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LMI APPROACH TO H_∞ FILTERING OF LINEAR NEUTRAL SYSTEMS WITH DELAY

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Abstract: An improved delay-dependent H_∞ filtering design is proposed for linear, continuous, time-invariant systems with time delay. The resulting filter is of the Luenberger observer type and it guarantees that the H_∞ -norm of the system, relating the exogenous signals to the estimation error, is less than a prescribed level. The filter is based on the application of the descriptor model transformation and Park's inequality for the bounding of cross terms. The advantage of the new filtering scheme is clearly demonstrated via simple examples. *Copyright C 2001 IFAC*

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1. INTRODUCTION

The H_∞ filtering problem for linear systems with delay-dependent (see Fattou *et al.*, 1998 and Palhares *et al.*, 2000) and (more conservative) delay-independent (see Ge *et al.*, 2000 and Mahmoud, 2000) designs have acquired a lot of attention recently. The prevailing methods are based on bounded real lemmas (BRLs) in terms of Riccati algebraic equations or Linear Matrix Inequalities (LMIs) which guarantee a prescribed attenuation level. Unfortunately, these criteria provide only sufficient conditions for the required attenuation and they may lead, in many cases, to conservative filter designs.

Recently, a new approach to H_∞ filtering has been introduced (Fridman and Shaked, 2001a). This approach is based on representing the system by a descriptor type model (see Fridman, 2001 and Fridman and Shaked, 2001b) and on deriving a BRL for the corresponding adjoint system. The new BRL was found to be very efficient and it considerably reduced the achievable attenuation level as compared to other results reported in the literature. By assuming a Leunberger type estimator, the new BRL was applied to the resulting estimation error system and provided the best filtering estimates. In spite of the advantage of the new filter design, it still entails a significant amount of conservatism stemming from the over-bounding of mixed terms in the proof of the BRL in (Fridman and Shaked, 2001a).

A new over-bounding technique has recently been proposed that produces tighter bounds (Park, 1999). In the present note, this technique is applied to reduce the over-design entailed in the approach of (Fridman and Shaked, 2001a). The treatment is also extended to the more general class of neutral type systems with multiple delays. It is shown, via simple examples, that the resulting schemes dramatically improve the estimation results.

Notation: Throughout the paper the superscript 'T' stands for matrix transposition, $\mathcal{R}^n$ denotes the n dimensional Euclidean space, $\mathcal{R}^{n\times m}$ is the set of all $n \times m$ real matrices, and the notation $P>0$, for $P \in \mathcal{R}^{n\times n}$ means that P is symmetric and positive definite. The space of functions in $\mathcal{R}^q$ that are square integrable over $[0 \quad \infty)$ is denoted by $\mathcal{L}_2^q[0,\ \infty)$.

2. PROBLEM FORMULATION

Consider the following system:

$$\dot{x} - F\dot{x}(t-g) = A_0x + A_1x(t-h) + Bw,$$
$$x(t) = 0, \forall t \leq 0, \qquad (1)$$

where $x(t) \in \mathcal{R}^n$ is the system state vector and $w(t) \in \mathcal{L}_2^q[0,\ \infty]$ is the exogenous disturbance signal. The time delays $h > 0,\ g > 0$ are assumed to be known. The matrices $A_0,\ A_1,\ F$ and B are constant matrices of appropriate dimensions. For simplicity only one delay is considered, however, the results can easily be generalized to any finite number of delays.

It is assumed :

A1 All of the eigenvalues of F are inside the unit circle.

Given the measurement equation

$$y = Cx + D_{21}w \qquad (2)$$

where $y(t) \in \mathcal{R}^r$ is the measurement vector and the matrices C and D_{21} are constant matrices of appropriate dimension, a filter of the following Luenberger observer form is sought:

$$\dot{\hat{x}} - F\dot{\hat{x}}(t-g) = A_0\hat{x} + A_1\hat{x}(t-h) + K(y - C\hat{x}) \qquad (3)$$

This filter must ensure that the performance index

$$J(w) = \int_0^\infty (z^Tz - \gamma^2 w^Tw)d\tau \qquad (4)$$

is negative $\forall w(t) \in \mathcal{L}_2^q[0,\ \infty]$, for a prescribed value of γ. The signal $z(t) \in \mathcal{R}^p$ is the state combination to be estimated and is given by:

$$z \triangleq L(x - \hat{x}) \qquad (5)$$

where L is a constant matrix.

