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ON TRANSPOSE JACOBIAN–BASED REGULATORS USING UNIT QUATERNIONS: AN ENERGY SHAPING APPROACH [1]

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Abstract: The regulation of end–effector pose of manipulators is addressed in this paper. Regarding nonredundant manipulators, we present an extension of transpose Jacobian–based regulators obtained by energy shaping, where orientation is represented by unit quaternion. *Copyright © 2002 IFAC*

Keywords: Robot control, Transpose Jacobian control, Regulation, Euler parameters, Unit quaternion, Stability, Energy shaping.

1. INTRODUCTION

Controllers based on transpose Jacobian offer an attractive approach to robot control in task space. These controllers attempt to drive the robot end-effector posture to a specified constant desired position and orientation without solving neither the inverse kinematics nor computing the robot inverse Jacobian.

A number of analysis and extensions of the transpose Jacobian control scheme in Cartesian space have been reported in the literature (Takegaki and Arimoto, 1981; Kelly and Coello, 1999; Kelly, 1999). Transpose Jacobian–based regulators have also been proposed in (Masutani *et al.*, 1989; Koditscheck, 1991) for the case when the end–effector position and orientation is defined in $\mathbb{R}^3 \times SO(3)$.

In this paper we focus in the regulation control of pose in task space using the energy shaping technique, —originally introduced by Takegaki and Arimoto (Takegaki and Arimoto, 1981)— and a non minimal representation of orientation —the so-called Euler parameters or unit quaternion—. The whole analysis has been done regarding nonredundant manipulators.

2. ROBOT MODEL AND REGULATION GOAL

The dynamics of a serial–chain n-link robot manipulator can be written in joint space as (Spong and Vidyasagar, 1989):

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau \qquad (1)$$

where q is the $n\times 1$ vector of joint displacements, $\dot{q}$ is the $n\times 1$ vector of joint velocities, τ is the $n\times 1$ vector of applied torque inputs, $M(q)$ is the $n\times n$ symmetric positive definite manipulator inertia matrix, $C(q,\dot{q})\dot{q}$ is the $n\times 1$ vector of centripetal and Coriolis torques and $g(q)$ is the $n\times 1$ vector of gravitational torques.

The manipulator output considered in this paper is the pose of the end–effector frame with respect to the robot base frame. The pose of the end–effector is characterized by its position vector $p \in \mathbb{R}^3$, and its orientation in terms of the rotation matrix $R \in SO(3)$. Both position and orientation of the end–effector are function of the joint displacements, i.e., $p(q)$, $R(q)$.

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Given a constant desired output pose $p_d \in \mathbb{R}^3$ and $R_d \in SO(3)$, the regulation aim is to ensure

$$\lim_{t\to\infty} p(t) = p_d, \tag{2}$$

$$\lim_{t\to\infty} R(t) = R_d. \tag{3}$$

Rotation matrices $R \in SO(3)$ have associated the so–called Euler parameter or unit quaternions $\pm\left[\eta \ \varepsilon^T\right]^T$ with $\eta \in \mathbb{R}$ and $\varepsilon \in \mathbb{R}^3$ given by (Sciavicco and Siciliano, 2000)

$$\eta = \frac{1}{2}\sqrt{r_{11}+r_{22}+r_{33}+1}, \tag{4}$$

$$\varepsilon = \frac{1}{2}\begin{bmatrix} \mathrm{sign}(r_{32}-r_{23})\sqrt{r_{11}-r_{22}-r_{33}+1} \\ \mathrm{sign}(r_{13}-r_{31})\sqrt{r_{22}-r_{33}-r_{11}+1} \\ \mathrm{sign}(r_{21}-r_{12})\sqrt{r_{33}-r_{11}-r_{22}+1} \end{bmatrix}, \tag{5}$$

where r_{ij} are the entries of matrix R, and conventionally $\mathrm{sign}(x) = 1$ for $x \geq 0$ and $\mathrm{sign}(x) = 0$ for $x < 0$.

