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A BOUNDED-ERROR APPROACH TO DESIGNING UNKNOWN INPUT OBSERVERS

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Abstract: This paper focuses on the problem of designing unknown input observers for both linear and non-linear stochastic systems. The main objective is to show how to employ the bounded-error state estimation technique and some transformations of the system equations to form a new bounded-error unknown input observer. It is shown how to extend the proposed approach to tackle the problem of state estimation of non-linear stochastic systems. The final part of this paper shows an example, concerning fault detection of an apparatus that is a part of the evaporation station at the Lublin sugar factory, which confirms the effectiveness of the approach.

Keywords: observers, bounded noise, non-linear systems, fault diagnosis

1. INTRODUCTION

The design and application of the model-based fault diagnosis has received considerable attention during the last few decades. In such a task, the model of the real system of interest is utilized to provide estimates of certain measured and/or unmeasured signals. Then, in the most usual case, the estimates of the measured signals are compared with their originals, i.e. a difference between the estimate and its original is used to form a residual signal. This residual signal can then be employed to detect and isolate system faults. No matter what identification method is used, there is always the problem of model uncertainty, i.e. the model-reality mismatch. This uncertainty may dramatically degrade the reliability of the fault diagnosis system. To overcome this problem, many approaches have been proposed (Chen and Patton, 1999; Patton *et al.*, 2000). Undoubtedly, the most common approach is to use robust observers, such as the Unknown Input Observer (UIO) (Alcorta and Frank, 1997; Chen and Patton, 1999; Chen *et al.*, 1996; Patton and Chen, 1997; Patton *et al.*, 2000), which can tolerate a degree of model uncertainty, and hence increase the reliability of fault diagnosis. In such an approach, the model-reality mismatch is represented by the so called unknown input and hence the state estimate, and consequently the output estimate, is obtained taking into account model uncertainty. Unfortunately, in this context, the research is strongly oriented towards linear deterministic systems. Indeed, the problem of detecting and isolating faults for systems with both modelling uncertainty and the noise has not attracted enough research attention, although most of the real systems suffer from both modelling uncertainty and the noise. The existing approaches (see (Chen and Patton, 1999; Chen *et al.*, 1996; Keller and Darouach, 1999) and the references therein), which can be applied to linear stochastic systems, rely on a similar idea to that of the classical Kalman Filter (KF) (Anderson and Moore, 1979). The main drawback to such techniques lies in their restrictive assumptions concerning the noise distribution, i.e. it is assumed that the process and measurement noises are zero mean white noise sequences. However, in many practical situations it is more natural to assume that only bounds on the noise signals are available (for a detailed description of such approaches we refer the reader to (Maksarow and Norton, 1996a; Maksarow and Norton, 1996b; Milanese *et al.*, 1996; Walter and Pronzato, 1997) and the references therein). This bounded-error approach describes the set of all the states that are consistent with the model, the measured data and the error (or the noise) bounds. All members of this feasible set are then possible solutions to the state estimation problem. Unfortunately, the set obtained in such a way may become extremely complex. For the sake of computational complexity, this feasible set is usually characterized by the smallest (in some sense) ellipsoid that encloses it. Although, in the case of the observers of this type, the so-called unknown input can be treated in a similar way as the process noise, i.e. the only requirements are the bounds of the unknown input, it seems especially attractive to employ the bounded-error approach to design an UIO for linear stochastic systems. This is especially true from the fault isolation point of view. Indeed, in order to design a fault diagnosis system, consisting in a bank of observers, each of the observers should be insensitive to one fault while sensitive to the others. This can be achieved by combining the classical UIO with bounded-error techniques, resulting in an observer for a wide class of linear stochastic systems.

Another problem arises from the application of fault diagnosis to non-linear stochastic systems. Unfortunately, the only existing approaches to this class of systems consist in the application of the Extended Kalman Filter (EKF) (Anderson and Moore, 1979). Indeed, the non-linear extensions of the UIO (Alcorta and Frank, 1997; Chen *et al.*, 1996; Chen and Patton, 1999; Patton and Chen, 1997; Seliger and Frank, 2000) can only be applied to non-linear deterministic systems. Moreover, they require a relatively complex design procedure, even for simple laboratory systems (Zolghardi *et al.*, 1996). To tackle this problem, Witczak and Korbicz proposed the so-called Extended Unknown Input Observer (Witczak and Korbicz, 2001). They used the UIO for linear stochastic systems (Chen and Patton, 1999) to form an UIO for non-linear deterministic systems. Moreover, it is shown that the above observer is convergent under certain conditions as well as its design procedure is almost as simple as that for linear systems. Unfortunately, this approach cannot be applied to non-linear stochastic systems. Thus, it seems to be especially attractive to extend the proposed bounded-error UIO in such a way as it can be applied to non-linear stochastic systems.

