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ANALYSIS OF THE PURSUIT-EVASION PROCESS FOR MOVING PLANTS UNDER UNCERTAIN OBSERVATION ERRORS DEPENDENT ON STATES

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Abstract: The paper presents a solution to the pursuit problem under the presence of bounded (non-stochastic) errors in state measurements for the evader and under uncertain evader's controls bounded within the given compact set. It also provides the worst-case solution conditions, meaning the the worst-case evader's controls and observation errors, and the methods aiming for fulfillment of these conditions. Finally, we calculate the worst-case estimate for the number of discrete-time steps (observations) required for bringing the pursuer in the given neighborhood of evader.

Keywords: uncertainty, set values, control system synthesis, optimal control, bounded disturbances, discrete systems, invariant set, pursuit problem.

INTRODUCTION

A number of publications (see particularly (Fredman, 1971; Hajek, 1975; Krasovski and Subbotin, 1988*b*; Pontryagin and Mishchenko, 1986; Hagedorn *et al.*, 1977; Krasovski and Subbotin, 1988*a*; Pshenitchnyi and Ostapenko, 1992; Olsder, 1995; Chikrii, 1997)) are devoted to analysis of approach (pursuit and evasion) processes for conflicting moving plants. The control synthesis problem for a pursuit-evasion process, which is complicated in itself, becomes even more complicated as measurements of phase states of the players are affected by uncertain errors. In the known publications except the paper (Kuntsevich and Pshenitchnyi, 1995), this problem is analyzed under the common assumption that observation errors are random values independent on systems dynamics and having a priori known probabilistic properties. However, the hypothesis on statistic nature of observation errors is not applicable generally, particularly it is not applicable when errors (disturbances) are known to be dependent on phase coordinates (states) of the moving plants. This is particularly true for the cases of 3-D measurements in the air with the radio aids and under water with the acoustic instrumentation since measurement errors are the larger in these cases the longer is the distance between the players. Obviously, the statistic hypothesis is not applicable as observation errors are a priori estimated by sets (see (Chernousko, 1994; Kurzhanski and Vályi, 1997) and (Walter and Pronzato, 1997, Ch. 5.4)).

The present paper continues the investigations of (Kuntsevich and Pshenitchnyi, 1995) and utilizes the definitions (particularly of a minimal

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invariant set) and theorems found in (Kuntsevich and Pshenitchnyi, 1996; Kuntsevich and Pshenitchnyi, 1998). It is devoted to solution of the pursuit-evasion problem for two controllable moving plants under uncertainty conditions which are (i) uncertain evader's controls and (ii) uncertain observation errors dependent on pursuer's and evader's states. In other words, the pursuit-evasion problem is solved for the worst-case scenario, meaning the evader takes optimal (and unknown for the pursuer) controls from the given bounded set and the observation errors are least favorable for the pursuer. The authors failed to find a single publication (except (Kuntsevich and Pshenitchnyi, 1995)) concerning solution of the problem stated this way.

1. THE PURSUIT PROCESS UNDER UNCERTAIN DISTURBANCES

The pursuit process will be considered further in discrete time, meaning discrete-time state measurements and controls for the both players.

Consider in the state space $\mathbb{R}^m$ the coordinate vectors x and y for the pursuer and the evader respectively. Assume that these vectors are measured at discrete time instances $t_n = n\tau$, where τ is the discretization period. Assume also that the values x_n are measured error-free and the measurements of y_n are affected by additive observation errors (disturbances) f_n:

$$z_n = y_n + f_n, \quad n = 0, 1, \dots , \tag{1}$$

where z_n is the measured state of the evader at the discrete-time instance n.

We also assume hereinafter that observation errors f_n are norm-bounded and the upper-bound estimate depends on the current distance between the evader and the pursuer:

$$f_n \in \mathbf{f}_n = \{f_n : \ \|f_n\| \le \lambda \|e_n\|\}, \tag{2}$$

where

$$e_n = y_n - x_n \tag{3}$$

and

$$0 \le \lambda < 1.$$

The constraint $\lambda < 1$ is obviously fulfilled in practice, otherwise it would not be possible to locate the evader with an appropriate precision and the pursuit problem would not be solvable. It is also worth to note that one can use any particular norm in (2), since this does not affect the analysis presented below.

