15th Triennial World Congress, Barcelona, Spain

AN ADAPTIVE FUZZY ROBOT CONTROL WITHOUT A FUZZY RULE BASE

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Abstract: A vexing problem in a conventional fuzzy control is the exponential growth in rules as the number of variables increases. This problem is avoided here by the introduction of a new, nonconventional analytic adaptive method for synthesising the fuzzy robot control. For this purpose a new adaptive analytic function is defined that determines the positions of the centres of the output fuzzy sets, instead of the definition of a fuzzy rule base. This function is adapted by chaning the free fuzzy-set parameters. The proposed analytic approach to the synthesis of an adaptive fuzzy logic control, has been tested by a numerical simulation of an analytic adaptive fuzzy control system for a robot with four degrees of freedom. *Copiright © 2002 IFAC*

Keywords: Fuzzy systems; Fuzzy control; Adaptive algorithms; Adaptive control; Discrete-time systems; Robot control.

1. INTRODUCTION

Fuzzy sets are a generalization of conventional set theory that were introduced by Zadeh (1965) as a mathematical way to represent vagueness in everyday life. The building block of a conventional fuzzy logic system is a rule that maps inputs to outputs: IF X IS A, THEN Y IS B ("If the room is cool, then open the valve of the radiator"). Currently, fuzzy systems often apply where the number of inputs and outputs is small and events happen relatively slowly. Fitting into that category are fuzzy washing machines, videocameras, and even subways. These applications have on the order of 20 to 100 rules. Meanwhile, there are applications such as robot control, videorich multimedia and virtual reality, which can require many more rules of greater complexity operating in milliseconds. It becomes necessary to suggest ways to cope with a vexing problem in fuzzy logic: the exponential growth in rules as the number of variables increases (Kosko, 1996). This paragraph gives the motivation in the paper for introducing of an adaptive analytic function that determines the positions of centres of output fuzzy sets in each mapping process, instead of definition of a rule base. Thus, the paper approach is without any fuzzy logic rule. Adaptive fuzzy membership functions are presented by Lotfi nad Tsoi (1996), and by Novakovic (1995). In the paper a new adaptive shape and a new adaptive distribution of input fuzzy sets have been proposed.

Some of the structures of fuzzy logic controllers, as a fuzzy proportional-derivative controller, a self-organizing fuzzy logic controller, a genetic algorithms based fuzzy logic controller, and the Sugeno or Mamdani type fuzzy logic controller are presented by Malki et. al. (1994), Park et al. (1995), Homaifar McCormick (1995) and Galichet and Foulloy (1995), respectively. Langari and Tomizuka (1990) proposed self organizing fuzzy linguistic control with application to arc welding. Adaptive fuzzy-based classification systems are discussed by Nozaki et al. (1996). An adaptive fuzzy controller, constructed with parallel combination of robust controllers, has been presented by Kim et aal. (1996). Adaptive fuzzy systems for target tracking are discussed by Pacini, and Kosko (1992), while a fuzzy model-reference adaptive control is presented by Yin and (1995). This paragraph motivates for developing of Fuzzy Logic Control (FLC) systems that have ability to adapt their parameters automatically. This motivation is resulted with the proposed parameter β-adaptation algorithms of FLC systems in this paper.

As the contributions of the paper, the following items can be cited. A new analytic function that determines the positions of centres of output fuzzy sets instead of the conventional definition of fuzzy logic rule base. A new parameter β-adaptation algorithm for adaptation of FLC system, during working or simulation process. The functionality of the proposed approach to synthesis of the adaptive FLC system was demonstrated with a numerical example, based on an adaptive FLC system of robot of RRTR - structure.

This paper is organized as follows. Section 2 presents the process of synthesis of FLC system, employing a new analytic algorithm without any fuzzy rule base. In Section 3, the parameter β-adaptation schemes for adaptation of FLC systems are constructed and discussed. Following the procedures from Sections 2 and 3, the related adaptive FLC system of robot of RRTR-structure is designed in Section 4. Finally, the comments and conclusions are emphasized by Section 5.