The paper is organized as follows. In Section 2, the problem of state estimation of linear systems with bounded system and measurement noises is formulated. Moreover, it is shown how to transform the original system with an unknown input into a system without it. Section 3 describes the bounded-error state estimation algorithm. In the next section, an extension of the proposed approach, which can be applied to non-linear stochastic systems is shown. Section 5 presents the experimental results. Finally, the last section is devoted to conclusions.

2. PROBLEM STATEMENT

Let us consider the following discrete-time linear system

$$\begin{aligned} \boldsymbol{x}_{k+1} &= \boldsymbol{A}_k\boldsymbol{x}_k + \boldsymbol{B}_k\boldsymbol{u}_k + \boldsymbol{E}_k\boldsymbol{d}_k + \boldsymbol{w}_k, \\ \boldsymbol{y}_k &= \boldsymbol{C}_k\boldsymbol{x}_k + \boldsymbol{v}_k, \end{aligned} \tag{1}$$

where $\boldsymbol{x}_{k+1} \in \mathbb{R}^n$ denotes the state vector, $\boldsymbol{u}_k \in \mathbb{R}^r$ is the input vector, $\boldsymbol{d}_k \in \mathbb{R}^q$ is an unknown input vector, $\boldsymbol{y}_k \in \mathbb{R}^m$ is the output vector, and $\boldsymbol{w}_k$ and $\boldsymbol{v}_k$ are the process and measurement noises (or errors), respectively. The matrices $\boldsymbol{A}_k$, $\boldsymbol{B}_k$, $\boldsymbol{C}_k$, $\boldsymbol{E}_k$ are assumed to be known and have appropriate dimensions. As has already been mentioned, the robustness to model uncertainty and other factors which may lead to an unreliable fault detection is of paramount importance. In the case of UIO, the robustness problem is tackled by means of introducing the concept of unknown input $\boldsymbol{d}_k$, and hence the term $\boldsymbol{E}_k\boldsymbol{d}_k$ may represent various kinds of modelling uncertainty. The remaining factors can be modeled by $\boldsymbol{w}_k$ and $\boldsymbol{v}_k$. Indeed, it is only necessary to know the bounds of $\boldsymbol{w}_k$ and $\boldsymbol{v}_k$, which can be defined by the following sets

$$\mathbb{W}_k = \bigcap_i \left\{ \boldsymbol{w}_k : -b_i \le w_k^i \le b_i \right\}, \tag{2}$$

and

$$\mathbb{V}_k = \bigcap_i \left\{ \boldsymbol{v}_k : -r_i \le v_k^i \le r_i \right\}. \tag{3}$$

In order to use the bounded-error algorithm described in (Maksarow and Norton, 1996a) for the state estimation problem of the system (1) it is necessary to introduce some modifications concerning the unknown input. In the existing approaches the unknown input is treated in two different ways. The first one (Chen and Patton, 1999) relies on introducing an additional matrix into the state estimation equation, which is then used for de-coupling the effect of the unknown input on the state estimation error (and consequently the residual signal). In the second one (Keller and Darouach, 1999), the system (1) is suitably transformed into a system without the unknown input. In the case of the algorithm (Maksarow and Norton, 1996a) it seems more convenient to use the second approach.

