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RECURSIVE STATE ESTIMATION FOR DISTRIBUTED PARAMETER UNCERTAIN SYSTEMS WITH INTEGRAL QUADRATIC CONSTRAINTS

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Abstract: The problem of recursive state estimation for a class of distributed parameter systems with an integral quadratic constraint in Hilbert spaces is addressed. Based on solving the linear tracking problem for time-varying systems in Hilbert spaces, necessary and sufficient conditions for robust state estimation problem involving construction of the set of all possible states at the finite interval time with given output measurements and integral quadratic constraints are derived.

Keywords: Robust control, state estimation, abstract systems, integral quadratic constraint

1. INTRODUCTION

Robust state estimation of linear uncertain systems has been for many years a subject of great interest to researchers in control engineering theory. This problem regarding as an extension of the Kalman filter to the case of uncertain systems is to find the set of all states consistent with the given measurements. The solution to this problem was found to be ellipsoid in state space which is defined by the standard Kalman filter equation. There has been a great deal of research effort regarding this problem, see (Bertsekas and Rhodes, 1971; Petersen and Savkin, 1999; Savkin and Petersen, 1998; Xie and Soh, 1994). However, little attention has been paid towards uncertain infinite-dimensional systems. The motivation of this is that the optimal control problem of infinite-dimensional systems is difficult both in mathematical theory and in practical engineering. From the 70's, many researchers began to develop the optimal control for distributed parameter systems and proposed the maximum principle, the generalized Riccati equation, etc., see (Anderson and Moore, 1990; Pritchard and Solomon, 1987; Phat, 1996). In these mentioned papers the system to be considered is either time-invariant or finite-dimensional. The aim of this paper is to extend the result of (Savkin and Petersen, 1995) to infinite-dimensional systems with an integral quadratic constraint. The main result of the paper shows that the recursive state estimation problem can be solved via the use of linear tracking problems in Hilbert spaces and then it can also be constructed by the solution of the infinite-dimensional Riccati differential equation.

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2. NOTATIONS AND DEFINITIONS

The following notations will be used in force throughout. R^n denotes the $n-$ dimensional Euclidean space. Let X, U, Y, W, Z denote Hilbert spaces with the norm $\|.\|_X$ and the inner product $\langle .,. \rangle_X$, etc. $\hat{X}$ denotes the space $X \times X$; $L(X,U)$ (respectively, $L(X)$) denotes the space of all linear bounded operators mapping X into U, (respectively, X into X). $L_2([t,s],X)$ denotes the set of all strongly measurable $2-$integrable $X-$valued functions on $[t,s]$. $C([t,s],X^+)$ denotes the set of all linear bounded non-negative definite operator functions in $L(X)$ continuous on $[t,s]$. $D(A), A^*$ and A^{-1} denote the domain, the adjoint and the inverse of the operator A, respectively. cl M denotes the closure of a set M. An operator $Q \in L(X)$ is self-adjoint if $Q = Q^*$. We recall that an operator $Q \in L(X)$ is non-negative definite (and denote $Q \geq 0$) if $\langle Qx, x\rangle \geq 0$, for all $x \in X$. If the inequality is strictly greater than 0 for $x \neq 0$, then Q is positive definite (and denote $Q > 0$). If $Q \in L(X)$ satisfies the following condition:

$$\exists c > 0 : \quad \langle Qx, x\rangle \geq c\|x\|^2, \quad \forall x \in X,$$

then Q is called a strictly positive definite operator (and denote $Q \gg 0$). It is obvious that for the finite-dimensional case, $X = R^n$, the positive definite operator is strictly positive definite. Consider a linear abstract uncertain system of the form

$$\begin{aligned} \dot{x} &= A(t)x + B(t)u + B_1(t)w, \quad t \in [0,T], \\ z(t) &= D(t)x(t) + C(t)u(t), \\ y(t) &= E(t)x(t) + v(t), \end{aligned} \tag{1}$$

