## WORST-CASE UNCERTAIN PARAMETER COMBINATIONS FOR FLIGHT CONTROL SYSTEMS ANALYSIS

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Abstract: This paper presents a new technique for generating linear fractional transformation based uncertainty models for use in the robustness analysis of flight control systems using the structured singular value  $\mu$ . The proposed approach does not require closed-form linear expressions for the aircraft dynamics – only a non-linear software model of the closed loop aircraft, which can be efficiently trimmed and linearized numerically, is required. The capability of the proposed approach to identify worst-case uncertain parameter combinations is demonstrated. The approach is compared with classical analysis methods via the robustness analysis of a control law for a Boeing 747 transport aircraft. *Copyright* © 2002 IFAC.

Keywords: Flight control, Models, Parameters, Structured singular value, Uncertainty

# 1. INTRODUCTION

The analysis of stability and performance robustness to variations in uncertain aircraft parameters represents a major issue in the flight control law analysis/ certification process. The complexity of current aircraft simulation models and control laws, together with the large number of different combinations of flight parameters (e.g. variations in mass, centre of gravity positions, inertia, non-linear aerodynamics, aerodynamic tolerances, air data system tolerances, structural modes and failure cases) which must be examined throughout the entire flight envelope, makes this analysis a costly and labour intensive task. As well as identifying the regions of the flight envelope where the aircraft is "safe to fly", a key issue in the control law analysis is to identify worst-case (in terms of the resulting closed loop stability or performance) combinations of the uncertain parameters. Current approaches which check stability and performance at each point in a gridding of the space of all possible uncertain parameter variations quickly become computationally inefficient as the number of uncertain parameters increases. Moreover, gridding approaches provide no guarantee that the true worst-case has in fact been found – in particular, situations where the worst case parameter combination occurs in the interior of the parameter space could easily be missed.

The technique of  $\mu$ -analysis (Doyle, 1982) has been proposed as a tool which may be used to improve both the efficiency and accuracy of the flight control law analysis task (Ferreres, 1999; Döll *et al.*, 1999). Unlike with the gridding approach,  $\mu$  provides guarantees that a particular stability or performance property is satisfied over a continuous range of values for each uncertain parameter. Moreover, worst-case values of these parameters may be computed on a frequency by frequency basis.

In order to apply  $\mu$ -analysis techniques to the flight control law certification problem, a so-called linear



Fig. 1 Upper LFT uncertainty description

fractional transformation (LFT)-based model of the uncertain closed loop system must first be generated, see Fig. 1. *M* represents the known part of the system (plant and controller) and  $\Delta$  represents the uncertainty present in the system. In effect, extra inputs and outputs are introduced so that the system uncertainty can be considered as part of an "external" feedback loop.  $\mu$  defines a stability-test for a closed loop system subject to *structured* uncertainty  $\Delta$  in terms of the maximum *structured* singular value (Doyle, 1982). The problem of calculating the exact value of  $\mu$  has been shown to be NP-hard (Braatz *et al.*, 1994), and so in practice upper and lower bounds are generally computed using various approaches (Ferreres, 1999).

In recent years much attention has been paid to the issue of how to efficiently generate accurate (and ideally minimal) LFT-based uncertainty models for complex uncertain systems - see Ferreres (1999) for an overview. A common assumption among almost all of the approaches suggested is that closed form analytical expressions relating the aircraft dynamics to the uncertain parameters of interest are available, from which LFT-based uncertainty models may be derived. The main drawbacks of this approach can be identified as the substantial modelling effort required to accurately relate all the uncertain parameters to the non-linear aircraft dynamics, and the fact that the symbolically linearized state-space models are generally valid only at and around the relevant operating point in the flight envelope.

In this paper, an alternative approach for generating LFT-based uncertainty models is presented, which does not require the availability of analytical expressions relating the aircraft dynamics to the uncertain parameters – only a non-linear software model of the closed loop aircraft, which can be efficiently trimmed and linearized numerically for different values of the uncertain parameters, is required.