2. FUZZY LOGIC CONTROL WITHOUT FUZZY RULE BASE

A conventional Fuzzy Logic Control (FLC) system is composed, as it is well known, by four principal elements: fuzzy rule base, fuzzification interface, fuzzy inference machine, and defuzzification interface. More details about conventional FLC systems can be found in many of the well known references, as well as in the references Malki et al. (1994) and Galichet and Foulloy (1995). In this paper it has been considered multi-input-single-output (MISO) fuzzy logic control system as a function $f : U \subseteq R^m \to R.$ That is because a multi-output system, which has no output-input interactions, can always be separated into a collection of single-output systems. In order to create a new fuzzification interface process, we first define a new type of the fuzzy membership functions $s_i(y)$, (i =1,..., n), on a universe of discourse Y, $y \in Y$, that can be adapted by employing an adaptation parameter ε_i. Let y_{oi} denotes the beginning, and y_{ei} the end of the i-th fuzzy set, on the abscise axes, y, where $s_i(y_{oi})=s_i(y_{ei})=0$. The points y_{ai} and y_{bi} are determined by $s_i(y)=1$, $y_{ai} \le y \le y_{bi}$. The fuzzy set centre is denoted by y_{ci}, while the fuzzy set basis is $T_i=y_{ei}-y_{oi}$. The adaptation parameter ε_i is defined by the equation:

$$\varepsilon_i = (y_{ei} - y_{oi})/(y_{bi} - y_{ai}), \qquad (\varepsilon_i > 1), \quad (1)$$

where ε_i can be adapted by employing some of the parameter adaptation schemes. Further, one can use the following calculation:

$$\begin{aligned} y_{oi} &= y_{ci} - T_i/2, \quad y_{ai} = y_{ci} - T_i/2\varepsilon_i, \\ y_{bi} &= y_{ci} + T_i/2\varepsilon_i, \quad y_{ei} = y_{ci} + T_i/2. \end{aligned} \quad (2)$$

The adaptive i-th fuzzy set shape is defined by the equations:

$$\begin{gathered} s_i(y_{oi}) = 0,\ s_i(y_{ai}) = 1,\ s_i(y_{bi}) = 1, \\ s_i(y_{ei}) = 0, \\ s_i(y) = (1 - \cos((2\pi\varepsilon_i(y - y_{ci} + T_i/2))/ \\ ((\varepsilon_i - 1)T_i)))/2, \quad y_{oi} \le y \le y_{ai}, \\ s_i(y) = 1, \quad y_{ai} \le y \le y_{bi}, \\ s_i(y) = (1 - \cos((2\pi\varepsilon_i(y_{ci} + T_i/2 - y))/ \\ ((\varepsilon_i - 1)T_i)))/2, \quad y_{bi} \le y \le y_{ei}. \end{gathered} \quad (3)$$

In the case that $\varepsilon_i \to \infty$, the fuzzy set shape is *cosine-triangle*. For $\varepsilon_i \to 1$, the fuzzy set shape is *cosine-rectangle*. Otherwise $(1 < \varepsilon_i < \infty)$, the fuzzy set shape is *cosine-trapezoid*.

The second step of the creation of the new fuzzification interface process, is the normalization of input variables. Let U be the universe of discourse of all input variables x_j (j =1,..., m) of FLC system, and $x_j \in U$. The centres of input fuzzy sets are denoted by x_{ci}^j, i = 1,..., n_j. The input variable x_j should be normalized by the equation:

$$\underline{x}_j = K_j x_j, \quad K_j = 1/|x_{j\,\max}|, \quad (4)$$

where $x_{j\,\max}$ is the maximum value of x_j on the universe of discourse U, and $\underline{x}_j$ is the normalization of x_j. In that case the new fuzzification interface uses the equations (1) to (4), with substitution y by $\underline{x}_j$, and y_{ci} by $\underline{x}_{ci}^j$, for calculation of the membership function $s_i^j(\underline{x}_j)$, i =1,..., n, of the j-th input variable, j =1, ..., m, of the system. The third and finale step of the creation of the new fuzzification interface process, is an introduction of the new ε-β distribution of input fuzzy sets. In order to create an adaptation of input fuzzy sets one can use a special distribution of input fuzzy sets. It has been done by the following modification of the fuzzy set shape from (3):

$$\underline{s}_i^j\left(\underline{x}_j\right)=1, \quad \underline{x}_{ai}^j \le \underline{x}_j \le \underline{x}_{bi}^j,$$
$$\underline{s}_i^j\left(\underline{x}_j\right)=s_i^j\left(\underline{x}_j\right)/\exp\left(\beta_j \varepsilon_i^j\left|\underline{x}_j\right|\right), \quad \underline{x}_j \le \underline{x}_{ai}^j, \tag{5}$$

