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TRAJECTORY CONTROLLER DESIGN OF MOBILE ROBOT BASED ON BACK-STEPPING PROCEDURE

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Abstract: In this paper, the constructive modeling procedure of nonholonomic mobile robot system is carried out with the help of controllability Lie algebra used in differential geometry field. And, a new trajectory controller is suggested to guarantee its convergence to reference trajectory. Design procedure of the suggested trajectory controller is back-stepping scheme which was introduced recently in nonlinear control theory. The performance of the proposed trajectory controller is verified via computer simulation. In the simulation the trajectory controller is applied to differentially driven mobile robot system on the assumption that the trajectory planner be given. *Copyright © 2002 IFAC*

Keywords: Mobile robot, Nonlinear control, Differential geometry, Trajectory, Controller, Tracking

1. INTRODUCTION

Generally, the wheel-driven mobile robot systems, by their structural property, have nonholonomic constraints. The Brockett's theorem says that such systems cannot be stabilized to an equilibrium point by smooth and time-invariant state feedback controllers (Brockett et al., 1983). Although nonholonomic systems have been studied in classical mechanics for more than 150 years. it is only recently that the study of control problems for such systems has been initiated (Wang, 1996; Bullo, 1999). The reason is as follows. Constraints of nonholonomic systems are not integrable and cannot be written as time derivatives of some function of the generalized coordinates. Hence, nonlinear approaches are required to solve the problems

In this paper, the constructive modeling procedure of nonholonomic mobile robot systems is carried out with the help of controllability Lie algebra used in differential geometry field, and their geometrical properties are also analyzed. And, a new trajectory controller is suggested to guarantee its convergence to reference trajectory. Design procedure of the suggested trajectory controller is back-stepping scheme which was introduced recently in nonlinear control theory. The back-stepping procedure guarantees the existence of Lyapunov function to verify the stability of the overall system, and provides alternative of the existing feedback linearization technique (d'Andrea-Novel et al., 1992), which often confronts the complexity of design procedure in MIMO system and some problems to make worse system's performance. The design procedure of the trajectory controller for mobile robot system is summarized as follows: the kinematic model of a mobile robot is transformed into the chained nonholonomic form via coordinate transformation and its input change. and then the chained nonholonomic system is used to design the trajectory controller based on the back-stepping procedure. The stability of the suggested trajectory controller is guaranteed by the existence of Lyapunov function and its asymptotical stability is also proved by applying the Habalat's lemma (Slotine. 1991)

The performance of the proposed trajectory controller is verified via computer simulation. In the simulation, the trajectory controller is applied 'to differentially driven mobile robot system on the assumption that the trajectory planner be given.

2. CONTROLLER DESIGN USING BACK-STEPPING SCHEME

In this chapter, the feedback controller design method of the nonlinear system using back-stepping scheme is proposed. Suppose that the controlled system is represented by

$$\dot{x} = f(x) + g(x) \quad (1a)$$
$$\dot{\xi} = u \quad (1b)$$

And suppose that smooth feedbak control laws $u = \alpha(x)$, $\alpha(0) = 0$ where input is $\xi \in R$ exist in (1a). and (1a) satisfies following condition,

$$\frac{\partial V}{\partial x}(x)[f(x) + g(x)\alpha(x)] \le -W(x) \le 0 \; \forall x \in \boldsymbol{R}^n \quad (2)$$

where $V: R^n \to R$ is positive definitive and radially unbounded smooth function, $W: R^n \to R$ is positive definitive or positive semi-definitive function.

First, if $W(x)$ is positive definitive function, the following function (3) is control Lyapunov function to the entire system, and thus there exists a feedback controller $u = \alpha_a(x, \xi)$ that globally and asymptotically stabilizes at equilibrium point $x = 0, \xi = 0$.

$$V_a(x, \xi) = V(x) + \frac{1}{2}(\xi - \alpha(x))^2 \quad (3)$$

For a example of the stable feedback controller, there is as follows.

$$u = -c(\xi - \alpha(x)) + \frac{\partial \alpha}{\partial x}[f(x) + g(x)\xi] - \frac{\partial V}{\partial x}(x) g(x), \quad c > 0 \quad (4)$$