### 2. LFT-BASED UNCERTAINTY MODELLING USING TRENDS & BANDS

As shown by Morton and McAfoos (1985), LFTbased parametric uncertainty models may be conveniently derived from a linear state space representation of the uncertain system of the form

$$\begin{aligned} x &= (A_0 + A_1 \delta_1 + \ldots + A_n \delta_n) x + (B_0 + B_1 \delta_1 + \ldots + B_n \delta_n) u \\ y &= (C_0 + C_1 \delta_1 + \ldots + C_n \delta_n) x + (D_0 + D_1 \delta_1 + \ldots + D_n \delta_n) u \end{aligned}$$
(1)

The matrices  $A_0$ ,  $B_0$ ,  $C_0$ ,  $D_0$  describe the nominal system, while  $A_k$ ,  $B_k$ ,  $C_k$ ,  $D_k$ , k = 1, ..., n, describe deviations from the nominal system depending on the normalized physical uncertain parameter  $\delta_k$  with  $-1 \le \delta_k \le 1$ .

In this paper, the problem of generating state space representations of the type given in (1) from the original non-linear simulation model of the aircraft is considered. One possible approach is the so-called "minmax" technique (Varga et al., 1997). Here, the values of the system-matrix elements are evaluated so that the minimum and the maximum value within the given parameter range are identified element wise. Note that for each varying element of the state space matrices A, B, C, D an individual  $\delta$  is needed. Note further that these  $\delta$ 's are "fictitious" and have no physical meaning. The method is straightforward to implement, but it can lead to conservatism since possible joint parametric dependencies in the state space model are ignored. Furthermore, since the "fictitious"  $\delta$ 's do not directly represent the physical uncertain parameters, it is not possible to identify the worst-case combination of these parameters in the resulting  $\mu$ -analysis.

In this paper, a new method for generating LFT-based uncertainty models is proposed, which is less conservative than the "min-max" approach and moreover allows the computation of worst-case uncertainty combinations in terms of physical uncertain parameters. As with the "min-max" approach, however, no closed form expressions (typically linearized equations of motion) involving the physical parameters are required. The key idea of the proposed method is to model the uncertainties using a curve fitting technique in a least squares sense. Consider as an example the element  $a_{ij} = f(\delta_k)$  of the matrix A depending on one parameter  $\delta_k$ . The formulation

$$a_{ij} = a_{ij_0} + a_{ij_k} \delta_k \tag{2}$$

then represents a linear approximation of the dependency of this element on the uncertain parameter  $\delta_k$ , assuming the coefficients  $a_{ij_0}$  and  $a_{ij_k}$  are derived using a least squares fit based on *m* data-pairs  $(a_{ij}^1, \delta_k^1)$ , ...,  $(a_{ij}^m, \delta_k^m)$ . Fig. 2 illustrates both the actual dependency  $a_{ij} = f(\delta_k)$  and the approximation  $a_{ij} = a_{ij_0} + a_{ij_k}\delta_k$ and shows the  $r^{th}$  data-pair  $(a_{ij}^r, \delta_k^r)$ , r = 1...m.

In the case of two uncertain parameters, the curve which was approximated by a line, as in Fig. 2, can then be interpreted as a surface which is approximated by a plane. In the case of n uncertain parameters, the approximation can be interpreted as a multi-dimensional regression plane. Note that the  $\delta$ 's are now all related to the physical uncertain parameters. The implementation of higher order polynomial fits under this approach is also straightforward. For the work presented in this paper, however, a linear fit was found



Fig. 2 Dependence of a matrix element on a parameter

to be adequate. For use in a  $\mu$ -analysis, these fits are transformed into an LFT-based uncertainty model as proposed by Morton and McAfoos (1985).

The method described above is a linear approximation of the uncertain system. Therefore, it does not cover nonlinearities in the dependence of state-space matrices on the uncertain parameters. Hence, some unstable combinations could be left out during the  $\mu$ -analysis, which would make the whole analysis invalid. For instance, assume that a matrix has a varying element  $a_{ii}$ . As shown in Fig. 2, the unstable region could be left out by the test, if only a linear approximation is used. To include deviations from the linear approximations, additional real nonlinearity compensation parameters (CP's) are added to the existing  $\delta$ -set (Varga *et al.*, 1997). These take the form of  $a_{ijCP}\delta_{a_{iiCP}}$  where  $a_{ijCP}$ represents the maximum deviation from the linear representation and  $\delta_{a_{iiCP}}$  is an additional normalized  $\delta$ . The compensation parameters add an uncertainty "band"-structure to the "trend" established by the dependence of the state-space elements on the physical uncertain parameter. Each compensation parameter independently acts on one state-space matrix element.