Let us assume that $\text{rank}(\boldsymbol{C}_k\boldsymbol{E}_k) = q$ and

$$\boldsymbol{T}_k = \boldsymbol{\alpha}(\boldsymbol{I} - \boldsymbol{H}_k^+\boldsymbol{H}_k), \tag{4}$$

where $\boldsymbol{H}_k = (\boldsymbol{C}_k\boldsymbol{E}_k)^+$ denotes the generalized inverse or pseudo-inverse of $\boldsymbol{C}_k\boldsymbol{E}_k$, and $\boldsymbol{\alpha}$ is an arbitrary matrix chosen such that $\boldsymbol{T}$ is a full-row rank matrix. Since $\text{rank}\left([\boldsymbol{H}_k\ \boldsymbol{T}_k]^T\right) = m$ the system (1) can be transformed into an equivalent form

$$\boldsymbol{x}_{k+1} = \boldsymbol{A}_k\boldsymbol{x}_k + \boldsymbol{B}_k\boldsymbol{u}_k + \boldsymbol{E}_k\boldsymbol{d}_k + \boldsymbol{w}_k, \tag{5}$$

$$\boldsymbol{H}_k\boldsymbol{y}_{k+1} = \boldsymbol{H}_k\boldsymbol{C}_k\boldsymbol{x}_{k+1} + \boldsymbol{H}_k\boldsymbol{v}_{k+1}, \tag{6}$$

$$\boldsymbol{T}_k\boldsymbol{y}_{k+1} = \boldsymbol{T}_k\boldsymbol{C}_k\boldsymbol{x}_{k+1} + \boldsymbol{T}_k\boldsymbol{v}_{k+1}. \tag{7}$$

Substituting the relation (5) into (6) leads to

$$\begin{aligned}\boldsymbol{H}_k\boldsymbol{y}_{k+1} = \boldsymbol{H}_k\boldsymbol{C}_k\left[\boldsymbol{A}_k\boldsymbol{x}_k + \boldsymbol{B}_k\boldsymbol{u}_k + \boldsymbol{w}_k\right] + \boldsymbol{d}_k \\ +\boldsymbol{H}_k\boldsymbol{v}_{k+1},\end{aligned} \tag{8}$$

or equivalently

$$\begin{aligned}\boldsymbol{d}_k = \boldsymbol{H}_k\left[\boldsymbol{y}_{k+1} - \boldsymbol{C}_k\left[\boldsymbol{A}_k\boldsymbol{x}_k + \boldsymbol{B}_k\boldsymbol{u}_k + \boldsymbol{w}_k\right]\right. \\ \left.-\boldsymbol{v}_{k+1}\right],\end{aligned} \tag{9}$$

Inserting (9) into (5) leads to an alternative form of the system (1)

$$\begin{aligned}\boldsymbol{x}_{k+1} &= \bar{\boldsymbol{A}}_k\boldsymbol{x}_k + \bar{\boldsymbol{B}}_k\boldsymbol{u}_k + \bar{\boldsymbol{E}}_k\boldsymbol{y}_{k+1} + \bar{\boldsymbol{w}}_k, \\ \bar{\boldsymbol{y}}_{k+1} &= \bar{\boldsymbol{C}}_k\boldsymbol{x}_{k+1} + \bar{\boldsymbol{v}}_{k+1},\end{aligned} \tag{10}$$

where

$$\bar{\boldsymbol{A}}_k = [\boldsymbol{I} - \boldsymbol{E}_k\boldsymbol{H}_k\boldsymbol{C}_k]\boldsymbol{A}_k, \tag{11}$$

$$\bar{\boldsymbol{B}}_k = [\boldsymbol{I} - \boldsymbol{E}_k\boldsymbol{H}_k\boldsymbol{C}_k]\boldsymbol{B}_k, \quad \bar{\boldsymbol{E}}_k = \boldsymbol{E}_k\boldsymbol{H}_k, \tag{12}$$

$$\bar{\boldsymbol{w}}_k = [\boldsymbol{I} - \boldsymbol{E}_k\boldsymbol{H}_k\boldsymbol{C}_k]\boldsymbol{w}_k - \bar{\boldsymbol{E}}\boldsymbol{v}_{k+1}, \tag{13}$$

$$\bar{\boldsymbol{y}}_{k+1} = \boldsymbol{T}_k\boldsymbol{y}_{k+1}, \quad \bar{\boldsymbol{C}}_k = \boldsymbol{T}_k\boldsymbol{C}_k, \tag{14}$$

$$\bar{\boldsymbol{v}}_{k+1} = \boldsymbol{T}_k\boldsymbol{v}_{k+1}. \tag{15}$$

The bounds of $\bar{\boldsymbol{w}}_k$ and $\bar{\boldsymbol{v}}_{k+1}$, i.e.

$$\bar{\mathbb{W}}_k = \bigcap_i \left\{\bar{\boldsymbol{w}}_k : -\bar{b}_i \le \bar{w}_k^i \le \bar{b}_i\right\}, \tag{16}$$

and

$$\bar{\mathbb{V}}_k = \bigcap_i \left\{\bar{\boldsymbol{v}}_k : -\bar{r}_i \le \bar{v}_k^i \le \bar{r}_i\right\}, \tag{17}$$

can easily be obtained using the equations (2) and (3).