Assuming that controls u_n and w_n are constant within time intervals of the length τ, describe the motion of the players x_n and y_n, $n = 0, 1, 2, \dots$, by the following difference equations,

$$x_{n+1} = x_n + \tau u_n, \tag{4}$$

$$y_{n+1} = y_n + \tau w_n. \tag{5}$$

Obviously, the controls (velocities) u_n and w_n are bounded. Without loss of generality, assume that

$$u_n \in \mathbf{u} = \{u : \|u\| \le 1\}, \tag{6}$$

$$w_n \in \mathbf{w} = \{w : \|w\| \le \beta\}. \tag{7}$$

Further, constant β will be assumed to satisfy the constraint $\beta < 1$. This means that the pursuer has a certain advantage in speed against the evader. In other conditions, the pursuer would not be able to approach the evader and the respective problem statement would be senseless.

From (3), (6) and (7), it is easy to obtain the equation for the reduced system:

$$e_{n+1} = e_n + \tau(w_n - u_n). \tag{8}$$

Next, one needs to calculate the estimate $\tilde{e}_n$ of the vector e_n, which depends on measurable coordinate vectors x_n and z_n. Obtain from (3) and (1) the expression for $\tilde{e}_n$:

$$\tilde{e}_n = z_n - x_n = e_n + f_n \tag{9}$$

and the norm bounds:

$$\|\tilde{e}_n\| \le (1 + \lambda) \|e_n\|. \tag{10}$$

The set estimate for $f_n(e_n)$ is expressed in terms of unknown (exactly) e_n. Therefore, it is also needed to introduce the upper-bound estimate $\tilde{f}(\tilde{e}_n)$ for $f_n(e_n)$ with the following properties,

$$\|f_n(e_n)\| \le \|\tilde{f}_n(\tilde{e}_n)\| \le \lambda\|\tilde{e}_n\| \le \lambda(1 + \lambda)\|e_n\|. \tag{11}$$

By virtue of (9) and (11), the set-valued estimate $\mathbf{e}_n$ for e_n (which is expressed in measured values) is as follows,

$$e_n \in \mathbf{e}_n = z_n - x_n - \tilde{\mathbf{f}}_n,$$

where

$$\tilde{\mathbf{f}}_n = \left\{\tilde{f}_n : \ \|\tilde{f}_n\| \le \lambda(1 + \lambda)\|z_n - x_n\|\right\}. \tag{12}$$

Synthesis of control u_n requires using a point estimate of vector $e_n \in \mathbf{e}_n$. Let us calculate this point estimate $\overset{\star}{e}_n$ as a solution to the following minimization problem,

$$\min_{\overset{\star}{e}_n} \left\{ \left\|e_n - \overset{\star}{e}_n\right\| = \left\|\tilde{e}_n - \tilde{f}_n - \overset{\star}{e}_n\right\| \right\}. \tag{13}$$

The problem formulation (13) includes the unknown exact value of error vector $\tilde{f}_n$, which has to be substituted with its worst-case set-valued estimate $\tilde{\mathbf{f}}_n$. Consequently, the problem (13) takes the form of minimax problem

$$\min_{\overset{\star}{e}_n} \max_{\tilde{f}_n \in \tilde{\mathbf{f}}_n} \left\|\tilde{e}_n - \tilde{f}_n - \overset{\star}{e}_n\right\|. \tag{14}$$

Lemma 1. Vector

$$\overset{\star}{e}_n = \tilde{e}_n$$

is the solution to the problem (14).

Proof. Consider the "internal" maximization problem of the objective function (14) for a particular value of $\overset{\star}{e}_n$. Recall that errors can take their values within the solid hyper sphere (12). Obviously, maximum of the objective function is reached if (i) the vector $\tilde{f}_n$ has the maximal feasible norm, i.e., it belongs to the boundary of set (12), the hyper sphere, and (ii) $\tilde{f}_n$ is collinear with the vector $\overset{\star}{e}_n - \tilde{e}_n$.