The proposed technique therefore tries to find trends, which represents the parameter variations, but allows for nonlinear variations by introducing compensation parameters. In the general case these trends are modelled by a multi-dimensional regression plane, which describes the linear variation, and a band-structure (compensation parameters), that is limited by planes parallel to the regression plane, above and below, to include nonlinear deviations. The actual value of the uncertain state-space element is then assumed to lie somewhere within this band-structure.

Compared with the element-wise "min-max" approach, which also works with bands, the size of the bands is now reduced, thus decreasing the conservatism of the analysis results. Furthermore, the trends introduce  $\delta$ 's with a physical interpretation, which can be used to identify the actual worst case combination of the physical uncertain parameters.

## 3. COMPUTATION OF WORST-CASE PARAMETER COMBINATIONS

To demonstrate the capability of the proposed method to find worst-case parameter combinations, a control system for the lateral axis of a civil transport aircraft (Ferreres, 1999) is considered. The model was chosen since an exact LFT representation (Kureemun et al., 2001), based on a physical model derived from symbolic equations, is available for comparison of worstcase parameter results. The nominal model is characterized by 4 states  $x = \begin{bmatrix} \beta & p & r & \phi \end{bmatrix}^T$ , 4 outputs  $y = \begin{bmatrix} n_y & p & r & \phi \end{bmatrix}^T$  and 2 control inputs  $u = \begin{bmatrix} \delta p & \delta r \end{bmatrix}^T$ . As suggested by Kureemun et al. (2001), uncertainties are introduced in the mass and in the 14 stability derivatives, see Table 1. Each uncertain parameter is allowed to vary within  $\pm 10\%$  of its nominal value. A constrained static output feedback law was synthesized using  $H_{\infty}$  loop shaping techniques (Bates and Kureemun, 2001).

LFT representations derived using three different methods are compared. The first LFT is referred to as the exact LFT since it is based on a representation of the symbolic equations and the uncertain parameters in SIMULINK block diagram form. The method is described in detail by Kureemun et al. (2001). Note that the process of generating an LFT-based uncertainty model using this method is quite tedious and time consuming, even for this simple academic aircraft model. The  $\Delta$  associated with the resulting LFT consists of 18 real  $\delta$ 's, of which 4, corresponding to the mass, are repeated. The second LFT is obtained using the "minmax" approach described in the previous section. The resulting  $\Delta$  consists of 19 real  $\delta$ 's without physical meaning. Finally, the proposed trends & bands (T&B) method yields an LFT with a  $\Delta$  matrix consisting of 38 real  $\delta$ 's. 19  $\delta$ 's correspond to the physical parameters, of which the mass is repeated 5 times. The remaining 19  $\delta$ 's are compensation parameters without physical interpretation.

In the following  $\mu$  calculations the MATLAB  $\mu$ -toolbox (Balas *et al.*, 1995) has been used to calculate the upper bound on  $\mu$ . Since the mixed- $\mu$  lower bound algorithms contained in the toolbox often yield poor results and can even fail to converge when applied to problems containing only real uncertainties, a method for computing lower bounds on real  $\mu$  proposed by Hayes *et al.* (2001) was used.

The resulting bounds on  $\mu$  for the three LFTs are shown in Fig. 3. As expected, the peak value of the  $\mu$ upper bound for the exact LFT is the smallest. The "min-max" based LFT  $\mu$ -peak value is approximately double the size of the exact value while the proposed T&B approach yields a value approximately 50% higher than the exact result.