Since the system (1) was transformed into an equivalent form (10) it is straightforward to use the state estimation algorithm described in (Maksarow and Norton, 1996a). The purpose of the subsequent section is to give an outline of the above algorithm.

3. STATE ESTIMATION

In a similar manner to the classical Kalman filtering procedure, the bounded-error approach consists in alternating two phases, i.e. the time and measurement updates. Unlike the classical approach where the initial state estimate $\hat{\boldsymbol{x}}_0$ is assumed to be a random variable, it is assumed that $\hat{\boldsymbol{x}}_0$ belongs to an ellipsoidal set defined as

$$\begin{aligned}\mathbb{E}_0 = \Big\{\boldsymbol{x}_0 : (\boldsymbol{x}_0 - \hat{\boldsymbol{x}}_0)^T \\ \boldsymbol{P}_0^{-1}(\boldsymbol{x}_0 - \hat{\boldsymbol{x}}_0) \le 1\Big\},\end{aligned} \tag{18}$$

where $\hat{\boldsymbol{x}}_k$ denotes the centre of the ellipsoid (the state estimate), and $\boldsymbol{P}_0$ is a positive definite matrix describing its size and orientation. Thus, the ellipsoid containing all the admissible states at time $k-1$ is

$$\begin{aligned}\mathbb{E}_{k-1} = \Big\{\boldsymbol{x}_{k-1} : (\boldsymbol{x}_{k-1} - \hat{\boldsymbol{x}}_{k-1})^T \\ \boldsymbol{P}_{k-1}^{-1}(\boldsymbol{x}_{k-1} - \hat{\boldsymbol{x}}_{k-1}) \le 1\Big\},\end{aligned} \tag{19}$$

As a result of the *time update*, being a consequence of transforming the $\mathbb{E}_{k-1}$ according to the state transition equation, the ellipsoid $\mathbb{E}_{k/k-1}$ is obtained. The centre of the new ellipsoid is

$$\hat{\boldsymbol{x}}_{k/k-1} = \bar{\boldsymbol{A}}_{k-1}\hat{\boldsymbol{x}}_{k-1} + \bar{\boldsymbol{B}}_{k-1}\boldsymbol{u}_{k-1} + \bar{\boldsymbol{E}}_k\boldsymbol{y}_k. \tag{20}$$

The matrix defining its size and orientation is successively computed by

$$\boldsymbol{P}_{k/k-1}^0 = \bar{\boldsymbol{A}}_{k-1}\boldsymbol{P}_{k-1}\bar{\boldsymbol{A}}_{k-1}^T, \tag{21}$$

$$\begin{aligned}\boldsymbol{P}_{k/k-1}^{i+1} &= (1+p_i)\boldsymbol{P}_{k/k-1}^i \\ &+(1+p_i^{-1})\bar{b}_i^2\boldsymbol{l}_i\boldsymbol{l}_i^T, \\ &i = 1,\ldots,n-1,\end{aligned} \tag{22}$$

where $l_i = [0,\ldots,\overbrace{1}^{i},\ldots,0]^T$. The value of the parameter $p_i > 0$ in (22) is the positive root of

$$np_i^2 + (n-1)\text{trace}(\boldsymbol{Q}_i)p_i - \text{trace}(\boldsymbol{Q}_i) = 0, \tag{23}$$

where

$$\boldsymbol{Q}_i = \bar{b}_i\left(\boldsymbol{P}_{k/k-1}^i\right)^{-1}\boldsymbol{l}_i\boldsymbol{l}_i^T. \tag{24}$$

Finally $\boldsymbol{P}_{k/k-1} = \boldsymbol{P}_{k/k-1}^n$.