In this way, the rotation matrix of the end–effector $R(t)$ and the desired rotation matrix R_d have associated the unit quaternions $\pm\left[\eta \ \varepsilon^T\right]^T$ and $\pm\left[\eta_d \ \varepsilon_d^T\right]^T$, respectively. So, the orientation part of the regulation goal (3) can be reformulated as

$$\lim_{t\to\infty}\begin{bmatrix}\eta(t)\\ \varepsilon(t)\end{bmatrix} = \pm\begin{bmatrix}\eta_d\\ \varepsilon_d\end{bmatrix}. \tag{6}$$

Expressions (2) and (6) can be rewritten as

$$\lim_{t\to\infty}\widetilde{y}(t) = 0, \tag{7}$$

where $\widetilde{y}$ is the output error defined as

$$\widetilde{y} = \begin{bmatrix}\widetilde{p}\\ e\end{bmatrix}, \tag{8}$$

with the position error vector $\widetilde{p} \in \mathbb{R}^3$ given by

$$\widetilde{p}(t) = p_d - p(q(t)), \tag{9}$$

and for the orientation error vector $e \in \mathbb{R}^3$, let us consider (Yuan, 1988)

$$e = \eta(q)\varepsilon_d - \eta_d\varepsilon(q) + S(\varepsilon(q))\varepsilon_d \tag{10}$$

where, for a given $x \in \mathbb{R}^3$, the skew–symmetric matrix $S(x)$ is defined by

$$S(x) = \begin{bmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{bmatrix}. \tag{11}$$

Such a definition of the orientation error e was first introduced explicitly by Yuan (Yuan, 1988) and then considered by several authors (Lin, 1995; Lizarralde and Wen, 1996; Caccavale *et al.*, 1999; Sciavicco and Siciliano, 2000). A detailed motivation can be found in those references.

The linear and angular velocities of the end–effector frame denoted by $\dot{p} \in \mathbb{R}^3$ and $\omega \in \mathbb{R}^3$ respectively, are given as functions of the joint position q and velocity $\dot{q}$ by (Sciavicco and Siciliano, 2000)

$$\begin{bmatrix}\dot{p}\\ \omega\end{bmatrix} = \begin{bmatrix}J_p(q)\\ J_o(q)\end{bmatrix}\dot{q} = J_G(q)\dot{q}, \tag{12}$$

where $J_p(q)$ and $J_o(q)$ are $3\times n$ matrices, and $J_G(q) \in \mathbb{R}^{6\times n}$ is the manipulator geometric Jacobian. Using (12), it can be shown that the time derivative of the orientation error (10) according with the regulation aim is given by (Campa *et al.*, 2001)

$$\dot{\widetilde{y}} = \begin{bmatrix}\dot{\widetilde{p}}\\ \dot{e}\end{bmatrix} = \begin{bmatrix} -J_p(q)\dot{q} \\ \left[-\frac{1}{2}\eta_e I - \frac{1}{2}S(e)\right]J_o(q)\dot{q}\end{bmatrix} \tag{13}$$

where $\eta_e = \eta\eta_d + \varepsilon_d^T\varepsilon$. This is equivalent to

$$\dot{\widetilde{y}} = -Q(\eta_e, e)J_G(q)\dot{q}, \tag{14}$$

with

$$Q(\eta_e, e) = \begin{bmatrix} I & 0 \\ 0 & \frac{1}{2}\eta_e I + \frac{1}{2}S(e)\end{bmatrix} \tag{15}$$

where $I \in \mathbb{R}^{3\times 3}$ is the identity matrix.

3. ENERGY SHAPING: A REVIEW

The energy shaping technique was introduced by Takegaki and Arimoto in 1981 (Takegaki and Arimoto, 1981). The basic idea of this methodology is to shape the manipulator natural potential energy in order to satisfy a specific control goal. By designing a controller that makes the closed–loop system have a desired potential energy, and adding velocity feedback, then we can have an asymptotically stable system.