Fig. 3 µ bounds for the civil transport aircraft

In column 1 and 2 of Table 1 the worst-case parameter combinations corresponding to the peak value of the  $\mu$  lower bound for the T&B and the exact LFT are given, both normalized to lie between -1 and 1. Comparing the worst-case parameter combination predicted by the T&B approach with the actual worst case parameter combination it can be noted that the values agree in most cases. To analyze the cases of disagreement, the  $\mu$ -sensitivities (Braatz and Morari, 1991) for both the T&B and the exact LFT are calculated.

Based on the  $\mu$ -sensitivities given in column 5 and 6 of Table 1 both the T&B and the exact LFTs are reduced in order to contain only "significant"  $\delta$ 's. The  $\mu$  calculation is then repeated for the reduced LFTs. The  $\mu$  bounds for the LFTs with all  $\delta$ 's and with "significant"  $\delta$  's only are nearly identical, confirming the result obtained by the  $\mu$ -sensitivities analysis. The selection of significant  $\delta$ 's and the resulting worst-case parameter combinations are given in column 3 and 4 of Table 1. Now all worst-case parameters suggested by the T&B LFT agree with the exact worst case parameters except for the value of  $C_{N_{\beta}}$ . A manual variation of this parameter, however, showed that the dependence of the closed loop system's eigenvalues' on  $C_{N_0}$  is negligible. This leads to the conclusion that the "worst-case" value of  $C_{N_{\rm R}}$  is mainly influenced by the initial guess in the calculations and is not really a function of the systems properties, as was already indicated by the very small value of the  $\mu$  -sensitivity for  $C_{N_{\rm R}}$ . As a consequence, it can be concluded that the proposed T&B approach allows the correct identification of all relevant worst case parameters for this example, without the need for the difficult and timeconsuming task of generating an exact LFT-based uncertainty model.

# 4. COMPARISON WITH THE CLASSICAL APPROACH

In order to compare the results based on the proposed method for generating parametric LFT-based uncertainty models with the classical analysis approach, a more realistic model of a large transport aircraft is

Table 1 Worst-case parameter combinations for T&B and exact LFT (columns 1,2), for T&B and exact LFT with reduced number of parameters (columns 3,4) and  $\mu$ -sensitivities for T&B and exact LFT (columns 5,6)

	$\delta_{T\&B}$	$\delta_{Exact}$	$\delta_{TB, red}$	$\delta_{Ex, red}$	$\delta_{\mu, T\&B}$	$\delta_{\mu, Exact}$
т	1.000	1.000	1.000	1.000	0.226	0.108
$C_{Y_{o}}$	-1.000	-1.000	-1.000	-1.000	0.237	0.200
$C_{Y}^{\mu}$	0.572	-0.409	-	-	0.000	0.000
$C_Y^{p}$	-0.742	-0.892	-	-	0.000	0.000
$C_{Y_{s}}$	0.981	1.000	1.000	1.000	0.022	0.020
$C_{Y_{S_n}}$	0.999	1.000	0.999	1.000	0.012	0.010
$C_{L_{0}}^{1 \text{ or}}$	1.000	1.000	1.000	1.000	0.219	0.167
$C_{L_{\mu}}^{P}$	-0.919	-0.971	-	-	0.000	0.000
$C_{L_n}^{p}$	-0.941	-0.889	-	-	0.001	0.001
$C_{L_{s_n}}$	-1.000	-1.000	-1.000	-1.000	0.139	0.116
$C_{L_s}^{op}$	-1.000	-1.000	-1.000	-1.000	0.075	0.059
$C_{N_0}^{-or}$	-0.946	-0.273	-0.969	-0.310	-0.001	0.011
$C_N^{p}$	0.681	0.111	-	-	0.000	0.000
$C_{N_{n}}^{p}$	-0.677	-0.801	-	-	0.000	0.000
$C_{N_{8}}$	-1.000	-0.949	-0.999	-0.912	0.007	0.006
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considered. This model was used to investigate the 1992 Amsterdam aircraft accident (Smaili, 2000) and provides an accurate simulation of all flying qualities, engine dynamics, characteristics and hydraulic system operations of a Boeing 747-100/200 aircraft throughout its flight envelope. Due to the complexity of the model, and the fact that lookup-tables are used to evaluate the aerodynamic parameters over the flight envelope, accurate closed form expressions relating the aircraft dynamics to the uncertain parameters of interest cannot easily be derived. As a consequence, methods to generate LFT-based uncertainty models based on symbolic equations were not considered. Instead, two alternative options are compared: a classical analvsis based on a gridding of the uncertain parameter space, and the proposed T&B approach to LFT-based uncertainty modelling.