where $x(t) \in X$ is the state, $u(t) \in U$ is the control input, $v(t) \in Y, w(t) \in W$ are the uncertain inputs, $z(t) \in Z, y(t) \in Y$ are the measured outputs; $A : X \to X; B \in L(U,X); B_1 \in L(W,X); C \in L(U,Z); D \in L(X,Z), E \in L(X,Y)$. The uncertainty in the above system is described by

$$[w(t), v(t)] = \psi(t, x(.)). \tag{2}$$

Let $M \in L(X)$, and $Q(t) \in L(W), R(t) \in L(Y)$ be given symmetric operators such that

$$M > 0, Q(t) \geq 0, R(t) \gg 0, \forall t \geq 0. \tag{3}$$

Definition 2.1. (System uncertainties) Let $x_0 \in X$ be a given state, $d > 0$ be a given positive number. For any finite time interval $[0,s], s \leq T$, the uncertainty (2) of the system (1) is admissible if $w(t) \in L_2([0,s],W), v(t) \in L_2([0,s],Y))$ and for any control input $u(t) \in L_2([0,s],U)$ and any corresponding solution of system (1) with the initial condition $x(0)$ the following condition holds

$$\langle M(x(0)-x_0), x(0)-x_0\rangle + \int_0^s \Big[\langle R(t)w(t), w(t)\rangle + \langle Q(t)v(t), v(t)\rangle\Big] dt \leq d + \int_0^s \|z(t)\|^2 dt \tag{4}$$

Let $u(t) = u_0(t), y(t) = y_0(t)$ be arbitrary fixed control input and measured output. We consider any finite time interval $[0,s]$ and denote

$$X_s[x_0, u_0, y_0, d]$$

the set of all possible solutions $x(s)$ of system (1) at time s for any admissible uncertain $w(.)$.

Definition 2.2. The uncertain system (1) is robustly verifiable if for any $x_0 \in X$, any time $s \in [0,T]$, any constant $d > 0$, any fixed control $u(t) = u_0(t)$, and fixed measured output $y(t) = y_0(t)$, the set $X_s[x_0, u_0, y_0, d]$ is bounded.

We will consider the following problem. Let $y(t) = y_0(t), u(t) = u_0(t)$ be fixed given measured output and input of the uncertain system (1), and let the finite time interval $[0,s]$ be given. Then, find the corresponding finite set $X_s[x_0, y_0, u_0, d]$ of all possible states $x(s)$ at time $s \in (0,T]$ for the uncertain system (1) with given initial condition and admissible uncertainty inputs (2).

3. LINEAR TRACKING PROBLEM

The main technique used in solving robust estimation problem is the standard linear regulator problem which is constructed by solving a Riccati differential equation. There are some results on the linear regulator problem for infinite-dimensional systems presented in (Bensoussan et al., 1992; Kuelen, 1993) and the optimal control conditions are derived from the solution of Riccati differential equations in Hilbert spaces. In this section we give optimal control conditions for a linear tracking problem of infinite-dimensional system. The obtained conditions will be implemented to solving the robust estimation control problem in next section.

Consider a linear time-varying control system in Hilbert space of the form

$$\begin{aligned} \dot{x}(t) &= A(t)x(t) + B(t)u(t), \quad t \in [t_0, s], \\ x(t_0) &= x_0. \end{aligned} \tag{5}$$

We make the following assumption on the system (5).

(i) $B(t) \in L(U,X)$ for every $t \geq t_0$ and $B(.)u, A(.)x$ is a continuous function on $[t_0, s]$ for all $u \in U, x \in X$.