Consider the "external" minimization problem. If $\overset{\star}{e}_n \neq \tilde{e}_n$, then one can always increase the objective function over the maximal value of $\|\tilde{f}_n\|$ by choosing $\tilde{f}_n$ as it is described above. Only with $\overset{\star}{e}_n = \tilde{e}_n$, the objective function reaches its minimum which is equal to the maximal value of $\|\tilde{f}_n\|$, i.e., $\lambda(1+\lambda)\|z_n - x_n\|$. □

2. CONTROL OF THE PURSUIT PROCESS

Consider now calculating the control u_n as function of $\overset{\star}{e}_n = \tilde{e}_n$.

Remark 1. Concerning controls w_n, we shall be assuming that $w_n \in \mathbf{w}$ and nothing else. We shall not be interested also in the strategy of choosing controls w_n as a strategy aiming for evasion of the system (5) from the system (4). We shall be concerned in the motions of the system (5) as soon as they will affect the approaching process of the two systems (4) and (5).

It is appropriate to calculate the control u_n as minimizer for the right-hand side of the equation (8). Since the value w_n is known to the extent of its set-valued estimate, the optimal control synthesis problem takes the minimax form:

$$\min_{u_n \in \mathbf{u}} \max_{w_n \in \mathbf{w}} \|e_n - \tau u_n + \tau w_n\|. \tag{15}$$

It is easy to see that the control

$$w_n = \beta \frac{e_n}{\|e_n\|} \tag{16}$$

is a maximizer for the problem (15). The minimizer for (15) is

$$u_n = \frac{e_n}{\|e_n\|}. \tag{17}$$

The optimal controls (16) and (17) are trivial: the pursuer makes every effort to catch the evader by moving with the maximal speed in the direction of the evader, and the evader tries to escape by moving in the same direction with the highest possible speed. According to the stated objective, one has to assume that the evader knows the exact coordinates x_n of the pursuer and his own exact coordinates y_n, therefore, the evader knows the exact vector e_n and the optimal control (16). In contrary, the pursuer does not know the exact coordinates y_n of the evader and consequently he has to use the estimate $\tilde{e}_n$ (9) of vector e_n (see Lemma 1) when calculating the control u_n:

$$u_n = \frac{\tilde{e}_n}{\|\tilde{e}_n\|} = \frac{e_n + f_n}{\|e_n + f_n\|}. \tag{18}$$

The control u_n (18) was applied in (Kuntsevich and Pshenitchnyi, 1995), where it was postulated without referring to solution of the minimax problem (15).

Substitute the expression (18) into (8) and obtain the equation

$$e_{n+1} = e_n - \tau \frac{\tilde{e}_n}{\|\tilde{e}_n\|} + \tau w_n. \tag{19}$$

2.1 *The case of error-free measurements*

With the aim of analyzing the main features of the approaching process for two controllable plants, particularly those features induced by the discrete-time model for measurements and controls, consider first the system (19) for the case of error-free measurements, i.e., for $\lambda = 0$ and consequently $\mathbf{f}(\cdot) = \emptyset$. The equation (19) is reduced to the following one,

$$e_{n+1} = \psi(e_n) + \tau w_n, \tag{20}$$

where

$$\psi(e_n) = e_n - \tau \frac{e_n}{\|e_n\|}. \tag{21}$$

For a while, assume $w_n = 0$, i.e., $\mathbf{w} = \emptyset$, and analyze the system (21), which takes the following form under the assumption made,

$$e_{n+1} = \psi(e_n). \tag{22}$$

Determine the conditions providing that the vector function $\psi(\cdot)$ realizes a contracted mapping, meaning the following inequality takes place,

$$\|\psi(e_n)\| = \left\| e_n - \tau \frac{e_n}{\|e_n\|} \right\| < \|e_n\|. \tag{23}$$

To avoid certain difficulties, which are specific for analysis of the vector function $\psi(\cdot)$ at $\|e_n\| \to 0$, let us exclude from our consideration a sufficiently small μ-neighborhood of the origin. Further, without mentioning this every time, we shall be analyzing the system (19) and its modifications (20)-(21) and (22) only for

$$\|e_n\| \geq \mu, \quad \mu = \text{const}, \tag{24}$$

assuming that one applies the linear control $u_n = \gamma \tilde{e}_n$ as soon as $\|e_n\| < \mu$. In this case, equation (19) transforms in the equation

$$e_{n+1} = e_n - \gamma\tau\tilde{e}_n + \tau w_n. \quad (25)$$

The case of infinite $\|e_0\|$ is impracticable, therefore we require hereinafter the condition $\|e_0\| \le \rho$, where ρ is a given constant. Use the notation $\mathbf{E}$ for the set $\{e : \|e\| \le \rho\}$.