If $W(x)$ is positive semi-definitive function, there exists a feedback control input $u = a_a(x, \xi) > 0$ that satiesfies $\dot{V}a \le -W(x, \xi) \le 0$ and becomes $W(x, \xi) > 0$ when $W(x) > 0$ or $\xi \neq \alpha(x)$. And the state variable $[x(t)^T, \xi(t)]^T$ of the overall feedback control system converges to the largest invariant set contained in the following set (Kritic. et al., 1995),

$$E_a = \{[x^T, \xi]^T \in R^{n+1} \mid W(x) = 0\}$$

Introducing the error variable $z = \xi - \alpha(x)$, the state equation of the control system is given bv

$$\dot{x} = f(x) + g(x)[\alpha(x) + z] \quad (5a)$$
$$\dot{z} = u - \frac{\partial \alpha}{\partial x}(x)[f(x) + g(x)(\alpha(x) + z)] \quad (5b)$$

Eqution (5a) satisfies the assumption given by (2), so using (2) the time derivative of the control Lyapunov function $V_a(x, \xi)$ is given by

$$\begin{aligned} vu &= \frac{\partial V}{\partial x}(f + g[\alpha + z]) + z[u - \frac{\partial \alpha}{\partial x}(f + g[\alpha + z])] \\ &\le W(x) + z[\frac{\partial V}{\partial x} g + u - \frac{\partial \alpha}{\partial x}(f + g[\alpha + z])] \end{aligned} \quad (6)$$

Thus selecting the control input U that satisfies $\dot{V}_a \le -W_a(x, \xi) \le -W(x)$, by the positive definitive character of W_a and LaSalle-Yoshizawa's theorem (Slotine, 1991), X, z and ξ is globally bounded and $W(x(t))$ and $z(t)$ converge to 0 along the time $t \to \infty$. And by the LaSalle's theorem. it is guaranted that $[x(t)^T, \xi(t)]^T$ converges to the largest invariant set contained in the set $E, = \{[x^T, \xi]^T \in \boldsymbol{R}^{n-1} \mid W(x) = 0\}$.

To satisfy the upper property, $\dot{V}_a$ has to be negative definitive function about z. Applying the control input U given by (4) to the equation (6), there is as follows.

$$\dot{V}_a \le -W(x) - cz^2 = -W_a(x, \xi) \le 0 \quad (7)$$

If $W(x)$ is positive definitive function, by the LaSalle-Yoshizawa's theorem, the equilibrium point $x = 0, z = 0$ is globally and asymptotically stable. Thus $x = 0, \xi = 0$ is also globally and asymptotically stable because of $z = \xi - \alpha(x)$ and $\alpha(0) = 0$.

Now we extend that the controlled system has the increased form by k integrator Then the feedback controller design of that system using back-stepping scheme is proposed. In this case, the controlled system can be written as

$$\begin{aligned} \dot{x} &= f(x) + g(x)\xi \\ \dot{\xi}_1 &= \xi_2 \\ &\vdots \\ \dot{\xi}_{k-1} &= \xi_k \\ \dot{\xi}_k &= u \end{aligned} \quad (8)$$

In the system given by (8), suppose that $\xi_1, \cdots, \xi_k$ are virtual inputs. And then applying repeatedly the back-stepping procedure explained previously, the Lyapunov function is given by

$$V_a(x,\ \xi_1,\cdots,\ \xi_k)=$$
$$V(x)+\frac{1}{2}\sum_{i=1}^{k}[\ \xi_i-\ \alpha_{i-1}(x,\ \xi_1,\cdots,\ \xi_{i-1})\ ]^2 \tag{9}$$

Similarly selecting the control input u that satisfies $\dot{V}_a\leq -W_a(x,\ \xi_1,\cdots\ \xi_{i-1})\leq 0$, $W(x)=0$ when $W_a(x,\ \xi_1,\cdots,\ \xi_{i-1})$ equals 0 and $\xi_i=\ \alpha_{i-1}(x,\ \xi_1,\cdots,\ \xi_{i-1})$ to all i, the state variable $[x^T(t),\xi_1(t),\cdots,\xi_{k(t)}]^T$ of the overall system is globally bounded and by the LaSalle-Yoshizawa's theorem converges to the largest invariant set M_a contained in the following set,

$$E_a=\{[x^T,\xi_1,\cdots,\xi_k]^T\in R^{n+k}\mid W(x)=0,$$
$$\xi_i=\ \alpha_{i-1}(x,\xi_1,\cdots,\ \xi_{i-1}),i=1,\cdots k\}$$

If $W(x)$ is positive definitive function, $x=0$ is globally and asymptotically stable by means of ξ_1. It means that equilibrium point $x=0$, $\xi_1=\cdots=\xi_k=0$ can be stable via control input u. That is, to the system contained with k integrator, we can also see that the feedback Controller design using back-stepping procedure guarantees simultaneously the stability and boundness of the overall system.