(ii) The operator $A(t) : D(A(t)) \subset X \to X$, cl$D(A(t)) = X$ generates a strong evolution operator $U(t,r) : \{(t,r) \in R^2 : t \geq r \geq t_0\} \to L(X)$

such that the system (5) has a unique solution, see (Bensoussan et al., 1992; Curtain and Zwart, 1995) given by

$$x(t) = U(t, t_0)x_0 + \int_{t_0}^{t} U(t, \tau)B(\tau)u(\tau)d\tau.$$

Consider control system (5) with the minimized cost functional

$$J(u) = \langle Mx(s), x(s)\rangle + \int_{t_0}^{s} [\langle Ru(t), u(t)\rangle + \langle Qx(t), x(t)\rangle]dt, \quad (6)$$

where $M \in L(X), R(t) \in L(U), Q(t) \in L(X)$ are self-adjoint operators satisfying the condition (3). Let

$$V(x(t_0), t_0) = \min_{u(.)} J(u(t), t_0).$$

For any $Q(t) \in L(X)$, we recall that the linear operator function $P(t) \in L(X)$ is a solution of the Riccati differential equation

$$\dot{P} + PA + A^*P - PBR^{-1}B^*P + Q = 0, \quad (7)$$

if the following relation holds for all $x \in D(A(t)), t \in [t_0, s]$:

$$\langle \dot{P}x, x\rangle + \langle PAx, x\rangle + \langle Px, Ax\rangle - \langle R^{-1}B^*Px, B^*Px\rangle + \langle Qx, x\rangle = 0.$$

Proposition 3.1. (Curtain and Zwart, 1995) *The optimal control problem (5)-(6) is solved such that the performance index $V(x(t_0), t_0)$ exists and is finite if and only if the Riccati differential equation (7) has a solution $P(t) \in C([t_0, s], X^+)$ with the boundary condition $P(t_0) = M$. Moreover, the optimal controller is given by*

$$u^*(t) = -R^{-1}(t)B^*(t)P(t)x(t),$$

and the optimal cost is

$$V(x(t_0), t_0) = \langle P(t_0)x(t_0), x(t_0)\rangle.$$

We now apply the linear regular problem to a linear time-varying tracking problem as follows. Consider the following linear time-varying observed control system

$$\dot{x}(t) = A(t)x(t) + B(t)u(t), \quad t \in [t_0, s],$$
$$y(t) = E(t)x(t), \quad (8)$$

Let $y_0(.)$ be a measure output of the uncontrolled system

$$\dot{z}(t) = \bar{A}(t)z(t), \quad t \in [t_0, s],$$
$$y_0(t) = F(t)z(t), \quad (9)$$

where $z(t) \in X, \bar{A} : X \to X, F \in L(X)$. Let $M \in L(X), R(t) \in L(U)$ and $Q(t) \in L(X)$ be given operators satisfying the condition (3). The optimal tracking problem is to find an optimal controller for the system (8)-(9) minimizing the cost functional

$$J(t_0, x, u) = \langle M(x(s) - z(s)), x(s) - z(s)\rangle + \int_{t_0}^{s} \Big\{\langle Ru, u\rangle + \langle Q(y_0 - y), y_0 - y\rangle\Big\}dt. (10)$$

We define the following linear operators $\hat{M}, \hat{Q} : \hat{X} \to \hat{X}$, by setting

$$\hat{M}\hat{x} = (Mx_1 - Mx_2, -Mx_1 + Mx_2),$$
$$\hat{Q}\hat{x} = (E^*QEx_1 - E^*QFx_2, -F^*QEx_1 + F^*QFx_2),$$
$$\hat{A}\hat{x} = (Ax_1, \bar{A}x_2),$$

and $\hat{B} : U \to \hat{X}$ defined by $\hat{B}u = (Bu, 0)$. Consider the linear operator $\hat{P}(t) \in L(\hat{X}), t \in [t_0, s]$ defined by

$$\hat{P}(t)\hat{x} = (P(t)x_1 + P_1(t)x_2, P_1(t)x_1 + P_2(t)x_2),$$

where $P(t), P_i(t) \in L(X), i = 1, 2$, satisfying the following system of Riccati differential equations

$$\dot{P} + PA + A^*P - PBR^{-1}B^*P + E^*QE = 0,$$
$$\dot{P}_1 + P_1\bar{A} + A^*P_1 - PBR^{-1}B^*P_1 - E^*QF = 0,$$
$$\dot{P}_2 + P_2\bar{A} + \bar{A}^*P_2 - P_1BR^{-1}B^*P_1 + F^*QF = 0, \quad (11)$$

with the boundary condition

$$P(s) = M, P_1(s) = -M, P_2(s) = M.$$

We can apply the linear regulator problem stated in Proposition 3.1 to obtain the following result.