In the following analysis only the longitudinal axis is considered. The control law used for the analysis is a multivariable glide-slope coupler (Härefors, 2001), designed for the model with all uncertain parameters at their nominal values. The equations of motion are characterized by 6 states  $x = \left[ q \ V \alpha \ \theta \ h_e \ x_e \right]^T$ . The inputs are the control column for the elevator and the thrust setting for the 4 engines. The outputs air speed  $V_T$  and flight path angle  $\gamma$  are fed back to the controller. Variations in the aircraft mass, as well as centre of gravity position in x - and z -directions are considered as uncertain parameters. The mass is allowed to vary between 300.000 kg and 350.000 kg, the range for the x-position of the centre of gravity is 11% to 31% of the mean aerodynamic chord and the z-position of the centre of gravity is allowed to vary within  $\pm 1$  meter of its nominal value. These uncertain parameters are represented by normalized the variables  $-1 \le \delta_m, \, \delta_{xcg}, \, \delta_{zcg} \le 1$ , where +1 corresponds to the maximum physical value.



Fig. 4 Definition of the stability margin  $\rho$ 

To quantify the "level of robustness" of the considered closed loop system in the analysis, a measure of robustness has to be defined. In order to meaningfully compare both analysis approaches, a classical stability margin is adopted, based on a Nichols exclusion region, commonly used in the aerospace industry to evaluate robustness of flight control systems. The closed loop is cut at the sensors. To avoid the shortcomings of measuring gain and phase margins separately, a criterion of simultaneous gain and phase margins is used (Muir, 1997). As shown in Fig. 4, the stability margin  $\rho$  is characterized by an exclusion zone in the Nichols plane for the open loop frequency response of each loop of the system. An allowable gain-offset of 4.5 dB and an allowable phase-offset of 35° is defined to correspond to a stability margin of  $\rho = 1$ . The exclusion zone can be scaled until it touches the Nichols curve resulting in a stability margin of  $\rho > 1$  satisfying or  $\rho < 1$  violating the criterion for each loop of the system. Since all uncertainty parameters are allowed to vary (and affect each loop) simultaneously, the above stability margin results in a true multi-variable robustness test. Note that the loops can be cut one at a time or simultaneously, and the loop can be broken on the actuator side as well as on the sensor side. Due to space restrictions in this paper only the results for the  $V_T$ -feedback loop cut at the sensor output are given.

Using the classical approach to study the influence of the uncertain parameters, a grid of 5 points was applied to each parameter, resulting in  $5^3 = 125$  uncertain parameter combinations, at which the stability margin  $\rho$  has been calculated. With this approach, the "worst-case" uncertainty parameter combination was identified to be the maximum possible mass and the centre of gravity at it's highest and aft limit position. For this particular choice of uncertain parameters this of course agrees with our expectations from flight dynamics. The corresponding Nichols curves are given in Fig. 5. In the nominal case the Nichols Exclusion criterion is fulfilled,  $\rho = 1.24$ , while the worst case uncertain parameter combination leads to a violation of the criterion with a stability margin of  $\rho = 0.82$ . Due to the gridding of the uncertain parameter space, however, no guarantee can be given that the actual worst case uncertain parameter combination has been found. This becomes particularly important in situations where the worst case is not produced by a combination of the parameters at their vertices.

For the purposes of comparison, both the "min-max" approach and the T&B technique are used to generate uncertainty models. The  $\Delta$  associated with the "min-max" based LFT uncertainty model consists of 27 fictitious "min-max"  $\delta$ 's. The T&B uncertainty model has an associated  $\Delta$  composed of 9 repeated  $\delta$ 's for the mass, 9 repeated  $\delta$ 's for  $x_{cg}$ , 3 repeated  $\delta$ 's for  $z_{cg}$  and an additional 27 fictitious  $\delta$ 's associated with the compensation parameters.