As examined above, in the case of the controller design of nonlinear system using back-stepping pi-ocedure always guarantees the stability of the overall feedback system on each step.

3. TRAJECTORY CONTROLLER DESIGN OF MOBILE ROBOT

Fig. 1 shows the appearance of differentially driven mobile robot used in this experiment. This mobile robot is Pioneer 1 made in Active Media Inc. The structure of this mobile robot is composed of two fixed wheel and one caster type wheel.

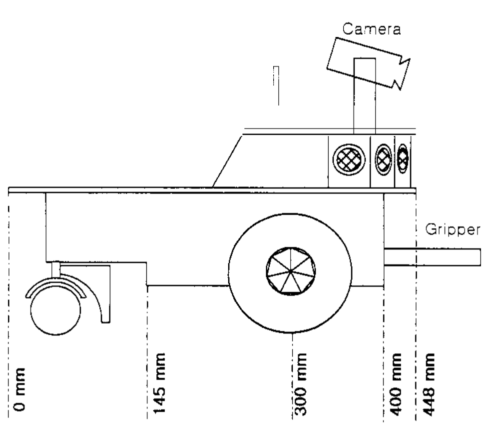

Fig. 1. Pioneer 1 mobile robot

The kinematic model of differentially driven mobie robot can be written as

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\end{bmatrix}=\begin{bmatrix}\cos(\theta) & 0\\ \sin(\theta) & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}v\\ \omega\end{bmatrix} \tag{10}$$

where v is line velocity of mobile robot and w is revolutionary angular velocity of mobile robot.

To the kinematic model of the mobile robot given by (10), we perform the following diffeomorphism coordinate transfomation (11) and input transfomation (12).

$$\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}=\begin{bmatrix}0 & 0 & 1\\ \cos(\theta) & \sin(\theta) & 0\\ \sin(\theta) & -\cos(\theta) & 0\end{bmatrix}\begin{bmatrix}x\\ y\\ \theta\end{bmatrix} \tag{11}$$

$$\begin{aligned}u_1&=\omega\\ u_2&=v-\omega x_3\end{aligned} \tag{12}$$

Then the kinematic model of the mobile robot can be transformed into the following chained nonholonomic form.

$$\begin{aligned}\dot{x}_1&=u_1\\ \dot{x}_2&=u_2\\ \dot{x}_3&=x_2\,u_1\end{aligned} \tag{13}$$

Thus the trajectory planner model of the mobile robot is designed as follows.

$$\begin{aligned}\dot{x}_{1d}&=\ u_{1d}\\ \dot{x}_{2d}&=\ u_{2d}\\ \dot{x}_{3d}&=\ x_{2d}\ u_{1d}\end{aligned} \tag{14}$$

From the equation (13) and (14), the error dynamics is given by

$$\begin{aligned}\dot{x}_{1e}&=\ u_1-\ u_{1d}\\ \dot{x}_{2e}&=\ u_2-\ u_{2d}\\ \dot{x}_{3e}&=\ x_{2e}\ u_{1d}+\ x_2(\ u_1-\ u_{1d})\end{aligned} \tag{15}$$

The trajectory control problem is identical with the feedback controller design problem that makes the state variable error converge to 0. Now we design the nonlinear controller that asymptotically stabilizes the equilibrium point of the error dynamics (15) using back-stepping procedure. Applying the diffeomorphism coordinate trans-fomation $\Phi_1(x_e;x_d):R^n\rightarrow R^n$, we can obtain new state variable (16) and state equation (17).

$$\begin{aligned}y_1&=x_{3e}-(x_{2e}+x_{2d})x_{1e}\\ y_2&=x_{2e}\\ y_3&=x_{1e}\end{aligned} \tag{16}$$

$$\begin{aligned}\dot{y}_1&=u_{1d}y_2-u_2y_3\\ \dot{y}_2&=u_2-u_{2d}\\ \dot{y}_3&=u_1-u_{1d}\end{aligned} \tag{17}$$