or $\underline{x}_j \ge \underline{x}_{bi}^j$, $\underline{x}_{ai}^j \ge \underline{x}_{ai}^j$, i = 1, ..., n_j, j =1,..., m, where ε_i^j and β_j are free adaptation parameters. In this distribution the all input fuzzy sets have the same centre position $\underline{x}_{ci}^j = 0$, and the same basis $T_i^j = \underline{x}_{ei}^j - \underline{x}_{oi}^j$, where $\underline{x}_{oi}^j = -1$, and $\underline{x}_{ei}^j = 1$, but different adaptation parameter ε_i^j, i =1,..., n_j. This type of distribution of input fuzzy sets is called ε-β distribution of input fuzzy sets. In order to create a new analytic inference algorithm, in this paper max/min operators have been replaced by sum/product operators. In this sense, the new distribution of input fuzzy sets (5) is suitable for definition of a new type of an analytic inference algorithm. For this purpose one can define an activation function ω_j by the equation:

$$\underline{s}_1^j\left(\underline{x}_j\right)s_{Bj} + \underline{s}_2^j\left(\underline{x}_j\right)s_{Bj} + \ldots + \underline{s}_{nj}^j\left(\underline{x}_j\right)s_{Bj} = \sum_{i=1}^{nj}\underline{s}_i^j\left(\underline{x}_j\right)s_{Bj} = \omega_j s_{Bj}. \tag{6}$$

In the equation (6) s_{Bj} is a membership function of an output fuzzy set B_j. Thus, the activation function ω_j of the j-th output fuzzy set B_j, can be computed by an analytic form:

$$\omega_j = \sum_{i=1}^{nj}\underline{s}_i^j\left(\underline{x}_j\right), \quad j=1,\ldots,m. \tag{7}$$

The activation function ω_j denotes the grade of membership of input $\underline{x}_j$ to all of the input fuzzy sets $A^j{}_i$, i=1,...nj, j=1,..., m. Instead of using of fuzzy rules it has been defined the function for an analytic determination of the centres of the output fuzzy sets. Thus, amplitudes of normalized positions of output fuzzy sets centres can be computed by the equation:

$$\left|\underline{y}_{cj}\right| = 1-\left(\sum_{i=1}^{nj}\underline{s}_i^j\left(\underline{x}_j\right)\right)/n_j = 1-\omega_j/n_j, \tag{8}$$

where j=1, ..., m. Taking into account that a sign of $\underline{y}_{cj}$ must be equal to a sign of $\underline{x}_j$, the normalized positions of centres of output fuzzy sets can be calculated by the equations:

$$\underline{y}_{cj} = \left(1-\omega_j/n_j\right)\mathrm{sgn}\left(\underline{x}_j\right), \quad j=1,\ldots,m, \tag{9}$$

where $\mathrm{sgn}\left(\underline{x}_j\right)=1$, if $\underline{x}_j > 0$, $\mathrm{sgn}\left(\underline{x}_j\right)=-1$, if $\underline{x}_j < 0$, and $\mathrm{sgn}\left(\underline{x}_j\right)=0$, if $\underline{x}_j = 0$. Since the input variables are normalized, it requires a determination of a gain K_{cj} of output fuzzy set centre position. In general, an adaptive gain of output fuzzy set centre position can be computed by the equation:

$$K_{cj} = U_m F_j\left(1+\left|\underline{x}_j\right|^{\alpha j}\right), \qquad U_m > 0, \; F_j > 0, \; \alpha_j \ge 0, \tag{10}$$

where U_m is a maximal value of a control variable u, and F_j and α_j are free parameters that should be obtained by simulation in each concrete case. Finally, the output fuzzy set center position y_{cj} is determined by a product of contents of (9) and (10):

$$y_{cj} = K_{cj}\underline{y}_{cj}, \qquad y_{cj} = U_m F_j\left(1+\left|\underline{x}_j\right|^{\alpha j}\right)\left(1-\omega/n_j\right)\mathrm{sgn}\left(\underline{x}_j\right). \tag{11}$$

In the discrete-time-point t the activation function ω_j activates the j-th output fuzzy set B_j, to a degree $\omega_j(t)$. At the same time the position of the centre of the output fuzzy set B_j is $y_{cj}(t)$:

$$y_{cj}(t) = K_{cj}(t)\left(1-\omega_j(t)/n_j\right)\mathrm{sgn}\left(\underline{x}_j(t)\right). \tag{12}$$