The results of a  $\mu$  stability test using both LFT based uncertainty models yield the following results: Both  $\mu$ upper bound peak values, 0.81 for the "min-max" and 0.78 for the T&B approach, guarantee robust stability of the closed loop system for the considered uncertainty parameter range, as was expected from the analysis using a gridding approach. The "phugoid peak" in the  $\mu$  curves is reduced by 30% with the T&B approach in comparison with the "min-max" technique.

To calculate the worst case uncertainty parameter combination with respect to the Nichols criterion, the exclusion zone is included in the  $\mu$ -analysis (Mannchen et al., 2001) for the T&B based LFT uncertainty model. The size of the exclusion zone is then iteratively scaled until a µ-peak value of unity is obtained. The final scaling factor for the Nichols exclusion zone corresponding to a  $\mu$ -peak value of unity is then equivalent to the minimum (worst-case) stability margin  $\rho$  which can be guaranteed for the considered uncertain parameter range. Using this approach the value of the stability margin calculated using the T&B based LFT uncertainty model was 0.31. Compared to the stability margin of 0.82, calculated with the parameter grid, this result is conservative. However, the associated worst case uncertainty parameter combination calculated from the  $\mu$  lower bound is  $\delta_m = 1$ ,  $\delta_{xcg} = 1$  and  $\delta_{zcg} = 1$ , which is exactly the same as the worst case uncertainty parameter combination identified with the parameter gridding approach. Thus, although the guaranteed stability margin is conservative, the worst case uncertainty parameter combination was again found exactly in this example. Furthermore,



Fig. 5 Nichols plots for a variation of parameters

the LFT-based uncertainty model covers all possible uncertainty parameter combinations, in contrast to the classical approach, which merely evaluates the criterion on a finite number of data points. Thus, situations where the worst case parameter combination occurred in the interior of the parameter space, which could easily be missed by the classical approach, would be found by the LFT-based uncertainty model.

### 5. CONCLUSIONS

This paper has introduced a new technique for the generation of linear fractional transformation based uncertainty models which are required as inputs for the analysis of flight control systems using the structured singular value  $\mu$ . In contrast to standard approaches which use symbolic linearized equations of motion, the proposed approach requires only a non-linear software model of the aircraft which can be efficiently trimmed and linearized numerically. In the proposed technique, linear dependencies ("trends") of the aircraft dynamics on the uncertain parameters are modelled by a multi-dimensional regression plane. Additional non-linear dependencies are modelled using a "band"-structure defined by nonlinearity compensation parameters. The capability of the proposed approach to identify the true worst-case combination of uncertain parameters, with significantly reduced modelling effort, was demonstrated via the analysis of a lateral axis controller for a simple civil transport aircraft model. The approach was also compared with classical gridding-based analysis methods via the robustness analysis of a multivariable glide-slope coupler control law for a detailed model of a Boeing 747 transport aircraft. For this example, consistent results were obtained in terms of identification of the worstcase parameter combination, although in general only the trends & bands method guarantees robustness for all possible combinations of the uncertain parameters. The major limitation of the proposed approach is that it is prone to be at least somewhat conservative, especially for systems with a strong non-linear dependence of the state-space representation on the uncertain parameters. The technique allows the process of generating LFT-based uncertainty models to be almost fully automated. More generally, the proposed approach opens up the possibility of applying the powerful  $\mu$  analysis theory to large-scale complex systems which cannot be satisfactorily described using simple differential equation based symbolic models.

### ACKNOWLEDGEMENTS

The authors are pleased to acknowledge the contributions to the design of the controllers used in this analysis from Ridwan Kureemun of Leicester University and Melker Härefors of Volvo Aero Corporation. Thanks also to Hafid Smaili at NLR and all at Delft University of Technology Aerospace Engineering Department for the Boeing 747 aircraft model. This work was completed while the first author was a visiting researcher at the University of Leicester, supported by EPSRC Grant No. GR/R18871/01.

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