0592    0.0890   -0.1195    3.1829
   0.0177   -0.0265    0.0397   -0.0596    0.0894   -0.1297
  -0.0118    0.0177   -0.0266    0.0398   -0.0597    0.0896
   0.0079   -0.0118    0.0177   -0.0266    0.0399   -0.0598
  -0.0053    0.0079   -0.0118    0.0177   -0.0266    0.0399
  -0.0009    0.0014   -0.0021    0.0031   -0.0046    0.0069
   0.0005   -0.0007    0.0011   -0.0016    0.0024   -0.0037

  Columns 7 through 12
        0         0         0         0         0         0
        0         0         0         0         0         0
        0         0         0         0         0         0
        0         0         0         0         0         0
        0         0         0         0         0         0
        0         0         0         0         0         0
   3.1958         0         0         0         0         0
  -0.1330    3.1999         0         0         0         0
   0.0897   -0.1341    3.2012         0         0         0
  -0.0598    0.0897   -0.1344    3.2017         0         0
  -0.0104    0.0156   -0.0234    0.0351    1.5208         0
   0.0055   -0.0082    0.0124   -0.0186   -0.8033   -1.2529
```

From the middle rows of this matrix it is observed that the impulse response of the compensator is approximated by

$$\mathsf{d} = [3.20, -0.134, .0897]'$$

to which corresponds the transfer function

$$D(z) = 3.20 - 0.134z^{-1} + 0.0897z^{-2}$$

The step responses of the closed-loop system with the two compensators are compared in Figure 3. Both designs are satisfactory, although neither achieves zero steady state error; this requirement has not been incorporated into the design specifications.

6. CONCLUSIONS

The transmission matrix method provides a framework for design of filters and control algorithms directly from empirically-measured data of linear systems without the need for first developing state-space models for the systems. The design algorithm, based on Cholesky factorization, are readily applicable to systems for which the measured impulse-response sequence can contain many hundred terms, since Cholesky factorization of large matrices is not an overwhelming task. The obvious alternative technique of using a state-space model in which the dimension of the state space equals the number of terms in the measured impulse response (as illustrated in the above example) and then solving the associated matrix Riccati equations would seem to be precluded by the sheer size of the matrices involved in the calculation.

The method as presented here is applicable only to single-input, single-output systems. Extension of the method to "square" (i.e., equal number of inputs and outputs) multi-input, multi-output systems appears feasible. The elements of the system transmission matrix H would be an array the elements of which are square matrices of the same dimension as the number of inputs and outputs. It is readily verified that the Cholesky factorization extends to such arrays, provided the matrices on the diagonal are nonsingular. Extension to non-square systems, however, appears problematical.

State space methods, of course, do not have these limitations. Moreover, the transmission-matrix method is inherently limited to linear systems, whereas state-space methods are not. Thus, when an appropriate state-space model for the process can be developed, state-space continues to provide the methods of choice.

APPENDIX

```
% function [K,T] = TDFIL(h,rho)
% Designs time-domain least-squares filter
%         using Cholesky factorization.
%
% h=impulse response vector of plant
% rho="signal-to-noise" variance ratio
%
function [K,T] = TDFIL(h,rho)
%
% Form transmission matrix from impulse response
n=length(h);
H=zeros(n);
H(:,1)=h';
%
for k=2:n
    temp=H(:,k-1);
    H(:,k)=[0;temp(1:n-1)];
end
%
Q=H*H';
P=eye(n)+rho*Q;
% Cholesky factorization
M=chol(P)';
B=inv(M);
%
C=Q*B';
% Throw away part above upper triangle.
C=tril(C)
% Filter transmission matrix
K=C*B;
% Feedback form
T=K*inv(eye(n)-K);
% end of TDFIL
```

7. ACKNOWLEDGMENT

An anonymous reviewer indicated the connection of Friedland's 1957 method with Dynamic Matrix Control, and simplified the Matlab coding of the design algorithm. The author is grateful for this contribution.

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