The inequality (23) is easily transformed to the following one,

$$|(\|e_n\| - \tau)| < \|e_n\|. \quad (26)$$

The inequality (26) is fulfilled under the condition

$$0 < \tau < 2\|e_n\|. \quad (27)$$

Therefore, for any set $\mathbf{E}$ (with arbitrary finite ρ) in the space $\mathbb{R}^m$ and under the condition (27), the vector function $\psi(\cdot)$ is Lipschitzian with Lipschitz constant $c < 1$, i.e.,

$$\|\psi(e_n)\| \le c\,\|e_n\|. \quad (28)$$

According to (Kuntsevich and Pshenitchnyi, 1996; Kuntsevich and Pshenitchnyi, 1998), if the condition (28) is fulfilled for some constant $c < 1$ and the set $\mathbf{w}$ is bounded, the system (20) has a minimal invariant set $\mathbf{M}$, meaning the inclusion $e_n \in \mathbf{M}$, leads to the inclusion $e_{n+1} \in \mathbf{M}$. (An invariant set is minimal if it does not include any other invariant sets.)

For the considered case, the values of function $\psi(e_n)$ are known only to the extent of the upper-bound estimate (28), hence one cannot calculate exactly the set $\mathbf{M}$. On the other hand, as it has been proven in (Kuntsevich, 1998; Kuntsevich and Kuntsevich, n.d.), one can calculate the upper-bound estimate for the radius

$$\mathrm{r}(\mathbf{M}) = \max_{e_n \in \mathbf{M}} \|e_n\|$$

of minimal invariant set $\mathbf{M}$ as one knows the upper-bound estimate for the vector-valued function $\psi(e_n)$. (The radius of a minimal invariant set is analogous to the dispersion for statistically described (random) observation errors.)

For the convenience, we present here Theorem 1 of the paper (Kuntsevich, 1998) (see also (Kuntsevich and Kuntsevich, n.d.)).

Theorem 1. (Kuntsevich, 1998) The upper-bound estimate of the radius of minimal invariant set of the system (20)-(21) for arbitrary compact set $\mathbf{E}$ and the Lipschitzian vector function $\psi(e_n)$ (with Lipschitz constant $c < 1$) is defined by the inequality

$$\mathrm{r}(\mathbf{M}) \le \tau\beta \sum_{k=0}^{\infty} c^k = \frac{\tau\beta}{1-c}. \quad (29)$$

In conclusion for the considered simplest case (error-free measurements), one can only ensure the existence of a bounded invariant set for the disretized model of measurements and controls (discretization is inevitable for practical applications). The dimensions of this set depend on the discretization period τ. One can secure the required approach distance between the pursuer (4) and the evader (5) by the respective choice of τ with any feasible controls of the evader.

2.2 *The case of measurements with errors*

Consider now the system (19) with $\mathbf{f} \ne \emptyset$. Primarily, define the conditions, under which the vector function

$$\varphi(e_n, f_n, w_n) = e_n - \tau\frac{e_n + f_n}{\|e_n + f_n\|} + \tau w_n \quad (30)$$

realizes a contracted mapping for $w_n \in \mathbf{w}$, i.e., the following inequality is fulfilled,

$$\|\varphi(e_n, f_n, w_n)\| < \|e_n\|. \quad (31)$$

Aiming to obtain the worst-case solution, meaning for any feasible $f_n \in \mathbf{f}$ and $w_n \in \mathbf{w}$, consider the problem

$$\max_{f_n \in \mathbf{f}_n,\ w_n \in \mathbf{w}} \|\varphi(e_n, f_n, w_n)\|, \quad (32)$$

which we intend to solve analytically.