According to the energy shaping method (Takegaki and Arimoto, 1981), a suitable control structure is given by

$$\tau = -\nabla_q\overline{\mathbf{U}}_a(\widetilde{y}(q)) - \nabla_{\dot{q}}\mathscr{F}(\dot{q}) + g(q) \tag{16}$$

where $\overline{\mathbf{U}}_a(\widetilde{y}(q))$: $\mathbb{R}^m \times \mathbb{R}^m \to \mathbb{R}$ is a continuously differentiable positive definite function called *artificial potential energy*, $\mathscr{F}(\dot{q})$: $\mathbb{R}^n \to \mathbb{R}$ is a continuously differentiable dissipation function, $\nabla_q\overline{\mathbf{U}}_a(\widetilde{y}(q))$ stands for the gradient with respect to q of $\overline{\mathbf{U}}_a(\widetilde{y}(q))$, and $\nabla_{\dot{q}}\mathscr{F}(\dot{q})$ stands for the gradient with respect to $\dot{q}$ of $\mathscr{F}(\dot{q})$.

For the dissipation function $\mathscr{F}(\dot{q})$ and for the artificial potential energy function $\overline{\mathbf{U}}_a(\widetilde{y}(q))$, the following properties must be satisfied (Kelly, 1999):

P1. $\widetilde{y}=0$ is an isolated minimum point of the artificial potential energy function $\overline{\mathbf{U}}_a(\widetilde{y})$.

P2. $\nabla_{\dot{q}}\mathscr{F}(\mathbf{0})=\mathbf{0}$.

P3. $\dot{q}^T\nabla_{\dot{q}}\mathscr{F}(\dot{q})>0 \quad \forall \dot{q}\neq\mathbf{0}$.

Since $\widetilde{y}$ is also a function of the joint displacement vector q and using the chain rule, the control law (16) can be written as

$$\tau=-\left[\nabla_q\widetilde{y}(q)\right]^T\nabla_{\widetilde{y}}\overline{\mathbf{U}}_a(\widetilde{y})-\nabla_{\dot{q}}\mathscr{F}(\dot{q})+g(q). \tag{17}$$

Equation (17) represents a family of transpose Jacobian–based controllers (Kelly, 1999).

Before beginning the analysis of the control law (17) it is worth to make the following assumptions:

A1. There exist $q_d\in\mathbb{R}^n$ such that

$$p_d=p(q_d),$$
$$\begin{bmatrix}\eta_d\\ \varepsilon_d\end{bmatrix}=\pm\begin{bmatrix}\eta(q_d)\\ \varepsilon(q_d)\end{bmatrix}.$$

A2. The robot is nonredundant and no self–motion exists at the desired pose p_d, $\pm\left[\eta_d\ \varepsilon_d^T\right]^T$.

A robot manipulator has no self–motion if there is not any continuous path in joint space for which the end–effector pose does not change. Nonredundant robots whether possess self–motion or not, it only may happen at singular configurations (Seng *et al.*, 1997). Therefore, according with the assumptions A1 and A2, that $q=q_d$ is an isolated solution of the forward kinematics given in terms of the position vector $p(q)$ and the unit quaternion $\left[\eta(q)\ \varepsilon(q)^T\right]^T$.

The closed–loop system obtained from the robot dynamics (1) and the control law (17) can be described by using the state space vector $[q^T \quad \dot{q}^T]^T$ as

$$\frac{d}{dt}\begin{bmatrix}q\\ \dot{q}\end{bmatrix}=$$
$$\begin{bmatrix}\dot{q}\\ -M(q)^{-1}[\nabla_q\overline{\mathbf{U}}_a(\widetilde{y}(q))+\nabla_{\dot{q}}\mathscr{F}(\dot{q})+C(q,\dot{q})\dot{q}]\end{bmatrix} \tag{18}$$