3. DELAY-DEPENDENT H_∞ FILTERING

From (1)-(3) it follows that the estimation error $e(t) = x(t) - \hat{x}(t)$ is described by the following model

$$\dot{e} - F\dot{e}(t-g) = (A_0 - KC)e + A_1e(t-h)$$
$$+ (B - KD_{21})w, \quad z = Le. \qquad (6)$$

The problem then becomes one of finding the filter gain K such that the H_∞ -norm of the system of (6) will be less than a prescribed value of γ.

3.1. H_∞ -norm of the 'adjoint' system.
Using the arguments of (Fridman and Shaked, 2001a) it can be shown that the H_∞ -norms of the system described by (6) and the following system are equal:

$$\dot{\xi} - F^T\xi(\tau-g) = (A_0^T - C^TK^T)\xi + A_1^T\xi(\tau-h)$$
$$+ L^T\tilde{z}, \quad \xi(\tau) = 0, \ \forall\tau < 0$$
$$\tilde{w} = (B^T - D_{21}^TK^T)\xi. \qquad (7)$$

where $\xi(t) \in \mathcal{R}^n$, $\tilde{z}(t) \in \mathcal{R}^p$ and $\tilde{w}(t) \in \mathcal{R}^q$. Note that the latter system represents the forward adjoint of (6) (as defined in (Bensoussan, 1992).

For the equivalent descriptor form representation of (3a):

$$\dot{\xi} = \zeta, \ 0 = -\zeta + F^T\zeta(t-g) + (\Sigma_{i=0}^1 A_i^T - C^TK^T)\xi$$
$$-A_1^T \int_{t-h}^t \zeta(s)ds + L^T\tilde{z},$$

the following Lyapunov-Krasovskii functional has been suggested in (Fridman and Shaked, 2002) :

$$V(t) = [\xi^T(t)\ \zeta^T(t)]EP\begin{bmatrix}\xi(t)\\ \zeta(t)\end{bmatrix}$$
$$+\int_{t-h}^t \xi^T(\tau)S\xi(\tau)d\tau + \int_{t-g}^t \zeta^T(\tau)U\zeta(\tau)d\tau$$
$$+\int_{-h}^0 \int_{t+\theta}^t \zeta^T(s)[0\ A_1]R\begin{bmatrix}0\\ A_1^T\end{bmatrix}\zeta(s)d\tau d\theta, \quad (8)$$

where

$$E = \begin{bmatrix} I_n & 0\\ 0 & 0\end{bmatrix},\ P = \begin{bmatrix} P_1 & 0\\ P_2 & P_3\end{bmatrix},\ P_1, U, S, R > 0 \qquad (9)$$

The first term of (8) corresponds to the descriptor system (see e.g. (Takaba *et al.*, 1995 and Masubuchu *et al.*, 1997), the third - to the delay-independent conditions with respect to the discrete delays of ζ , while the second and the fourth terms - to the delay-dependent conditions with respect to the distributed delays.

Based on a similar functional, a BRL was derived in (Fridman and Shaked, 2001a) which provided an LMI sufficiency condition for the H_∞ -norm of (3) to be less than γ. This condition, though still efficient compared to other methods in the literature, is still conservative, due to the bounding of a mixed term in the proof of the BRL in (Fridman and Shaked, 2001a). Recently, an improved BRL was proposed by (Fridman and Shaked, 2002), which considerably reduces the over-design entailed in the over bounding of the above mixed term. It is based on the fact that for any $2n \times 2n$-matrices $R > 0$ and M, the following holds (see Park, 1999): $-2\int_{t-h}^{t} b^T(s)a(s)ds \leq$

$$\int_{t-h}^{t} \begin{bmatrix} a(s) \\ b(s) \end{bmatrix}^T \begin{bmatrix} R & RM \\ M^TR & \Upsilon \end{bmatrix} \begin{bmatrix} a(s) \\ b(s) \end{bmatrix} ds \qquad (10)$$

for $a(s) \in \mathcal{R}^{2n}$, $b(s) \in \mathcal{R}^{2n}$, and for a given $s \in [t-h, t]$. Here $\Upsilon \triangleq (M^TR+I)R^{-1}(RM+I)$.