Proposition 3.2 (Time-varying linear tracking problem). *The optimal tracking problem (8)-(10) is solved such that the optimal performance index $V(t_0, .)$ is finite if the system of Riccati differential equations (11) has the solution $P(t), P_i(t) \in C([t_0, s], X^+), i = 1, 2$. Moreover, the optimal controller and the optimal cost are given by*

$$u(t) = -R^{-1}[B^*Px + B^*P_1z],$$
$$V(t_0, x_0) = \langle P(t_0)x(t_0), x(t_0)\rangle + 2\langle b(t_0), x(t_0)\rangle + c(t_0), \quad (12)$$

where $-\dot{b}(t) = (A^* - PBR^{-1}B^*)b - E^*Qy_0, b(s) = -Mz(s); \dot{c}(t) = \langle BR^{-1}B^*b, b\rangle - \langle Qy_0, y_0\rangle, c(s) = \langle Mz(s), z(s)\rangle$.

Sketch of the proof. We define a new variable $v(t) = [x(t), z(t)] \in \hat{X}$. This variable together with the operators $\hat{M}, \hat{A}(t), \hat{Q}(t)$ are so constructed that we can therefore reduce to the time-varying linear regular problem (8)-(10) in $\hat{X}$ requiring minimizing the quadratic index

$$J(t_0,v) = \langle \hat{M}v(s), v(s)\rangle + \int_{t_0}^{s}\{\langle R(t)u(t),u(t)\rangle) + \langle \hat{Q}(t)v(t),v(t)\rangle\}dt, \quad (13)$$

with the relationship

$$\dot{v}(t) = \hat{A}v(t) + \hat{B}u(t), v(s) = [x(s), z(s)] \in \hat{X}.$$

Assume that there are operators $P(t), P_i(t) \in C([t_0,s], X^+)$ satisfying the system of Riccati differential equations (11). It is easy to verify that the operator $\hat{P}(t)$ is a solution of the Riccati equation

$$\dot{\hat{P}} + \hat{P}\hat{A} + \hat{A}^*\hat{P} - \hat{P}\hat{B}R^{-1}\hat{B}^*\hat{P} + \hat{E}^*\hat{Q}\hat{E} = 0,$$

with $\hat{P}(s) = \hat{M}$. Therefore, we can apply the linear regulator problem stated in Proposition 3.1 and conclude that the optimal controller is defined by

$$u(t) = -R^{-1}(t)\hat{B}^*(t)\hat{P}(t)v(t).$$

Moreover, we can see that

$$V(t_0,v_0) = \langle P(t_0)x(t_0), x(t_0)\rangle + 2\langle P_1(t_0)z(t_0), x(t_0)\rangle + \langle P_2(t_0)z(t_0), z(t_0)\rangle.$$

Define the function $b(t), c(t)$ by

$$b(t) = P_1(t)z(t), \quad c(t) = \langle P_2(t)z(t), z(t)\rangle.$$

Differentiating $b(t), c(t)$ gives

$$\dot{b}(t) = -(A^* - PBR^{-1}B^*)b + E^*Qy_0,$$

$$\dot{c}(t) = \langle BR^{-1}B^*b, b\rangle - \langle Qy_0, y_0\rangle.$$

Therefore

$$V(t_0,x_0) = \langle P(t_0)x(t_0), x(t_0)\rangle + 2\langle b(t_0), x(t_0)\rangle + c(t_0).$$