Assuming that $[q^T \quad \dot{q}^T]^T=[\overline{q}^T \quad 0^T]^T$ is an equilibrium point of (18), it is proven in (Kelly, 1999) that the stability of this equilibrium can be studied with the Lyapunov function candidate:

$$V(\overline{q}-q,\dot{q})=\frac{1}{2}\dot{q}^TM(q)\dot{q}+\overline{\mathbf{U}}_a(\widetilde{y}(q)) -\overline{\mathbf{U}}_a(\widetilde{y}(\overline{q})). \tag{19}$$

The first right hand side term of (19) is a globally definite positive function with respect to $\dot{q}$ because of the positive definiteness of the inertia matrix $M(q)$. The remaining terms define a locally positive definite function with respect to $\overline{q}-q$. Using the property $\dot{q}^T[\frac{1}{2}\dot{M}(q)-C(q,\dot{q})]\dot{q}=0$, the time derivative of the Lyapunov function candidate yields

$$\dot{V}(\overline{q}-q,\dot{q})=-\dot{q}^T\nabla_{\dot{q}}\mathscr{F}(\dot{q}). \tag{20}$$

As a consequence of property P3, (20) is a globally negative semidefinite function; therefore, the Lyapunov's direct method allows the conclusion of stability. Asymptotic stability can be proven by LaSalle's theorem (Vidyasagar, 1993).

4. A CLASS OF TRANSPOSE JACOBIAN REGULATORS WITH JOINT SPACE DAMPING

It can be inferred from (14) that the gradient with respect to q of the error function $\widetilde{y}$ is given by (García, 2001)

$$\nabla_q\widetilde{y}(q)=\frac{\partial\widetilde{y}(q)}{\partial q}=-Q(\eta_e,e)J_G(q). \tag{21}$$

Hence, equation (17) becomes

$$\tau=J_G(q)^TQ(\eta_e,e)^T\nabla_{\widetilde{y}}\overline{\mathbf{U}}_a(\widetilde{y})-\nabla_{\dot{q}}\mathscr{F}(\dot{q})+g(q). \tag{22}$$

By choosing the dissipation function $\mathscr{F}(\dot{q})$ as $\mathscr{F}(\dot{q})=\frac{1}{2}\dot{q}^TK_V\dot{q}$, where $K_V\in\mathbb{R}^{n\times n}$ is a positive definite matrix, and a suitable artificial potential functions $\overline{\mathbf{U}}_a(\widetilde{y})$, from (22) we get a wide family of transpose Jacobian–based regulators with joint space damping, using unit quaternions to represent orientation.

4.1 *Example 1*

A suitable artificial potential energy function in terms of the error function $\widetilde{y}=[\widetilde{p}^T \quad e^T]^T$ is given by the quadratic form

$$\overline{\mathbf{U}}_a(\widetilde{y})=\frac{1}{2}\widetilde{y}^TK_P\widetilde{y} \tag{23}$$

where $K_P\in\mathbb{R}^{6\times 6}$ is a symmetric positive definite matrix. Equations (22) and (23) yield the controller

$$\tau=J_G(q)^TQ(\eta_e,e)^TK_P\widetilde{y}-K_V\dot{q}+g(q) \tag{24}$$

where $K_V\in\mathbb{R}^{n\times n}$ is a positive definite matrix. Substituting the control law (24) into (1) we obtain the closed–loop system

$$\frac{d}{dt}\begin{bmatrix}q\\ \dot{q}\end{bmatrix}=$$
$$\begin{bmatrix}\dot{q}\\ M(q)^{-1}[J_G(q)^TQ(\eta_e,e)^TK_P\widetilde{y}-K_V\dot{q}-C(q,\dot{q})\dot{q}]\end{bmatrix} \tag{25}$$

It is possible to demonstrate that (García, 2001)

$$\widetilde{y}(q_d) = \mathbf{0}, \tag{26}$$

and hence, $[q^T \ \dot{q}^T]^T = [q_d^T \ \mathbf{0}^T]^T$ is an isolated equilibrium point of the closed–loop system (25).