In the proof of the BRL in (Fridman and Shaked, 2001a), $M = 0$ was chosen. Taking $M \neq 0$ the following result is obtained (see Fridman and Shaked, 2002):

Lemma 1. Consider the system of (6). Given $\gamma > 0$ and $K \in \mathcal{R}^{n\times r}$, the cost function (4) achieves $J(w) < 0$ for all nonzero $w \in \mathcal{L}_2^q[0, \infty)$ and for all positive delay g, if there exist $n \times n$-matrices $0 < P_1,\ P_2,\ P_3,\ S,\ U$ and $2n \times 2n$-matrices $W,\ R$ that satisfy the following LMI:

$$\begin{bmatrix} \Psi & P^T\begin{bmatrix} 0 \\ L^T \end{bmatrix} & h\Phi & -W^T\begin{bmatrix} 0 \\ A_1^T \end{bmatrix} & P^T\begin{bmatrix} 0 \\ F^T \end{bmatrix} & \begin{bmatrix} B-KD_{21} \\ 0 \end{bmatrix} \\ * & -\gamma^2 I & 0 & 0 & 0 & 0 \\ * & * & -hR & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & -S & 0 & 0 \\ * & * & * & * & -U & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0 \quad (11)$$

where P is given by (9) and

$\Phi = W^T + P^T,$

$$\Psi \triangleq P^T\begin{bmatrix} 0 & I \\ \Sigma_{i=0}^{1} A_i^T - C^TK^T & -I \end{bmatrix} + \begin{bmatrix} 0 & \Sigma_{i=0}^{1} A_i - KC \\ I & -I \end{bmatrix} P$$
$$+ \begin{bmatrix} S & 0 \\ 0 & U + h[0\ A_1]R\begin{bmatrix} 0 \\ A_i^T \end{bmatrix} \end{bmatrix} + W^T\begin{bmatrix} 0 & 0 \\ A_1^T & 0 \end{bmatrix} + \begin{bmatrix} 0 & A_1 \\ 0 & 0 \end{bmatrix} W$$

Remark 1. For

$$W = -P,\ R = \frac{\epsilon I_{2n}}{h} \qquad (12)$$

the LMI of (11) produces for $\epsilon \to 0^+$ the following BRL condition that is delay-independent:

$$\begin{bmatrix} \hat{\Psi} & P^T\begin{bmatrix} 0 \\ L^T \end{bmatrix} & P^T\begin{bmatrix} 0 \\ A_1^T \end{bmatrix} & P^T\begin{bmatrix} 0 \\ F^T \end{bmatrix} & \begin{bmatrix} B-KD_{21} \\ 0 \end{bmatrix} \\ * & -\gamma^2 I & 0 & 0 & 0 \\ * & * & -S & 0 & 0 \\ * & * & * & -U & 0 \\ * & * & * & * & -I_p \end{bmatrix} < 0$$

where

$$\hat{\Psi} = P^T\begin{bmatrix} 0 & I \\ A_0^T - C^TK^T & -I \end{bmatrix} + \begin{bmatrix} 0 & A_0 - KC \\ I & -I \end{bmatrix} P + \begin{bmatrix} S & 0 \\ 0 & U \end{bmatrix}.$$

3.2. The case of instantaneous measurements

Restricting the discussion to the case of $W_i = \epsilon_i P,\ i = 1, 2$, where $\epsilon_i \in \mathcal{R}$ is a scalar parameter, enables the formulation of an LMI. Note that for $\epsilon_i = 0$, LMI (11) implies the delay-dependent conditions of (Fridman, 2001) and (Fridman and Shaked 2001b), while for $\epsilon_i = -1$, LMI (11) yields the delay-independent condition of Remark 1. It is obvious from the requirement of $0 < P_1$, and the fact that the $(2,2)$ block in Ψ is negative definite, that $-(P_3 + P_3^T)$ must be negative definite, and thus P is nonsingular. Defining

$$P^{-1} \triangleq Q = \begin{bmatrix} Q_1 & 0 \\ Q_2 & Q_3 \end{bmatrix},\ \Delta \triangleq diag\{Q,\ I_{q+p+7n}\}, \quad (13)$$