Remark 3.1. Note that by the same arguments that used in the proof of Proposition 3.2, we can extend the linear tracking problem to the systems (8), (9) with respect to two measure outputs:

$$y(t) = E(t)x(t), \quad y_0(t) = D(t)x(t),$$
$$w(t) = F(t)z(t), \quad w_0(t) = N(t)z(t),$$

with the cost functional

$$J(t_0,x,u) = \langle M(x(s)-z(s)), x(s)-z(s)\rangle + \int_{t_0}^{s}\Big\{\langle R(t)u(t),u(t)\rangle + \langle Q(t)[y_0-y],[y_0-y]\rangle + \langle Q_1(t)[w_0-w], w_0-w]\rangle\Big\}dt.$$

The condition (12) holds true for the functions $b(t)$ and $c(t)$ respectively replaced by

$$-\dot{b}(t) = [A^* - PB^*R^{-1}B]b - E^*Qy_0 - D^*Q_1w_0,$$

$$\dot{c}(t) = \langle BR^{-1}B^*b, b\rangle - \langle Qy_0, y_0\rangle - \langle Q_1w_0, w_0\rangle.$$

In this case the system of Riccati equations is similarly defined by (11).

4. MAIN RESULT

The solution to the robust state estimation problem involves the following system of Riccati equations

$$\dot{P} + PA + A^*P - PB_1R^{-1}B_1^*P + E^*QE - D^*D = 0, \quad (14)$$

where $P(0) = M$. Also, consider the following state estimator equation

$$\dot{e} = [A + H(E^*QE - D^*D)]e + f(t,u_0,y_0),$$

where $e(0) = x_0, H(t) = P^{-1}(t)$ and

$$f(t,u_0,y_0) = (B - HD^*C)u_0 - HE^*Qy_0.$$

Let us denote

$$\eta_s(u_0,y_0) = \int_0^s\Big\{\langle Q(y_0-Ee),(y_0-Ee)\rangle - \|De + Cu_0\|^2\Big\}dt.$$

The main result is the following theorem.

Theorem 4.1. *Assume that* $E^*QE - D^*D \geq 0$. *If the system (1) is robustly verifiable, then the Riccati differential equation (14) has a solution* $P(t) \in C([0,T], X^+)$. *Conversely, if the Riccati equation (14) has the solution* $P(t) \in C([0,T], X^+)$, *which is strictly positive definite for all* $t \in [0,T]$, *then the system (1) is robustly verifiable. Moreover, let* $s \in [0,T], x_0 \in X, d > 0, u_0(t), y_0(t)$ *be given, then*

$$X_s[x_0,u_0,y_0,d] = \Big\{x_s \in X : \langle P(s)[x_s - e(s)], [x_s - e(s)]\rangle \leq d + \eta_s(u_0,y_0)\Big\}.$$

Sketch of the proof : (i) Assume that the system (1) is robustly verifiable. Let $s \in [0,T]$. By definition of $X_s[x_0,u_0,y_0,d]$ that $x_s \in X_s[u_0,u_0,y_0,d]$ if and only if there exist functions $x(.), v(.)$ satisfying (1) such that $x(s) = x_s$, the constraint (4) holds. Due to the condition (4) it follows that $x_s \in X_s[u_0,y_0,d]$ if and only if there is an input $w(.) \in L_2[0,s]$ such that

$$J(x_s, w(.)) \leq d, \quad (15)$$

where $J(x_s,w(.))$ is defined by

$$J(x_s,w(.)) = \langle M(x(0)-x_0), x(0)-x_0\rangle + \int_0^s\{\langle Rw(t),w(t)\rangle + \langle Qv(t),v(t)\rangle - \|z(t)\|^2\}dt,$$

and $x(.)$ is the solution of system (1) with input $w(.)$ and boundary condition $x(s) = x_s$. Taking $x_0 = 0, u_0(t) = 0, y_0(t) = 0, d = 1$, the above cost functional is then rewritten in the form (13), where $\hat{Q} = E^*QE - D^*D$. By the assumption, the set $X_s[0,0,0,1]$ is bounded, there is a positive number $a > 0$ such that all $x_s \in X$ with $\|x_s\| = a$ do not belong to the set $X_s[0,0,0,1]$. Therefore, $J(x_s, w(.)) > 1$, for all $x_s \in X$ such that $\|x_s\| = a$ and for all $w(.) \in L_2([0,s], W)$. Since $J(x_s, w(s))$ is a homogeneous quadratic functional, it is easy to check that