According to (19), the stability analysis is carried out with the Lyapunov function candidate given by

$$V(q_d - q, \dot{q}) = \frac{1}{2}\dot{q}^T M(q)\dot{q} + \frac{1}{2}\widetilde{y}^T(q) K_P \widetilde{y}(q). \tag{27}$$

The time derivative of (27) along the trajectories of the closed–loop equation (25) yields

$$\dot{V}(q_d - q, \dot{q}) = -\dot{q}^T K_V \dot{q}. \tag{28}$$

The time derivative is globally negative semidefinite in virtue of positive definiteness of K_v; therefore, the Lyapunov's direct method allows the conclusion of stability of equilibrium point $[q^T \ \dot{q}^T]^T = [q_d^T \ \mathbf{0}^T]^T$. Due to the autonomous nature of the closed–loop system (25), LaSalle's theorem (Vidyasagar, 1993) can be invoked to show asymptotic stability.

In order to find the largest invariant set in a neighborhood of $[q^T \ \dot{q}^T]^T = [q_d^T \ \mathbf{0}^T]^T$, we need the solution of

$$J_G(q)^T Q(e(q))^T K_P \widetilde{y}(q) = \mathbf{0}. \tag{29}$$

It can be shown that $q = q_d$ is an isolated minimum point of $\frac{1}{2}\widetilde{y}^T K_P \widetilde{y}$, and therefore, its gradient with respect to q, i.e., $-J_G(q)^T Q(e)^T K_P \widetilde{y}$ must vanish only at $q = q_d$ in a neighborhood of $q = q_d$. These arguments lead to claim that $q = q_d$ is the largest invariant set in a neighborhood of this equilibrium point, and we can conclude asymptotic stability. Therefore

$$\lim_{t\to\infty} q(t) = q_d \tag{30}$$

provided that $\|q_d - q(0)\|$ and $\|\dot{q}(0)\|$ are sufficiently small. As a direct consequence of assumption A1, the control objective (7) is achieved.

It is important to remark that if assumptions A1 and A2 are satisfied, the desired joint position $q = q_d$ is allowed to be a singular configuration where the manipulator geometric Jacobian $J_G(q_d)$ is singular. In other words, it is still possible to achieve the control objective even though the desired pose (p_d and $\pm\left[\eta_d \ \varepsilon_d^T\right]^T$) corresponds to a singular robot configuration.

4.2 *Example 2*

Another suitable artificial potential energy function in terms of the error function $\widetilde{y} = [\widetilde{p}^T \quad e^T]^T$ is given by the following function

$$\overline{\mathbf{U}}_a(y_d, \widetilde{y}) = \sum_{i=1}^{6} \frac{k_{p_i}}{\lambda_i} \ln[\cosh(\lambda_i \widetilde{y}_i)] \tag{31}$$

where k_{p_i} and λ_i are positive constants. This selection produces the control law

$$\tau = J_G(q)^T Q(\eta_e, e)^T K_P \tanh(\Lambda\widetilde{y}) - K_V\dot{q} + g(q) \tag{32}$$

where $K_P \in \mathbb{R}^{6\times 6}$, $K_V \in \mathbb{R}^{n\times n}$ are positive definite matrices, Λ is a diagonal matrix defined as $\Lambda = \mathrm{diag}\{\lambda_1, \ldots, \lambda_6\} \in \mathbb{R}^{6\times 6}$ with λ_i positive constants; and for a vector $x \in \mathbb{R}^m$, the hyperbolic tangent function is given by $\tanh(x) = [\tanh(x_1) \ \cdots \ \tanh(x_m)]^T$.