The *measurement update* intersects the predicted ellipsoid $\mathbb{E}_{k/k-1}$ with the pairs of parallel hyperplanes defined using (17), i.e.

$$\mathbb{O}_k = \bigcap_i \left\{\boldsymbol{x}_k : \bar{y}_k^i - \bar{r}_i \le (\bar{\boldsymbol{c}}^i)^T\boldsymbol{x}_k \le \bar{y}_k^i + \bar{r}_i\right\}, \tag{25}$$

where $\bar{\boldsymbol{C}}^T = [\bar{\boldsymbol{c}}_1,\ldots,\bar{\boldsymbol{c}}_m]$. As a result the ellipsoid $\mathbb{E}_k \subset \mathbb{E}_{k/k-1} \cap \mathbb{O}_k$ is obtained. The centre, size and orientation defining matrix is successively computed as

$$\hat{\boldsymbol{x}}_k^0 = \hat{\boldsymbol{x}}_{k/k-1},\ \boldsymbol{P}_k^0 = \boldsymbol{P}_{k/k-1}, \tag{26}$$

$$\hat{\boldsymbol{x}}_k^{i+1} = \hat{\boldsymbol{x}}_k^i + q_i\frac{\boldsymbol{S}_k^i\bar{\boldsymbol{c}}_ie_i}{d_i^2}, \tag{27}$$

$$\boldsymbol{P}_k^{i+1} = \left(1 + q_i - \frac{q_ie_i^2}{d_i^2 + q_ig_i}\right)\boldsymbol{S}_k^i \tag{28}$$

where

$$e_i = \sqrt{g_i}\frac{\alpha_i^+ + \alpha_i^-}{2}, \quad d_i = \sqrt{g_i}\frac{\alpha_i^+ - \alpha_i^-}{2}, \tag{29}$$

$$g_i = \bar{\boldsymbol{c}}_i^T\boldsymbol{P}_k^i\bar{\boldsymbol{c}}_i,\ i = 0,\ldots,m-1. \tag{30}$$

In the standard procedure any hyperplane bound which does not intersect $\mathbb{E}_k^i$ is replaced by the closest parallel hyperplane touching $\mathbb{E}_k^i$. The parameters α_i^+ and α_i^- denote the normalized distances from the centre of the ellipsoid $\mathbb{E}_k^i$ to each of the i-th hyperplane after such a replacement. The above parameters can be obtained in the following way.

Let the i-th hyperplane bound be

$$\mathbb{V}_k^i = \left\{ \bar{\boldsymbol{v}}_k : \beta_i^+ \le \bar{\boldsymbol{c}}_i^T \boldsymbol{x}_k \le \beta_i^- \right\}. \tag{31}$$

For each ellipsoid $\mathbb{E}_k^j$, $j = 0, \ldots, i$, the normalized distances are

$$\alpha_j^+ = \frac{\bar{y}_k^i - \bar{\boldsymbol{c}}_i^T \hat{\boldsymbol{x}}_k^j + \bar{r}_k^i}{\sqrt{\bar{\boldsymbol{c}}_i^T \boldsymbol{P}_k^i \bar{\boldsymbol{c}}_i}}, \tag{32}$$

$$\alpha_j^- = \frac{\bar{y}_k^i - \bar{\boldsymbol{c}}_i^T \hat{\boldsymbol{x}}_k^j - \bar{r}_k^i}{\sqrt{\bar{\boldsymbol{c}}_i^T \boldsymbol{P}_k^i \bar{\boldsymbol{c}}_i}}. \tag{33}$$

In the next step

$$\text{If } \alpha_j^+ > 1 \text{ then } \beta_j^+ = \bar{\boldsymbol{c}}_i^T \hat{\boldsymbol{x}}_k^j + \sqrt{\bar{\boldsymbol{c}}_i^T \boldsymbol{P}_k^i \bar{\boldsymbol{c}}_i}, \tag{34}$$

$$\text{If } \alpha_j^- < -1 \text{ then } \beta_j^- = \bar{\boldsymbol{c}}_i^T \hat{\boldsymbol{x}}_k^j - \sqrt{\bar{\boldsymbol{c}}_i^T \boldsymbol{P}_k^i \bar{\boldsymbol{c}}_i}, \tag{35}$$

$$\text{If } -1 \le \alpha_j^+ \le 1 \text{ then } \beta_j^+ = \bar{y}_k^i + \bar{r}_k^i, \tag{36}$$

$$\text{If } -1 \le \alpha_j^+ \le 1 \text{ then } \beta_j^+ = \bar{y}_k^i - \bar{r}_k^i, \tag{37}$$