$$\inf_{u\in L_2[0,s]} J(x_s, w(s)) > 0,$$

for all $s \in [0,T], x_s \neq 0$. This minimizing problem subject to the system (1) is a linear regulator problem in which the time is reversed, using Proposition 3.1, there exists operator $P(t) \in C([0,s], X^+)$, which is the solution of the equations (14). Moreover, since the cost functional is strictly positive and the terminal value $P(s)$ is positive definite, we conclude that $P(t) > 0$ for all $t \in [0,T]$.

(ii) To prove the converse part, we first note that $x_s \in X_s[x_0, u_0, y_0, d]$ for any $s \in [0,T]$ if and only if there is an input $w(.) \in L_2([0,s], W)$ such that the condition (15) holds for the cost functional satisfying the condition (4). The minimizing problem

$$\min_{u\in L_2[0,s]} J(x_s, w(.)),$$

where the minimum is taken over all the solution $x(.)$ and $w(.)$ connected by the system (1) with the boundary condition $x(s) = x_s$. We wish to convert this optimal control problem into a tracking problem by setting

$$\tilde{x}(t) = x(t) - x_1(t),$$

where $x_1(t)$ is the solution of equation

$$\dot{x}_1(t) = A(t)x_1(t) + B(t)u_0(t), \quad x_1(0) = 0.$$

We can verify that that $\tilde{x}(t)$ is a solution of the following system

$$\dot{\tilde{x}}(t) = A(t)\tilde{x}(t) + B_1(t)w(t), \quad \tilde{x}(0) = x_0).(16)$$

Therefore, the cost functional can be rewritten in the form

$$J(\tilde{x}_s, w(.)) = \langle M[\tilde{x}(0) - x_0], [\tilde{x}(0) - x_0]\rangle$$
$$+ \int_0^s \{\langle Rw, w\rangle + \langle Qa(\tilde{x}, x_1, y_0), a(\tilde{x}, x_1, y_0)\rangle$$
$$-\|b(\tilde{x}, x_1, u_0)\|^2\} dt$$

where

$$a(\tilde{x}, x_1, y_0) = y_0 - E[\tilde{x}(t) + x_1(t)],$$
$$b(\tilde{x}, x_1, u_0) = D[\tilde{x}(t) + x_1(t)] + Cu_0.$$

We now consider control system (16) with the measure outputs:

$$v_0(t) = E\tilde{x}(t), \quad v_1(t) = -D\tilde{x}(t).$$

Hence, equation (16) and the cost functional

$$J(\tilde{x}_s, w(.)) = \langle M[\tilde{x}(0) - x_0], [\tilde{x}(0) - x_0]\rangle +$$
$$\int_0^s \{\langle Rw, w\rangle + \langle Q(\bar{y}_0 - v_0), (\bar{y}_0 - v_0)\rangle - \|\bar{y}_1 - v_1\|^2\} dt,$$

where $\bar{y}_0(t) = y_0 - E(t)x_1(t), \bar{y}_1(t) = Dx_1(t) + Cu_0(t)$, define a linear tracking problem, where $y_0(.), u_0(.), x_1(.)$ are all treated as reference inputs and taking $\bar{A}(t) = A(t), Q_1(t) = -I, z(t_0) = x_0$ and $N(t), F(t) \in L(X,Y)$ such that

$$\bar{y}_0(t) = F(t)z(t), \bar{y}_1(t) = N(t)z(t).$$

By the assumption there is a strictly positive definite operator function $P(t) \gg 0, t \in [0,T]$, which is the solution of the Riccati equation (14). Knowing $P(t)$, we can define the operators $P_i(t), i = 1,2$ satisfying the Riccati equations (11). Thus we can apply the tracking problem, Proposition 3.2 and Remark 3.1, for systems (16) such that this problem is solved in the reversed time and the optimal cost $V(s,.)$ exists and satisfies the condition