Substituting the control law (32) into (1) we obtain the closed–loop system

$$\frac{d}{dt}\begin{bmatrix} q \\ \dot{q} \end{bmatrix} = \begin{bmatrix} \dot{q} \\ M(q)^{-1}[J_G(q)^T Q^T K_P \tanh(\Lambda\widetilde{y}) - K_V\dot{q} - C(q,\dot{q})\dot{q}] \end{bmatrix} \tag{33}$$

According to (19), the Lyapunov function candidate for system (33) is

$$V(q_d - q, \dot{q}) = \frac{1}{2}\dot{q}^T M(q)\dot{q} + \sum_{i=1}^{6} \frac{k_{p_i}}{\lambda_i} \ln[\cosh(\lambda_i \widetilde{y}_i)]. \tag{34}$$

The time derivative of (34) along the trajectories of the closed–loop equation (33) yields

$$V(q_d - q, \dot{q}) = -\dot{q}^T K_V \dot{q}. \tag{35}$$

The time derivative is globally negative semidefinite in virtue of positive definiteness of K_V; therefore, the Lyapunov's direct method allows the conclusion of stability of equilibrium point $[q^T \ \dot{q}^T]^T = [q_d^T \ \mathbf{0}^T]^T$. Due to the autonomous nature of the closed–loop system (25), LaSalle's theorem (Vidyasagar, 1993) can be invoked to show asymptotic stability.

In order to find the largest invariant set in a neighborhood of $[q^T \ \dot{q}^T]^T = [q_d^T \ \mathbf{0}^T]^T$, we need the solution of

$$J_G(q)^T Q(e(q))^T K_P \tanh(\lambda\widetilde{y}(q)) = \mathbf{0}. \tag{36}$$

The gradient of $\sum_{i=1}^{6} \frac{k_{p_i}}{\lambda_i} \ln[\cosh(\lambda_i \widetilde{y}_i)]$ with respect to q is $-J_G(q)^T Q(e)^T K_P \tanh(\lambda\widetilde{y})$ and it vanishes only at $q = q_d$ in a neighborhood of $q = q_d$. Following similar arguments to those of example 1, we can conclude asymptotic stability; and therefore, the achievement of the control objective (7).

5. EXPERIMENTAL RESULTS

The experimental evaluation of the transpose Jacobian–based regulators was carried out on a three degrees–of–freedom spherical wrist manufactured using direct–drive technology (Figure 1). This wrist was built at

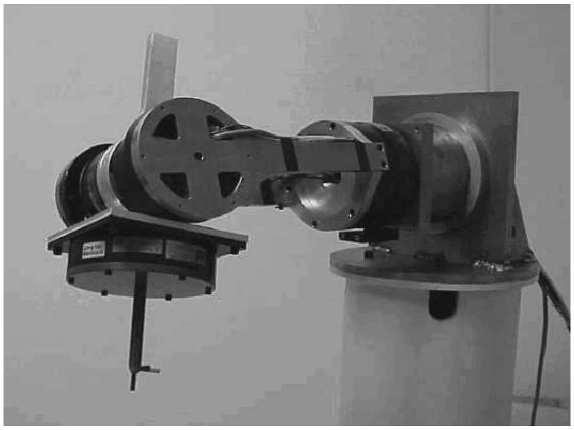

Fig. 1. Three degrees–of–freedom spherical wrist.

CICESE Research Center and it is equipped with joint position sensors, motor drivers, a host computer and software environment which provides a user–friendly interface.

For our wrist, position and orientation cannot be specified arbitrarily, then only orientation is of concern. This means that $\tilde{y} = e$. In this situation the transpose Jacobian control laws (24)and (32) reduce respectively to

$$\tau = J_o(q)^T Q_o^T(\eta_e, e) K_{P_o} e - K_v \dot{q} + g(q), \quad (37)$$

for the controller corresponding to example 1, and

$$\tau = J_o(q)^T Q_o^T(\eta_e, e) K_{P_o} \tanh(\Lambda e) - K_v \dot{q} + g(q) \quad (38)$$

for the controller corresponding to example 2. The orientation part of the wrist geometric Jacobian is given by

$$J_o(q) = \begin{bmatrix} 0 & -\sin(q_1) & \cos(q_1)\sin(q_2) \\ 0 & \cos(q_1) & \sin(q_1)\sin(q_2) \\ 1 & 0 & \cos(q_2) \end{bmatrix}. \quad (39)$$

Wrist singularities occur at $q_2 = n\pi$ with $n = 0, \pm 1, \pm 2, \ldots$ Moreover, our wrist possesses self–motion at these configurations. Therefore, the desired orientation is not allowed to be a singular configuration. In order to compare both controllers, the experiments were executed under similar conditions, that is, initial configuration, desired orientation, and gain matrices for both controllers are the same. Also, the regulators include viscous friction compensation.