If $\alpha_j^+ < -1$ or $\alpha_j^- > 1$ then $\mathbb{V}_k^i$ does not intersect the j-th intermediate ellipsoid at time k. This may correspond to an inaccurate selection of the noise bounds. Such a property makes it possible to check the consistency of the whole model with the measured data. Moreover, faults can be detected in a similar way. Indeed, a fault occurrence may (in some sense) be equivalent to the model inconsistency with the measured data. The parameters β_i^+ and β_i^+ in (31) are defined as

$$\beta_i^+ = \min_j \beta_j^+, \; \beta_i^- = \max_j \beta_j^-, \; j = 0, \ldots, i, \tag{38}$$

and finally the parameters α_i^+ and α_i^- in (29) are defined as

$$\alpha_i^+ = \frac{\beta_i^+ - \bar{\boldsymbol{c}}_i^T \hat{\boldsymbol{x}}_k^i}{\sqrt{\bar{\boldsymbol{c}}_i^T \boldsymbol{P}_k^i \bar{\boldsymbol{c}}_i}}, \quad \alpha_i^- = \frac{\beta_i^- - \bar{\boldsymbol{c}}_i^T \hat{\boldsymbol{x}}_k^i}{\sqrt{\bar{\boldsymbol{c}}_i^T \boldsymbol{P}_k^i \bar{\boldsymbol{c}}_i}}. \tag{39}$$

If $\alpha_i^+ \alpha_i^- \le -1/n$, then $\mathbb{E}_k^{i+1} = \mathbb{E}_k^i$, else the parameter q_i minimizing the volume of $\mathbb{E}_k^{i+1}$ should be obtained as positive root of

$$a_1 q_i^2 + a_2 q_i + a_3 = 0, \tag{40}$$

where

$$a_1 = (n-1) g_i^2, \tag{41}$$

$$a_2 = \left((2n-1) d_i^2 - g_i + e_i^2 \right) g_i, \tag{42}$$

$$a_3 = \left(n(d_i^2 - e_i^2) - g_i \right) d_i^2. \tag{43}$$

Finally $\hat{\boldsymbol{x}}_k = \hat{\boldsymbol{x}}_k^m$, and $\boldsymbol{P}_k = \boldsymbol{P}_k^m$.

4. AN EXTENSION TO NON-LINEAR SYSTEMS

As has already been mentioned, the application of the EKF to the state estimation of non-linear systems has received considerable attention during the last two decades. This is mainly because the EKF can be directly applied to a large class of non-linear systems as well as the implementation procedure is almost as simple as that for linear systems. The main drawback to such an approach is that its performance strongly depends on the difference between the non-linear system and the model linearized around the current state estimate. This is mainly because of the fact that in the EKF case the linearization errors are neglected. In the proposed approach, as in the EKF, the state equation is linearized around the current state estimate. The main difference between these two approaches is that in the proposed technique the linearization errors are taken into account as additional disturbances.

Let us consider a class of non-linear systems which can be modeled by the following equations

$$\begin{aligned} \boldsymbol{x}_{k+1} &= \boldsymbol{g}(\boldsymbol{x}_k) + \boldsymbol{h}(\boldsymbol{u}_k) + \boldsymbol{E}_k \boldsymbol{d}_k + \boldsymbol{w}_k, \\ \boldsymbol{y}_{k+1} &= \boldsymbol{C}_{k+1} \boldsymbol{x}_{k+1} + \boldsymbol{v}_{k+1}, \end{aligned} \tag{44}$$

where $\boldsymbol{g}(\boldsymbol{x}_k)$ is assumed to be continuously differentiable with respect to $\boldsymbol{x}_k$. In order to linearize the system (44) around the current state estimate, let us define the following matrix

$$\boldsymbol{A}_k = \left. \frac{\partial \boldsymbol{g}(\boldsymbol{x}_k)}{\partial \boldsymbol{x}_k} \right|_{\boldsymbol{x}_k = \hat{\boldsymbol{x}}_k}, \tag{45}$$

then the state equation of the system (44) can be transformed into an equivalent form