$$V(s, \tilde{x}_s) = \langle P(s)\tilde{x}(s), \tilde{x}(s)\rangle + 2\langle b(s), \tilde{x}(s)\rangle + c(s),$$

where the functions $b(t), c(t), t \in [0,s]$ satisfy the differential equations in Remark 3.1. Let $H(t) = P^{-1}(t), t \in [0,s]$, we define $v(t) = H(t)b(t)$. The optimal index can be rewritten as

$$V(\tilde{x}_s, s) = \langle P(s)[\tilde{x} + v], [\tilde{x} + v]\rangle - \eta_s(x_0, u_0, y_0),$$

where $\eta_s(x_0, u_0, y_0) = \langle H(s)b(s), b(s)\rangle - c(s)$. On the other hand, by integrating the function $\langle H(t)b(t), b(t)\rangle$, and $c(t)$ from 0 to s, and we can find the form of the function $\eta_s(.)$. Let us set $e(t) = x_1(t) - v(t)$. Then $e(0) = -v(0) = x_0$ and note that

$$\tilde{x}(t) + v(t) = x(t) - x_1(t) + v(t) = x(t) - e(t),$$

therefore, we have

$$V(x_s, s) = \langle P(s)[x(s) - e(s)], [x(s) - e(s)]\rangle$$
$$-\eta_s(u_0, y_0) \leq d.$$

Consequently, the set $X_s[x_0, u_0, y_0, d]$ defined by

$$X_s[x_0, u_0, y_0, d] = \{x_s \in R^n : \langle P(s)[x_s - e(s)],$$
$$[x_s - e(s)]\rangle \leq d + \eta_s(u_0, y_0)\},$$

is obviously bounded.

Remark 4.1. Note that if the system (1) is finite-dimensional, then the solution matrix $P(t)$ being positive definite for all $t \in [0,T]$ is invertible, and the condition stated in Theorem 4.1 is necessary and sufficient for the robust verifiability.

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REFERENCES

Anderson B.O and J.B. Moore (1990), *Optimal control: Linear Quadratic Methods.* Prentice Hall, New Jersey.

Bertsekas D.P. and I.B. Rhodes (1971), Recursive state estimation for a set-membership description of uncertainty. *IEEE Trans. Aut. Contr.,* vol.16, 117-128.

Bensoussan A., G.Da Prato, M.C. Delfour and S.K. Mitter (1992), *Representation and Control of Infinite-dimensional Systems,* vol. II, Birkhauser.

Curtain R.F. and H.J. Zwart (1995), *An Introduction to Infinite Dimensional Linear Systems Theory.* Springer-Verlag, Berlin.

van Keulen B. (1993), *H-infinity Control for Distributed Parameter Systems: A State-Space Approach.* Birkhauser, Boston.

Lion J.P. (1971), *Optimal Control of Systems Governed by Partial Differential Equations.* Springer-Verlag, Berlin.

Pertesen I.R. and A.V. Savkin (1999), *Robust Kalman Filter for Signals and Systems with Large Uncertainties.* Birkhauser, Boston.

Phat V.N. 1996), *Constrained Control Problems of Discrete Processes.* World Scientific, Singapore.

Pritchard J. and D. Solomon (1987), The linear quadratic control problem for infinite-dimensional systems with unbounded input and output operators. *SIAM J. Contr. Optim.,* vol. 25, 121-144.

Savkin A.V. and I. Petersen (1995), Recursive state estimation for uncertain systems with an integral quadratic constraint. *IEEE Trans. Aut. Contr.,* vol.40, 1080-1083.

Savkin A.V. and I. Petersen (1998), Robust state estimation and model validation for discrete-time uncertain systems with a deterministic description of noise and uncertainty. *Automatica,* vol.34, 271-274.

Xie L. and Y.C. Soh (1994), Robust Kalman filtering for uncertain systems. *Systems and Control Letters,* vol.22, 123-130.