By empolying a correlation-product inference for determination of the output fuzzy set′s shape s_{oj}, one can multiply the output fuzzy set shape s_{Bj} by ω_j:

$$s_{oj}(y,t) = \omega_j(t)s_{Bj}(y). \tag{13}$$

Thus, each input variable $\underline{x}_j(t)$ activates the related output fuzzy set B_j, to the degree $\omega_j(t)$, and this yields the output fuzzy set′s shape $s_{oj}(y,t)$. The FLC system then sums $s_{oj}(y,t)$ to form the combined output fuzzy set′s shape:

$$s_o(y,t) = \sum_{j=1}^{m} s_{oj}(y,t), \tag{14}$$

and with that the inference algorithm is finished. In order to generate a non fuzzy output (a crisp value) of the FLC system the centroid defuzzification method (Pacini and Kosko, 1992) is employed at the discrete time point t:

$$u(t+1) = \int y\, s_o(y,t)dy \Big/ \int s_o(y,t)dy, \tag{15}$$

where the limits of integration correspond to the entire universe of discoirse Y of control output values. Starting with the equation (15), one can develop an analytic method for computing a non fuzzy output of the FLC system. Further, the synthesis result is stated in the form of the proposition.

Proposition 1: If $x_j(t)$ (j=1,..., m) are FLC inputs in the discrete time point t that are normalized by (4), and FLC input fuzzy sets are defined by (1) to (5), and $\omega_j(t)$ is an activation function from (7) that activates the j-th output fuzzy set B_j to a degree $\omega_j(t)$, and $y_{cj}(t)$, T_j, and ε_{oj} are centre, basis and adaptation parameter of the j-th output fuzzy set B_j, respectively, then a non fuzzy output of the FLC system u(t+1), can be computed by the centroid defuzzification method (15) in the analytic form:

$$u(t+1)=\left(\sum_{j=1}^{m}\left(\omega_j(t)y_{cj}(t)T_j\left(\varepsilon_{oj}+1\right)/2\varepsilon_{oj}\right)\right)/ \left(\sum_{j=1}^{m}\left(\omega_j(t)T_j\left(\varepsilon_{oj}+1\right)/2\varepsilon_{oj}\right)\right), \qquad (16)$$

where $y_{cj}(t)$ has to be calculated by (12). The proof of the proposition 1 one can be obtained by following the corresponding proof algorithm given by Pacini and Kosko (1992). The proposed FLC synthesis procedures are very simply, because FLC system uses only three analytic scalar equations ((7), (11), and (16)) for calculation of output of FLC system.

3. SYNTHESIS OF AN ADAPTIVE FUZZY LOGIC CONTROL USING PARAMETER β - ADAPTATION SCHEME

During the FLC system working (or simulation) process, there is a possibility of adaptation of its free parameters α, β, F, T and ε. For this purpose, one can employ an optimal, or heuristic, or any of the remaining parameter adaptation schemes, including their combinations. In this Section the adaptive FLC system has been developed by adaptation of parameter β_j, j=1, ... ,d ≤ m. The other four free parameters α, F, T and ε, have been used as constants, obtained from the previous simulation process. Starting with the analysis of the equations (5) and (8), one can conclude, that the parameter β_j corresponds to the closed loop gain of the j-th input variable of the FLC system. Namely, the increasing of β_j parameters has the consequence in inccreasing of the amplitudes of normalized positions of centres of output fuzzy sets $|y_{cj}|$ and vice-versa. In this section, a new heuristic parameter adaptation algorithms, for adaptation of parameter β_j has been employed. For this purpose one can introduce discrete-time-variable parameter β^i_j (t) in each i-th mapping process, i = 1, ... , N, j=1, ... ,d ≤ m. For determination of parameter β^i_j (t) one can employ the following heuristic approach, starting with the equations:

$$\beta_j^i(t)=\beta_j^i(t_s)+\delta_j^i\; G_j^i(t), \quad t=t_s+1,\ldots,t_f, \qquad (17)$$

where $G^i_j(t)$ is a discrete - time - variable normalized gain of the j-th mapping process, δ^i_j is a related scaling factor (0.1 < δ^i_j < 1), and t_s and t_f are respectively, the starting and ending discrete time points of the adaptation period. The normalized gain, $G^i_j(t)$, is a function of a set of initial stables poles λ_{o1} ... λ_{on} of a referent time-continuous system, and a positive definite constrained function $\sigma^i(t)$:

$$G_j^i(t)=f_j^i\left(\lambda_{01}-\sigma^i(t),\ldots,\lambda_{0n}-\sigma^i(t),G_{j\max}\right), \quad 0\le\sigma^i(t)\le\sigma_{\max}, \qquad (18)$$