LMI (11) is multiplied by Δ^T and Δ, on the left and on the right, respectively. Applying Schur's formula to the quadratic term in Q, the following inequality results:

$$\begin{bmatrix} \Xi & \begin{bmatrix} 0 \\ L^T \end{bmatrix} & h(\epsilon I + I) & \epsilon\begin{bmatrix} 0 \\ A_1^T \end{bmatrix} & \begin{bmatrix} 0 \\ F^T \end{bmatrix} & Q^T\begin{bmatrix} B-KD_{21} \\ 0 \end{bmatrix} & \begin{bmatrix} Q_1 \\ 0 \end{bmatrix} & \begin{bmatrix} Q_2^T \\ Q_3^T \end{bmatrix} & h\begin{bmatrix} 0 & Q_2^T A_1 \\ 0 & Q_3^T A_1 \end{bmatrix} \\ * & -\gamma^2 I_p & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -hR_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -S & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -U & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -I_q & 0 & 0 & 0 \\ * & * & * & * & * & * & -S^{-1} & 0 & 0 \\ * & * & * & * & * & * & * & -U^{-1} & 0 \\ * & * & * & * & * & * & * & * & -hR^{-1} \end{bmatrix} < 0 \quad (14)$$

where

$$\Xi = \begin{bmatrix} 0 & I_n \\ \Sigma_{i=0}^{1} A_i^T + \epsilon A_1^T - C^TK^T & -I_n \end{bmatrix} Q + Q^T\begin{bmatrix} 0 & * \\ I_n & -I_n \end{bmatrix}.$$

Denoting Q_1K by Y, we obtain the following:

Theorem 2. Consider the system of (1) and the cost function of (4). For a prescribed $0 < \gamma$, $J(w) < 0$ for all nonzero $w \in \mathcal{L}_2^q[0, \infty)$ if for some prescribed scalar ϵ, there exist $Q_1 > 0,\ \bar{S},\ \bar{U},\ Q_2,\ Q_3, \in \mathcal{R}^{n\times n}$, $\bar{R} \in \mathcal{R}^{2n\times 2n}$ and $Y \in \mathcal{R}^{n\times r}$ that satisfy the following LMI:

$$
\begin{bmatrix}
Q_2+Q_2^T & \Xi_1 & 0 & h(\epsilon+1)\bar{R}_1 & h(\epsilon+1)\bar{R}_2 \\
* & -Q_3-Q_3^T & L^T & h(\epsilon+1)\bar{R}_2^T & h(\epsilon+1)\bar{R}_3 \\
* & * & -\gamma^2 I_q & 0 & 0 \\
* & * & 0 & -h\bar{R}_1 & -h\bar{R}_2 \\
* & * & * & * & -h\bar{R}_3 \\
* & * & * & * & * \\
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* & * & * & * & * \\
* & * & * & * & *
\end{bmatrix.
$$

$$
\left.\begin{matrix}
0 & Q_1 & Q_1B-YD_{21} & 0 & Q_2^T & 0 & hQ_2^TA_1 \\
\epsilon A_1^T\bar{S} & 0 & 0 & F\bar{U} & Q_3^T & 0 & hQ_3^TA_1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\bar{S} & 0 & 0 & 0 & 0 & 0 & 0 \\
* & -\bar{S} & 0 & 0 & 0 & 0 & 0 \\
* & * & -I & 0 & 0 & 0 & 0 \\
* & * & * & -\bar{U} & 0 & 0 & 0 \\
* & * & * & * & -\bar{U} & 0 & 0 \\
* & * & * & * & * & -h\bar{R}_1 & -h\bar{R}_2 \\
* & * & * & * & * & * & -h\bar{R}_3
\end{matrix}\right] < 0
$$

where $\bar{R}_1$, $\bar{R}_2$ and $\bar{R}_3$ are the $(1,1)$, $(1,2)$ and $(2,2)$ blocks of $\bar{R}$, respectively, and where

$$\Xi_1 = Q_3 - Q_2^T + Q_1(A_0 + A_1 + \epsilon A_1) - YC.$$

The filter gain is then given by

$$K = Q_1^{-1}Y.$$

Note that in the latter LMI $\bar{S}$, $\bar{U}$ and $\bar{R}$ are the inverses of S, U and R of (14), respectively. If this LMI possesses a solution for $h > 0$ then, because of the special dependence of its matrix entries on the delay length, it will also posses a solution for all $0 < \bar{h} < h$.