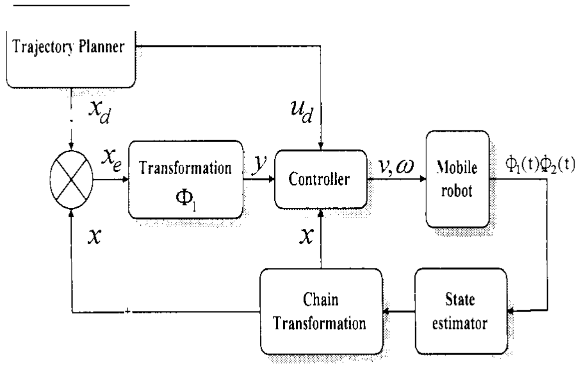

Fig 2 Block diagram of trajectory controller

From the equation (17), designing the controller using back-stepping procedure, the controller can he obtained as

$$
\begin{aligned}
u_2 &= u_{2d} - c_2 \tilde{y}_2 - u_{1d}\tilde{y}_1 \\
u_1 &= u_{1d} - c_3 y_3 + \tilde{y}_1 u_2
\end{aligned} \tag{18}
$$

where c_1, c_2 and c_3 are design variables. Applying the equation (12) to (18), the equation (18) can be transformed as follows.

$$
\begin{aligned}
v &= u_{2d} - c_2\tilde{y}_2 - u_{1d}\tilde{y}_1 + x_3[u_{1d} - c_3 y_3 + \\
&\qquad \tilde{y}_1(u_{2d} - c_2\tilde{y}_2 - u_{1d}\tilde{y}_1)] \\
\omega &= u_{1d} - c_3 y_3 + \tilde{y}_1(u_{2d} - c_2\tilde{y}_2 - u_{1d}\tilde{y}_1)
\end{aligned} \tag{19}
$$

The velocity v and angular velocity ω of the mobile robot are given by the composition of revolutionary veiocity at each wheel.

Synthesizing the procedure of the controller design, the trajectory controller of the mobile robot proposed in this paper is shown as Fig. 2.

4. COMPUTER SIMULATION AND RESULTS

To verify the validity of the trajectory controiler proposed in this paper, we perform the computer simulation. The trajectory used in this simulation is given by table 1.

Table 1 Specification of trajectory used in simulation

Trajectory name	Trajectory form	Initial position	Average velocity	Average angular velocitv
	Straight motion	$0, \frac{\pi}{4})$	0.1 m/s	0 rad/s
B	Rotational motion	(0,0.0)	0.1 m/s	0.2rad/s

In each case, suppose that the initial posture of the mobile robot has the error The design variables used in the simulation are given by