where $\lambda_k^i(t)=\lambda_{0k}-\sigma^i(t),\ k=1,\ldots,n,$ are time-variable stable poles, and n-is a predicted order of the referent system, respectively. Parameter $G_{j\,max}$ is the maximal value of the gains G^i_j (t) , i=1,..., N, of the j-th input variable. The set of the initial stable poles, $\lambda_{01}\ldots\lambda_{0n}$, of the referential time-continuous system can be obtained from the set of initial stable poles, $\lambda_{01}^d\ldots\lambda_{0n}^d$, of the corresponding referential time-discrete system, and vice-versta, by employing the following relationships:

$$\lambda_{0k}=1n\left(\lambda_{0k}^d\right)/T_s, \quad k=1,\ldots,n, \quad \lambda_{0k}^d=\exp\left(\lambda_{0k}T_s\right), \qquad (19)$$

where T_s is the sampling period. The set of initial stable poles of a referential system should be selected on the condition that the closed loop system (controlled plan + FLC system) must be stable. The function G^i_j (t), from (18), should be determined in each concrete case. For the class of mechanical systems, such as robots, machine tools, and so on, the following G^i_j (t) functions can be employed:

$$G_{1j}^i(t)=\left(\lambda_{0j1}-\sigma^i(t)\right)\left(\lambda_{0j2}-\sigma^i(t)\right)/G_{1j\max},$$
$$G_{2j}^i(t)==-\left(\lambda_{0j1}+\lambda_{0j2}-2\sigma^i(t)\right)/G_{2j\max},$$
$$i=1,\ldots,N, \quad j=1,\ldots,g, \quad t=t_s+1,.,t_f, \qquad (20)$$

where g-denotes the number of freedom of motions of mechanical system. In such a manner, G^i_{1j} (t) is connected with normalized gains of position errors, while G^i_{2j} (t) is connected with normalized gains of velocity errors of the mechanical system. In this case the set of initial stable poles of referential time-continuous system has the form:

$$\left(\lambda_{011},\lambda_{012},\lambda_{021},\lambda_{022},\ldots,\lambda_{0g1},\lambda_{0g2}\right), \quad \lambda_{0i1}\ne\lambda_{0i2}, \quad i=1,\ldots,g. \qquad (21)$$

The positive definite functions σ^i (t) are state and control constrained functions, which have to be adapted in the region:

$$0 \le \sigma^i(t) \le \sigma_{\max}, \tag{22}$$

where σ_{max} is the maximal value of the functions $\sigma^i(t)$. In this sense the following unified direct adaptation procedure for adaptation of functions $\sigma^i(t)$ can be applied:

$$\text{IF: } \left|\underline{x}_j^i(t)\right| > \underline{x}_{j\max},$$

$$\text{OR/AND: } \left|y^i(t)\right| > y_{\max},$$

$$\text{THEN: } \sigma^i(t) = 0,$$

$$\text{ELSE:}$$

$$\sigma^i(t) = \sigma_{\max}\left(p + 1 - \sum_{j=1}^{p}\left(R_j\left|\underline{x}_j^i(t)\right| / \underline{x}_{j\max}\right)\right.$$
$$\left. - R_y\left|y^i(t)\right| / y_{\max}\right)/(p+1),$$

$$(\sigma_{\max}, \underline{x}_{j\max}, y_{\max}) > 0, \quad 0 \le (R_j, R_y) \le 1,$$
$$i = 1,\dots,N, \quad j = 1,\dots,p, \quad p < m, \tag{23}$$

where y_{max} is the maximal value of the fuzzy logic control variables $y^i(t)$, and R_j and R_y are corresponding weighting parameters. In the case of mechanical systems p = 2, for each of g-degrees of freedom of motions of mechanical system.

As it can be seen from the equations (23), the increasing of FLC errors $\underline{x}_j^i(t)$, or/and the previous control variable $y^i(t)$, has the consequence in decreasing of the function $\sigma^i(t)$, and vice-versa. At the same time, the position and velocity gains, $G_{1j}^i(t)$ and $G_{2j}^i(t)$ from (20), will be decreased by decreasing of the function $\sigma^i(t)$. It means, by increasing of FLC errors $\underline{x}_j^i(t)$, or/and the previous control variable $y^i(t)$, the position and velocity gains, $G_{1j}^i(t)$ and $G_{2j}^i(t)$ from (20), will be decreased. Following the previous consideration one can conclude that the function $\sigma^i(t)$ is the gain adaptation function, which depends on the state errors and previous control variables, and acts as a system-stabilizing factor. Thus, the functions $G_{1j}^i(t)$ and $G_{2j}^i(t)$ from (20), become adaptive gains of the 1j-th and 2j-th inputs, respectively, in the i-th mapping process. Consequently, the parameters λ_{0k}, σ_{max}, R_j, and R_y (k=1, ... ,n; j=1, ... ,p) should be selected in order to obtain the practical closed loop stability, and the desired exponential stability of the controlled plant, taking into account the state and control constraints.