The result of the Theorem 1 is applied to the following example.

Example 1: Consider the same system as found in (de Souza and Lee, 1999) to which a state-feedback has been applied. Assuming that the measurement equation is the same as in (2), an observer which achieves a minimum estimation level is sought. The matrices corresponding to (1), (2) and (5) are as follows:

$$A_0 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \; A_1 = \begin{bmatrix} -1 & -1 \\ 0 & -.9 \end{bmatrix}, \; B = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix},$$

$$L = \begin{bmatrix} 1 & 0 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 \end{bmatrix}, \; D_{21} = .01, \; h = 0.999 \; secs.$$

Note that the system is unstable. Using the method of (Fridman and Shaked, 2001a), a minimum value of $\gamma = 22.8784$ was obtained with a filter gain matrix of $K = \begin{bmatrix} 4790 & 18139 \end{bmatrix}^T$.

On the other hand, applying Theorem 1, for $h = 0.999$ *seconds*, a minimum value of $\gamma = .0823$ was achieved by using $\varepsilon = -0.28$. The resulting filter gain was $K = 10^4 \begin{bmatrix} 6.158 & 6.1594 \end{bmatrix}^T$. Furthermore, while it was impossible to obtain a solution for $h \geq 1$, using the method of (Fridman and Shaked, 2001a), it was found that, by applying the LMI of Theorem 1, a solution for all $h \leq 1.295$ was available. For, say, $h = 1.25 sec.$ and $\varepsilon = -0.33$, a minimum value of $\gamma = 0.61$, with $K = 10^5 \begin{bmatrix} 2.2354 & 2.2358 \end{bmatrix}^T$ was obtained.

3.3. The case of delayed measurements

The above results were obtained for the case where no delay is encountered in the measurement. In case the measurement includes delayed state information of the form:

$$y = col\{C_0 x, \; C_1 x(t-h)\} + D_{21} w, \quad (15)$$

where $C_0 \in \mathcal{R}^{r_1 \times n}$ and $C_1 \in \mathcal{R}^{r_2 \times n}$ are constant matrices and $r_1 + r_2 = r$, an additional component is placed in series with the delayed component of y. The state space model of this component is given by:

$$\dot{\eta} = -\rho I_{r_2} \eta + \begin{bmatrix} 0 & \rho I_{r_2} \end{bmatrix} y \quad (16)$$

for $1 << \rho$. Denoting the augmented state vector by $\xi = col\{x, \; \eta\}$, the augmented system is then described by:

$$\dot{\xi} - \tilde{F}\dot{\xi}(t-g) = \tilde{A}_0 \xi + \tilde{A}_1 \xi(t-h) + \tilde{B} w \quad (17)$$

where

$$\tilde{A}_0 = \begin{bmatrix} A_0 & 0 \\ 0 & -\rho I_{r_2} \end{bmatrix}, \; \tilde{A}_1 = \begin{bmatrix} A_1 & 0 \\ \rho C_1 & 0 \end{bmatrix}, \; \tilde{F} = \begin{bmatrix} F \\ 0 \end{bmatrix}$$

$$\text{and } \tilde{B} = \begin{bmatrix} B \\ \rho[0 \;\; I_{r_2}] D_{21} \end{bmatrix}.$$

The following augmented filter is considered:

$$\dot{\hat{\xi}} - \tilde{F}\dot{\hat{\xi}}(t-g) = \tilde{A}_0 \hat{\xi} + \tilde{A}_i \hat{\xi}(t-h) + \tilde{K}(col\{[I_{r_1} \; 0]y, \; \eta\} - \tilde{C}\hat{\xi}) \quad (18)$$

where $\tilde{C} = diag\{C_0,\ I_{r_2}\}$.