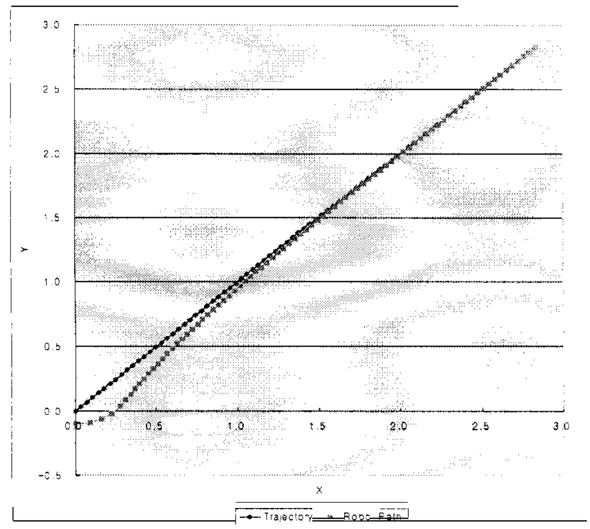

Fig 3(a) Tracking of mobile robot to trajectory A

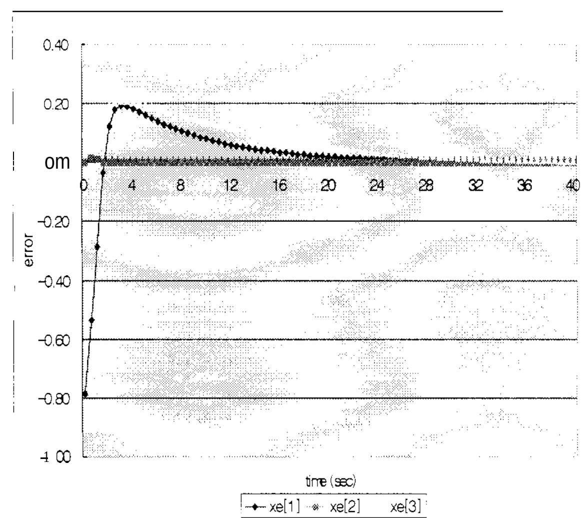

Fig. 3(b). The error on tracking the trajecory A

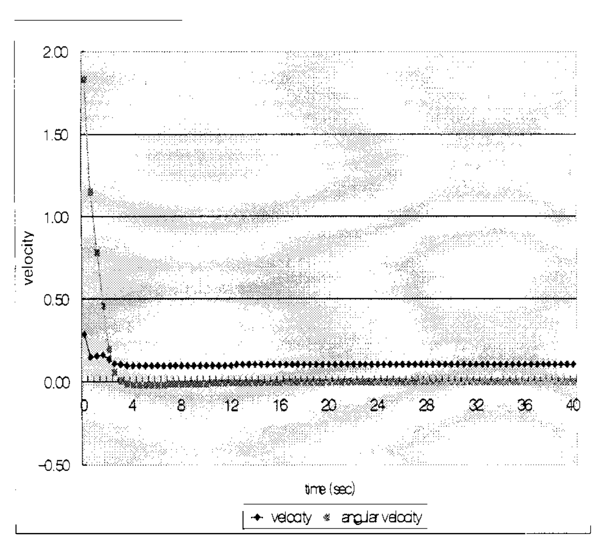

Fig 3(c) Control input of mobile robot on tracking the trajectory A

$$c_1=5,\ c_2=5,\ c_3=2$$

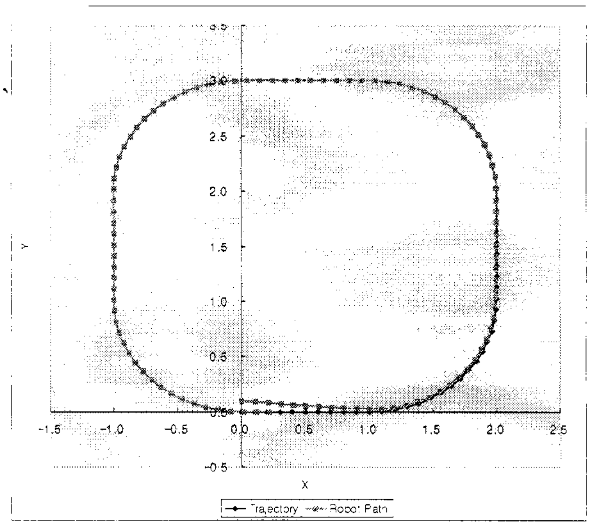

Fig. 4(a). Tracking of mobile robot to trajectory B

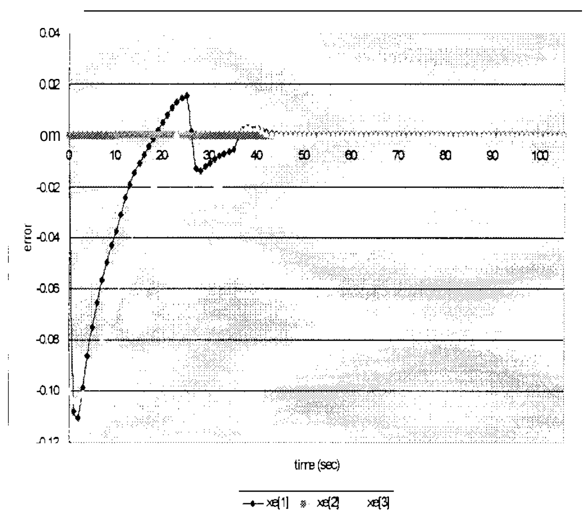

Fig. 4(b) The error on tracking the trajecory B

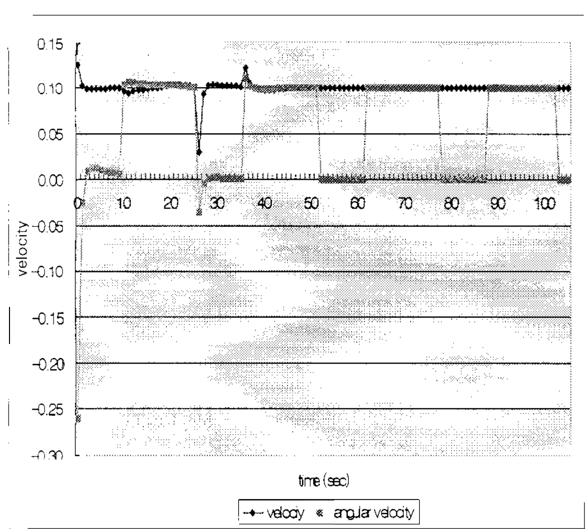

Fig 4(c) Control input of mobile robot on tracking the trajectory B

Fig. 3 shows the tracking features of the mobile robot to the trajectory A. In Fig. 3(a), the initial position of the mobile robot is $(0, -0.1, 0)$ and the trajectory generates to the angle of 45° with respect to x axis. In this case, we can see that the tracking of the mobile robot does'nt show the overshoot and the mobile robot converges to reference trajectory by the trajectory controller. The controller designed by back-stepping scheme is a little slow in the converging speed than Kanayama's controller but doesn't generate overshoot in the trajectory tracking(Kanayama, et al., 1991). Fig. 3(b) shows the state variable error of the mobile robot on tracking the trajectory A. Fig. 3(c) shows the control input of the mobile robot on tracking the trajectory A.

Fig. 4 shows the tracking features of the mobile robot to the trajectory B. In this case, the trajectory is generated for the mobile robot to do straight and rotational motion repeatedly. After the mobile robot converges to reference trajectory, we can see that the mobile robot performs perfect tracking if the change of curvature in the trajectory is bounded.

Fig. 5 shows the change of trajectory tracking response according to the gain change of the designed controller. In this case, the gain values to be selected are given by