4. ADAPTIVE FUZZY LOGIC CONTROL OF ROBOT OF RRTR STRUCTURE

The RRTR-structure of robot, which is considered in this section, is equal to the structure of the well-known Stanford Manipulator. A dynamic model of this robot is presented by Novakovic (1996). The physical parameters of a robot are given by Table 1.

Table 1. Parameters of robot of RRTR-structure

Robot link	1(R)	2(R)	3(T)	4(R)
Link Length [m]	0.64	0.19	min.1.10 max.1.48	0.22
Link mass [kg]	6.0	2.3	13.1	1.0
Movement [°, m]	-135 to135	0 to180	0 to 0.38	-90 to 90

The initial parameters used in FLC synthesis are shown in Table 2.

Table 2. Initial parameters used in fuzzy-control synthesis

Robot link no	1(R)	2(R)	3(T)	4(R)
Starting position [rad, m]	0	0.785	0.01	0
Final position [rad, m]	2.267	2.965	0.3	1.395
Starting and final velocity [rad/s, m/s]	0	0	0	0
Profile of velocity	cosine	cosine	cosine	cosine
Starting time point [s]	0	0	0	0
Final time point [s]	1.5	1.5	1.5	1.5
Sampling period [s]	0.015	0.015	0.015	0.015
Expected max. position errors [rad, m]	0.05	0.05	0.05	0.02
Expected max. velocity errors [rad/s, m/s]	0.3	0.3	0.3	0.3
Expected max. control variable [V]	35	25	10	15

The FLC system of this robot, maps robot position errors e(t), velocity errors $\dot{e}(t)$, and previous control variables u(t) (inputs of fuzzy logic controllers) to DC - motor control variables u(t+1) (voltages, as outputs of FLC system). The nominal (desired) robot velocities have cosine shapes in the inner space. Four DC - motors adjust the robot positions and velocities, following the nominal (desired) robot positions and velocities. Following the procedures in sections 2 and 3, and using the parameters from Tables 1 and 2, the adaptive FLC of robot of RRTR structure is realised and tested through the simulations. The corresponding simulation results of the parameter β-adaptation scheme are shown in Table 3.

Table 3. Simulation results of β-adaptation scheme

Robot link no.	1(R)	2(R)	3(T)	4(R)
Position errors at end-point ($t=t_f$)	0.332 $*10^{-3}$ rad	-0.682 $*10^{-3}$ rad	-0.382 $*10^{-3}$ m	0.895 $*10^{-3}$ rad
Max. transient position errors	0.0205 rad	0.017 rad	-0.003 m	0.007 rad
Velocity error at end-point ($t=t_f$)	-3.000 $*10^{-3}$ rad/s	6.500 $*10^{-3}$ rad/s	2.200 $*10^{-3}$ m/s	3.800 $*10^{-3}$ rad/s
Max. transient velocity errors	-0.148 rad/s	-0.137 rad/s	0.028 m/s	0.072 rad/s

The simulation results, presented in Table 3, show that the proposed FLC system, adapted with the heuristic β - adaptation scheme, can follow the desired robot trajectory with a high stage of accuracy.

5. CONCLUSION

In the paper the positions of centres of output fuzzy sets are determined by introducing of an analytic function of input variables, instead of definition of fuzzy rule base. The number of input and output variables and the number of fuzzy sets of FLC system are not limited, because there are no fuzzy rules in the paper approach. On this way the vexing problem in fuzzy logic (the exponential growth in rules as the number of variables increases) is solved. Starting with the parameter β-heuristic adaptation algorithm the adaptive FLC system has been designed. Finally, the proposed adaptive FLC synthesis procedures are applied to the adaptive FLC system of robot of RRTR-structure. Future directions of work opened up by these results include comparative studies of this FLC system synthesis approach with other algorithms in variety of applications (for an example Ordonez et al. (1997) and Kim et al. (1997). Some comparable advantages and disadvantages of this paper approach have been presented at the 15th IFAC World Congress.

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