The task is completely specified with the desired unit quaternion

$$\eta_d = 0.5566, \quad \varepsilon_d = [0 \quad 0.4226 \quad 0.7848]^T, \quad (40)$$

which has associated the rotation matrix

$$R_d = \begin{bmatrix} -0.3802 & -0.8738 & 0.4705 \\ 0.8738 & -0.0230 & 0.6634 \\ -0.4705 & 0.6634 & 0.8518 \end{bmatrix}. \quad (41)$$

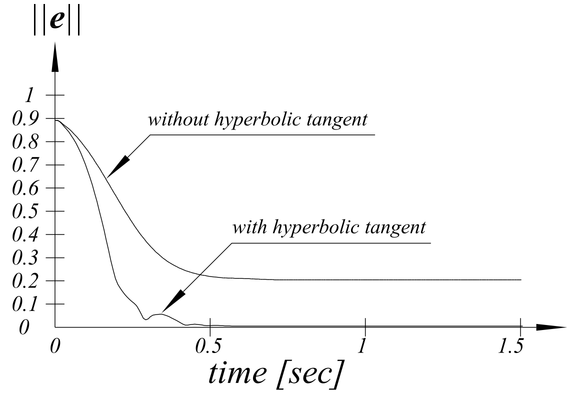

Fig. 2. Norm of orientation error $||e||$ for the transpose Jacobian–based regulators.

The experiments were carried out using the initial joint configuration $q(0) = [0 \ 0 \ 0]^T$ [rad], which has associated the initial unit quaternion $\eta(0) = 1$, $\varepsilon(0) = [0 \quad 0 \quad 0]^T$, with the corresponding initial rotation matrix

$$R(0) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (42)$$

The gain matrices were set to

$$K_{P_o} = \mathrm{diag}\{4.5, 20.5, 17.5\} \text{ [Nm]} \quad (43)$$

$$K_{V_o} = \mathrm{diag}\{1, 1, 1\} \text{ [Nm} \cdot \text{sec/rad]} \quad (44)$$

$$\Lambda = \mathrm{diag}\{30, 30, 30\} \quad (45)$$

A disappointing feature of the transpose Jacobian–based controller (37) is the lack of a procedure for tuning their gains, thus, the gain matrices were obtained through trial and error maneuvers. Also, re–tuning may be needed for different desired orientations. In contrast, the regulator with hyperbolic tangent function (38) does not have those features, it allows to specify several desired orientations without re–tuning of the gain matrices.

Figure 2 shows the experimental results in terms of the time history of the norm of orientation error for the control laws (37) and (38). The regulator that includes the hyperbolic tangent function in the control law (38) has better performance than the controller (37). For the former regulator the asymptotic value of the orientation error norm is $||e|| = 0.2043$ whereas for the latter regulator is $||e|| = 5.3184 \times 10^{-3}$. From a practical viewpoint, the second regulator achieves the control goal. In the first case the large error is due to uncompensated Coulomb friction. The steady state errors can be reduced if the gain matrices are increased, however, this action can saturate the vector of applied joint torques τ. The gain matrices were selected considering this constraint.

6. CONCLUSIONS

Transpose Jacobian–based controllers solve the regulation of manipulator pose without requiring neither the solution of the inverse kinematics nor the computation of robot inverse Jacobian. In this paper we have presented an extension of this control scheme using the energy shaping technique, when the orientation is specified with a non minimal representation —the so–called Euler parameters or unit quaternion— . Asymptotic stability has also been proven with none assumption on the Jacobian singularities. Further research can include Coulomb friction compensation in controllers were unit quaternions are used. Also, gain tuning policies still remain open.

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