Analyze the "worst" strategy of the evader (in view of the pursuer's objective), which is the choice of control (16) at each step n. (Recall that the evader presumably knows the exact value of e_n and the pursuer has to rely on its estimate $\tilde{e}_n$.) Rewrite the expression for $\varphi(\cdot)$ by taking into account the optimal control (16) for the evader:

$$\varphi(e_n, f_n) = e_n - \tau\frac{e_n + f_n}{\|e_n + f_n\|} + \tau\beta\frac{e_n}{\|e_n\|}.$$

Instead of solving the problem (32), consider solution of the reduced problem

$$\max_{f_n \in \mathbf{f}_n} \|\varphi(e_n, f_n)\|. \quad (33)$$

Maximize the function $\varphi^2(\cdot) = \varphi^T(\cdot)\varphi(\cdot)$, which is

$$\varphi^2(\cdot) = (\|e_n\| + \tau\beta)^2 + \tau^2 - 2\tau(\|e_n\| + \tau\beta)\frac{e_n^T(e_n + f_n)}{\|e_n\|\|e_n + f_n\|}, \quad (34)$$

with respect to $f_n \in \mathbf{f}_n$. The first two items in the right-hand side of the expression (34) are obviously positive. The third item is negative due to the positiveness of all multipliers, which this item consists of, particularly, the inner product $e_n^T(e_n + f_n)$ is positive by virtue of the definition of set $\mathbf{f}_n$ (2). Hence, the expression (34) reaches its maximum at the minimal value of the function

$$\xi_n(f_n) = \frac{e_n^T(e_n + f_n)}{\|e_n + f_n\|}$$

for the set $\mathbf{f}_n$, equivalently, maximum is reached at the maximal feasible angle between the vectors e_n and $e_n + f_n$. This angle is maximal as soon as the following two conditions are fulfilled,

(i) the vectors f_n and $e_n + f_n$ are orthogonal and
(ii) $\|f_n\| = \lambda\|e_n\|$.

Condition (i) provides the following equalities,

$$\|e_n\|^2 = \|f_n\|^2 + \|e_n + f_n\|^2,$$
$$f_n^T(e_n + f_n) = 0.$$

Make use of these equalities and condition (ii) for obtaining $\min\{\xi_n(f_n) : \quad f_n \in \mathbf{f}_n\}$. Omitting few simple transformations, express the required maximum as follows,

$$\max_{f_n \in \mathbf{f}_n} \varphi^2(e_n, f_n) = (\|e_n\| + \tau\beta)^2 + \tau^2 - 2\tau\sqrt{1-\lambda^2}\,(\|e_n\| + \tau\beta)\,. \tag{35}$$

Next, find the range of values τ, for which the vector function $\varphi(\cdot)$ realizes a contracted mapping, i.e., the range of values τ that ensure the fulfillment of inequality (31). With this aim in view, raise (31) to the second power and substitute $\varphi^2(\cdot)$ with its maximum (35). Resulting from this, find the upper-bound estimate for τ.

Lemma 2. With τ in the range

$$0 < \tau < 2\|e_n\| \frac{\sqrt{1-\lambda^2} - \beta}{1 + \beta^2 - \beta\sqrt{1-\lambda^2}}, \tag{36}$$

the vector function $\varphi(\cdot)$ (30) satisfies the condition (31), i.e., it realizes a contracted mapping.

Remark 2. Substitution of the values $\lambda = 0$ and $\beta = 0$, which correspond to the above-considered simplest case of error-free measurements and evader's static position, into the right-hand side of the inequality (36) results in obtaining the inequality (27), which presents a particular case of (36).

The upper-bound constraint (36) for τ determines also the corresponding upper bound for the value β which defines the dimensions of set $\mathbf{w}$. In fact, the value $\sqrt{1-\lambda^2} - \beta$ must be positive, hence

$$\beta < \sqrt{1-\lambda^2}. \tag{37}$$

Remark 3. Aiming to satisfy the condition (37) in the case the inequality is not fulfilled, one has no other options but decreasing λ, meaning enhancement of the accuracy of measurements of the evader's coordinates.