$$\begin{aligned} \boldsymbol{x}_{k+1} = \boldsymbol{g}(\hat{\boldsymbol{x}}_k) &+ \boldsymbol{A}_k(\boldsymbol{x}_k - \hat{\boldsymbol{x}}_k) + \boldsymbol{h}(\boldsymbol{u}_k) + \boldsymbol{E}_k \boldsymbol{d}_k \\ &+ \boldsymbol{w}_k + \boldsymbol{r}(\boldsymbol{x}_k, \hat{\boldsymbol{x}}_k), \end{aligned} \tag{46}$$

with the linearization error $\boldsymbol{r}(\boldsymbol{x}_k, \hat{\boldsymbol{x}}_k)$ satisfying

$$\|\boldsymbol{r}(\boldsymbol{x}_k, \hat{\boldsymbol{x}}_k)\|_\infty \le \gamma \|\boldsymbol{x}_k - \hat{\boldsymbol{x}}_k\|_\infty, \boldsymbol{x}_k, \hat{\boldsymbol{x}}_k \in \mathbb{E}_k, \tag{47}$$

where $\|\boldsymbol{x}\|_\infty = \max_{i=1,\ldots,n} |x_i|$. The equation (46) can be expressed in a more convenient form

$$\boldsymbol{x}_{k+1} = \boldsymbol{A}_k \boldsymbol{x}_k + \boldsymbol{u}_{k,f} + \boldsymbol{E}_k \boldsymbol{d}_k + \boldsymbol{w}_k + \boldsymbol{r}(\boldsymbol{x}_k, \hat{\boldsymbol{x}}_k), \tag{48}$$

where

$$\boldsymbol{u}_{k,f} = \boldsymbol{g}(\hat{\boldsymbol{x}}_k) - \boldsymbol{A}_k \hat{\boldsymbol{x}}_k + \boldsymbol{h}(\boldsymbol{u}_k), \tag{49}$$

For the purpose of further considerations, it will be more convenient to express the ellipsoid $\mathbb{E}_k$ as a deviation from its centre

$$\bar{\mathbb{E}}_k = \{\boldsymbol{z} : \hat{\boldsymbol{x}}_k + \boldsymbol{z} \in \mathbb{E}_k\}. \tag{50}$$

Let

$$\phi_k = \sup_{\boldsymbol{z} \in \bar{\mathbb{E}}_k} \|\boldsymbol{z}\|_\infty = \|\sqrt{\boldsymbol{P}_k(1,1)}, \ldots, \sqrt{\boldsymbol{P}_k(n,n)}\|_\infty, \tag{51}$$

then, using (47), the following relation is satisfied

$$\|\boldsymbol{r}(\boldsymbol{x}_k, \hat{\boldsymbol{x}}_k)\|_\infty \le \gamma \phi_k, \tag{52}$$

which implies that

$$-\gamma \phi_k \le r(\boldsymbol{x}_k, \hat{\boldsymbol{x}}_k)^i \le \gamma \phi_k, \quad i = 1, \ldots, n. \tag{53}$$

This means that the linearization error $\boldsymbol{r}(\boldsymbol{x}_k, \hat{\boldsymbol{x}}_k)$ can be treated as an additional disturbance vector with known bounds (53). Finally, the system (44) can be put in the following form

$$\begin{aligned} \boldsymbol{x}_{k+1} &= \boldsymbol{A}_k \boldsymbol{x}_k + \boldsymbol{u}_{k,f} + \boldsymbol{E}_k \boldsymbol{d}_k + \breve{\boldsymbol{w}}_k, \\ \boldsymbol{y}_{k+1} &= \boldsymbol{C}_{k+1} \boldsymbol{x}_{k+1} + \boldsymbol{v}_{k+1}, \end{aligned} \quad (54)$$

where

$$\breve{\boldsymbol{w}}_k = \boldsymbol{w}_k + \boldsymbol{r}(\boldsymbol{x}_k, \hat{\boldsymbol{x}}_k), \quad (55)$$

The bounds of $\breve{\boldsymbol{w}}$

$$\breve{\mathbb{W}}_k = \bigcap_i \left\{ \breve{\boldsymbol{w}}_k : -\breve{b}_i \leq \breve{w}_k^i \leq \breve{b}_i \right\}, \quad (56)$$

can easily be computed using (2) and (53). Since the system (44) is expressed in the form (54), it is straightforward to perform the system transformation detailed in Section 2 and then to use the state estimation algorithm described in Section 3.