The resulting estimation error vector is denoted by $\tilde{e}(t) = \xi(t) - \hat{\xi}(t)$ with the following state space representation:

$$\dot{\tilde{e}} - \tilde{F}\dot{\tilde{e}}(t-g) = (\tilde{A}_0 - \tilde{K}\tilde{C})\tilde{e} + \tilde{A}_1\tilde{e}(t-h) + \tilde{B}w - \tilde{K}\begin{bmatrix} I_{r_1} & 0 \\ 0 & 0 \end{bmatrix} D_{21}w.$$

Letting $\tilde{z} = \tilde{L}\tilde{e},\ \tilde{L} = [L\ \ 0]$ and considering

$$J_1 = \int_0^\infty (\tilde{z}^T\tilde{z} - \gamma^2 w^T w)d\tau \qquad (20)$$

one could apply Lemma 1 to obtain $\tilde{K}$ via an LMI that corresponds to the one in Theorem 1. The problem is, however, that due to the $O(\rho)$ entries in $\tilde{A}$ and $\tilde{B}$ the restriction of $W = \epsilon P,\ \epsilon \in \mathcal{R}$, which was made in order to obtain the LMI of Theorem 1, forces ϵ to be $O(\rho^{-1})$ and thus the solution that will be achieved for this scalar ϵ will tend to the one obtained in (Fridman and Shaked, 2001a).

In order to utilize the extra freedom provided by Park's over-bounding method, diagonal matrices $\bar{\epsilon}_i$ are sought that satisfy $W = \bar{\epsilon}P$. Denoting

$$\bar{\epsilon} = diag\{\bar{\epsilon}_1,\ \bar{\epsilon}_2\},\ \bar{\epsilon} \in \mathcal{R}^{n \times n}, \quad \bar{n} = n + r$$

and applying the method of Section 3.2, results in the following theorem:

Theorem 3. Consider the system of (17), (15) and (18) and the cost function (20). For a prescribed $0 < \gamma$ and for $\rho >> 1$, $J_1 < 0$ for all nonzero $w \in \mathcal{L}_2^q[0,\ \infty)$ if for some prescribed diagonal matrices $\bar{\epsilon}_1,\ \bar{\epsilon}_2 \in \mathcal{R}^{\bar{n} \times \bar{n}}$ there exist $Q_1 > 0,\ \bar{S},\ \bar{U},\ Q_2,\ Q_3 \in \mathcal{R}^{\bar{n} \times \bar{n}},\ \bar{R} \in \mathcal{R}^{2\bar{n} \times 2\bar{n}}$ and $Y \in \mathcal{R}^{(\bar{n}+r) \times r}$ that satisfy:

$$\left[\begin{matrix} Q_2 + Q_2^T & \tilde{\Xi}_1 & 0 & h(\bar{\epsilon}_1 + I_{\bar{n}})\bar{R}_1 & h(\bar{\epsilon}_1 + I_{\bar{n}})\bar{R}_2 \\ * & -Q_3 - Q_3^T & \tilde{L}^T & h(\bar{\epsilon}_2 + I_{\bar{n}})\bar{R}_2^T & h(\bar{\epsilon}_2 + I_{\bar{n}})\bar{R}_3 \\ * & * & -\gamma^2 I_q & 0 & 0 \\ * & * & 0 & -h\bar{R}_1 & -h\bar{R}_2 \\ * & * & * & * & -h\bar{R}_3 \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \end{matrix}\right.$$

$$\left.\begin{matrix} 0 & Q_1 & Q_1\tilde{B} - Y\begin{bmatrix} I_{r_1} & 0 \\ 0 & 0 \end{bmatrix} D_{21} & 0 & Q_2^T & 0 & hQ_2^T\tilde{A}_1 \\ \bar{\epsilon}_2\tilde{A}_1^T\bar{S} & 0 & 0 & \tilde{F}\bar{U} & Q_3^T & 0 & hQ_3^T\tilde{A}_1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\bar{S} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & -\bar{S} & 0 & 0 & 0 & 0 & 0 \\ * & * & -I_q & 0 & 0 & 0 & 0 \\ * & * & * & -\bar{U} & 0 & 0 & 0 \\ * & * & * & * & -\bar{U} & 0 & 0 \\ * & * & * & * & * & -h\bar{R}_1 & -h\bar{R}_2 \\ * & * & * & * & * & * & -h\bar{R}_3 \end{matrix}\right] < 0$$

where $\bar{R}_i,\ \bar{R}_2$ and $\bar{R}_3$ are the $(1,1),\ (1,2)$ and $(2,2)$ blocks of $\bar{R}$ and

$$\tilde{\Xi}_1 = Q_3 - Q_2^T + Q_1(\Sigma_{i=0}^1\tilde{A}_i + \tilde{A}_1\bar{\epsilon}_2) - Y\tilde{C}.$$