$$\text{gain\_1}:\ c_1=10,\ c_2=10,\ c_3=5$$
$$\text{gain\_2}:\ c_1=5,\ c_2=5,\ c_3=2$$
$$\text{gain\_3}:\ c_1=2,\ c_2=2,\ c_3=1$$

If the gain values of the controller are low, we can see that the converging speed to the trajectory may be slow. On the other hand, in proportion as the gain values are high, the control values are increased and the speed of response will be fast

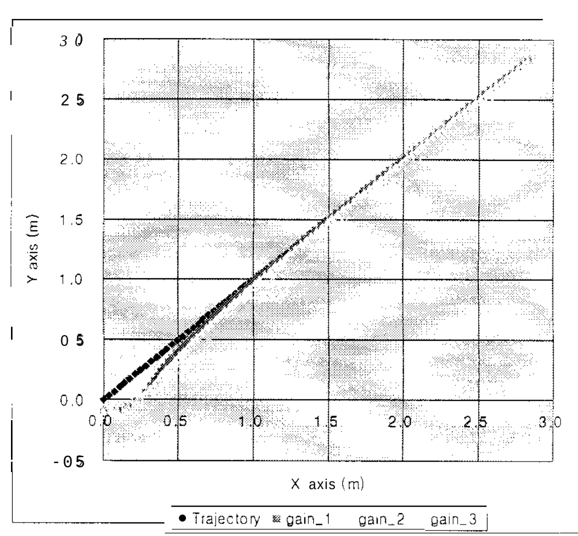

Fig. 5. Change of tracking response according to the change of gain

5. CONCLUSION

The dynamic characteristics of a mobile robot was analyzed through the analysis of the nonholonomic system. And, a new trajectory controller of a mobile robot was suggested to guarantee its convergence to reference trajectory. Design procedure of the suggested trajectory controller is back-stepping scheme which was introduced recently in nonlinear control theory. The back-stepping procedure guarantees the existence of Lyapunov function to verify the stability of the overall system. The kinematic model of a mobile robot was transformed into the chained nonholonomic form via coordinate transformation and its input change. And then the chained nonholonomic system was used to design the trajectory controller based on the back-stepping procedure.

The performance of the proposed trajectory controller was verified via computer simulation. In the simulation, the controller of a mobile robot generates no overshoot and converges better to rotational trajectory that has many change of the curvature. The proposed trajectory controller can be applied to every type mobile robot that can be transformed into chained form. We can see that this method is more systematic and efficient than the trajectory controller based on feedback linearization technique proposed by d'Andrea-Novel (d'Andrea-Novel, 1992).

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