5. EXPERIMENTAL RESULTS

The purpose of this section is to design a fault detection system for an apparatus that is a part of the evaporation station. A detailed description of the above plant and its model can be found in (Witczak and Korbicz, 2002). Unfortunately, the modelled system is an MISO one and hence it is impossible to design any UIO. Indeed, if $\boldsymbol{d}_k \in \mathbb{R}^q$, $q \leq m = 1$, then the matrix $\boldsymbol{T}_k$ equals zero, i.e. $\boldsymbol{T}_k = \boldsymbol{0}$, which makes it impossible to perform the system transformation described in Section 2. The same problem occurs with the decoupling approach presented in (Chen and Patton, 1999). Nevertheless, this drawback pertains to all unknown input observers, not only those presented in this work. Irrespective of the above consideration, it is possible to use the model described in (Witczak and Korbicz, 2002). Indeed, the second system output $y_{2,k}$ can be simulated by the model while the first output $y_{1,k}$ remains original.

The unknown input distribution matrix was obtained using the approach described in (Chen and Patton, 1999) (assuming that $q = 1$) and, as a result, the matrix $\boldsymbol{E}_k$ was $\boldsymbol{E}_k = [11, 95.8]^T$. The constant γ was assumed to be $\gamma = 0.2$.

To demonstrate the effectiveness of the obtained fault detection scheme, the following fault scenarios were considered:

Case 1: An abrupt fault of an actuator

$$f_{a,k} = \begin{cases} 0, & k < 250, \\ -0.25u_{1,k}, & \text{otherwise}, \end{cases} \quad (57)$$

Case 2: An abrupt fault of an actuator

$$f_{a,k} = \begin{cases} 0, & k < 100, \\ 0.3u_{2,k}, & \text{otherwise}, \end{cases} \quad (58)$$

As can be seen from Fig. 1, the residual is sensitive to all the faults. Moreover, detection of the considered faults was performed relatively fast.

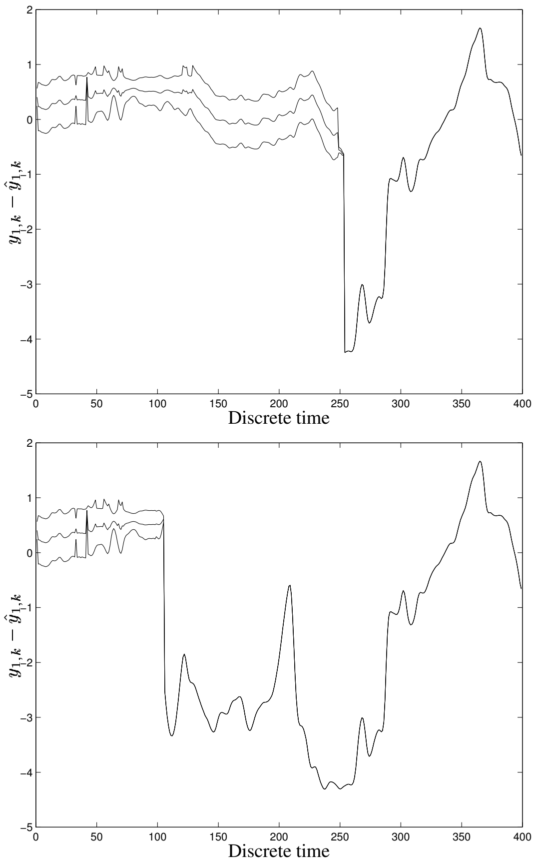

Fig. 1. A residual and its bounds for actuator faults: Case 1 (left), Case 2 (right).

6. CONCLUDING REMARKS

The main purpose of this paper was to propose a new unknown input observer for linear stochastic systems. This was achieved with the use of the bounded-error state estimation technique and a suitable transformation of the system equations. An extension of the proposed observer, which can be applied to the state estimation of non-linear stochastic systems was proposed as well. This was performed by applying the linearization technique similar to that of the classical EKF. Unlike in the case of the EKF, the proposed approach does not neglect the linearization errors. Indeed, these errors are taken into account as additional disturbances.

The drawbacks and advantages of the proposed approach were discussed during its application to fault detection of the apparatus that is a part of the evaporation station at the Lublin sugar factory in Poland.

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