Assume now that the inequality (36) is fulfilled and consequently the condition (31) is satisfied, meaning the function $\varphi(\cdot)$ is Lipschitzian (with Lipschitz constant $c < 1$) in arbitrary compact set $\mathbf{E}$:

$$\|\varphi(e_n)\| \le c\,\|e_n\|\,. \tag{38}$$

The boundedness of the vector function $\varphi(\cdot)$ (38) and the boundedness of the set $\mathbf{w}$ provide the existence of a bounded invariant set for the system (19). The upper-bound estimate of the radius of minimal invariant set is equivalent to the estimate (29) to the extent of constant value c. As it results from (29), this estimate is directly proportional to the discretization period τ and it decreases with the decrease of λ, i.e., with the enhancement of accuracy of measurements of y_n.

The final stage of analysis of the system (19) is calculation of the worst-case estimate for the number N of discrete-time steps (observations) required for bringing the pursuer, which is initially positioned at the distance $\|e_0\|$ from the evader, in the given ε-neighborhood of the evader, i.e., the number of steps N required for satisfying the condition $\|e_N\| \le \varepsilon$. Since we assume as above that the optimal pursuer's strategy is the choice of control $u_n = \tilde{e}_n/\|\tilde{e}_n\|$ and the optimal evader's control is $w_n = e_n/\|e_n\|$ for the step n, the lower bound for $\|e_n\| - \|e_{n+1}\|$, which is the decrease of distance between the players, for the step n is calculated as follows,

$$\min_{f_n \in \mathbf{f}_n} \{\Delta e_{n+1} = \|e_n\| - \|e_{n+1}\|\} = \min_{f_n \in \mathbf{f}_n} \frac{e_n^T(e_n + f_n)}{\|e_n\|\|e_n + f_n\|} - \beta = \sqrt{1-\lambda^2} - \beta.$$

Derivation of this formula is completely identical to maximization of (34) (see the proof for Lemma 2).

Theorem 2. With the pursuer's controls $u_n = \tilde{e}_n/\|\tilde{e}_n\|$ for each n and with any feasible evader's controls $w_n \in \mathbf{w}$, the pursuer appears in the ε-neighborhood of the evader under the condition (37) after the number of steps

$$N \le \frac{\|e_0\| - \varepsilon}{\sqrt{1-\lambda^2} - \beta},$$

where $\|e_0\|$ is the distance between the pursuer and the evader at the step $n = 0$.

The obtained results provide the answers for the following questions:

(1) Assume that the required terminal set is a solid sphere of the given radius ε. Does the given terminal set include the minimal invariant set?
(2) In the case this inclusion is not fulfilled, what particular steps should be taken with the aim of fulfillment of the inclusion?
(3) If the inclusion takes place, what is the worst-case estimate for the number N of discrete-time steps required for bringing the pursuer in the given ε-neighborhood of the evader, meaning satisfying the condition $\|e_N\| \leq \varepsilon$, with the initial distance $\|e_0\|$ between the pursuer and the evader and with any feasible controls $w_n \in \mathbf{w}$ by the evader?

CONCLUSION

The observation errors f_n have been assumed above to take their worst-case values (with respect to the pursuer's control objective). However, observation errors rarely take their boundary values in practice. This fact taken into account in certain particular cases could significantly reduce the estimates $\|e_n\|$ at every step n.

Analysis of another interesting case when the mathematical model of observation errors f_n takes the form $f_n \in \mathbf{f}_n = \{f : \|f\| \leq \Delta + \lambda \|e_n\|\}$, where Δ is a given constant, can be performed without significant difficulties. The presented above qualitative results remain still actual for this case. It can be easily proven that the presence of constant Δ in the definition of feasible errors leads to a certain increase of the radius of minimal invariant set of the system (19), i.e., this results in a certain increase of the guaranteed approaching distance between the pursuer and the evader.

Analysis of the considered pursuit-evasion problem for the case of two dynamic systems described by the difference equations

$$x_{n+1} = Ax_n + Bu_n,$$
$$y_{n+1} = Gy_n + Hw_n,$$

where, as above, x_n and y_n are m-dimensional coordinate vectors of the pursuer and the evader respectively, A and G are matrices of the dimension $(m \times m)$, B are H are matrices of the dimension $(m \times l)$, and u_n and w_n are l-dimensional control vectors satisfying the constraints (14) and (15) respectively, is worth of independent thorough investigation. Despite the apparent similarity of the pursuit-evasion problems stated respectively for two moving points and for two dynamic systems, generalization of the above-obtained *analytical* results to the case of dynamic systems seems to be rather complicated.

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