The filter gain is then given by

$$\tilde{K} = Q_1^{-1}Y.$$

Remark 2. The problem of choosing $\bar{\epsilon}_i,\ i = 1,2$ is now more involved. One way to reduce the complexity is to choose zeros for those diagonal elements of $\bar{\epsilon}_2$ that correspond to the $O(\rho)$ elements in $\tilde{A}_1$ and the same scalar for all the other diagonal elements in $\bar{\epsilon}_i,\ i = 1,2$.

The existence of a solution to the LMI of Theorem 2 guarantees that the filter built from the series connection of (16) and (18) will achieve the required performance as long as $0 < \rho$. Considering, however, $1 << \rho$ and denoting

$$\tilde{K} = \begin{bmatrix} K_{00} & K_{01} \\ K_{10} & K_{11} \end{bmatrix},$$

it follows from Theorem 2 that if the LMI is feasible, then the estimate of $x(t)$ is given by:

$$\dot{\hat{x}} - F\dot{\hat{x}}(t-g) = A_0\hat{x} + A_i\hat{x}(t-h) + K_{00}([I_{r_1}\ \ 0]y - C_0\hat{x}) + K_{01}(\eta - \hat{\eta}).$$

When $1 << \rho$ and $r_1 = 0$ (namely, when all of the measurements are delayed) the latter equation, together with the one obtained from (18) for $\hat{\eta}$, lead to the following filter:

$$\dot{\hat{x}} - F\dot{\hat{x}}(t-g) = A_0\hat{x} + A_i\hat{x}(t-h) + \bar{K}(y(t-h) - C_1\hat{x}(t-h)) + \phi$$

where

$\bar{K} = (\rho I_r + K_{11})^{-1}\rho K_{01} + O(\rho^{-1})$

and where $\phi = O(\rho^{-1}, s)$. The latter filter, with $\phi = 0$, will achieve the required estimation accuracy if ρ is chosen to be large enough.

The use of the results of Theorem 2 are demonstrated by the following example:

Example 2: Consider the system of Example 1 with a delay $h = 0.9sec.$. The measurement equation is as in (15) with

$$C_0 = 0,\ C_1 = \begin{bmatrix} 0 & 1 \end{bmatrix} \text{ and } r_2 = r = 1.$$

This example was solved in (Fridman and Shaked, 2001a) where, for $\rho = 10^{10}$, a minimum value of $\gamma = 128.406$ was obtained for the gain matrix $\bar{K} = \begin{bmatrix} -0.8450 & 0.2045 \end{bmatrix}^T$. Applying the result of Theorem 2, for $\bar{\epsilon}_1 = -0.22 I_3$ and $\bar{\epsilon}_2 = diag\{-.28,\ 0,\ 0\}$, a minimum value of $\gamma = 51.67$ is achieved for $\bar{K} = \begin{bmatrix} -0.9054 & 0.2089 \end{bmatrix}^T$. Taking $\bar{\epsilon}_2 = diag\{-0.22,\ 0,\ 0\}$,as was suggested in Remark 2, a slightly higher minimal value of $\gamma = 54.45$ is obtained with $\bar{K} = \begin{bmatrix} -0.9166 & 0.2094 \end{bmatrix}^T$.

4. CONCLUSIONS

A solution to the problem of H_∞ filtering for linear, continuous, time-invariant systems with time delay has been presented. The solution procedure is based on applying an observer type filter and it provides a sufficient condition for achieving a prescribed estimation accuracy. Since the results are only sufficient, the question arises as to how large an over-design is entailed in this method and whether or not it is smaller than the one encountered in other designs appearing in the literature. To answer this question one has to bear in mind that the filter designs are based, one way or another, on a related BRL that provides the sufficient condition for a system with delay to possess an H_∞ -norm that is less than a prescribed value. The over-design of the corresponding filter design approach will therefore strongly depend on the conservatism of the BRL used. In this note, the BRL utilized, is, in our opinion, the least conservative of all the other finite dimensional BRLs appearing in the literature, and therefore provides the best filtering solution so far published.

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