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A METHODOLOGY FOR SCHEDULING COMMODITIES IN A MULTI-PRODUCT PIPELINE

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Abstract: This paper addresses the problem of developing an optimisation model to aid the operational decision-making process in pipeline systems. The model is applied on a real world pipeline oil distribution scenario, which connects an inland refinery to a harbour, conveying multiple types of commodities. The optimisation model was developed based on mixed integer linear programming (MILP) with uniform time discretization. The MILP well-known computational difficulty was avoided by the problem domain decomposition. Simulation examples demonstrated that the optimisation model was able to define new operating points, providing significant cost saving. *Copyright © 2002 IFAC*

Keywords: operations research, mathematical programming, integer programming, mathematical models, process automation, scheduling.

1. INTRODUCTION

As the economy moves towards an increasingly global market, companies are forced to focus on production effectiveness under a highly dynamic market. In order to reduce costs and provide better services, the industrial structure modelling has become a fundamental tool.

The oil industry has a strong influence upon the economic market. Research in this area may provide highly profit solutions and also avoid environmental damages. The oil distribution-planning problem is within this context. A wide net with trains, tankers, and pipelines are used to link harbours, refineries and consumers. According to Kennedy (1993), pipelines provide an efficient way to transport oil and gas. The maximum utilisation efficiency of this transportation medium becomes interesting to the oil industry.

According to Lee *et al.* (1996), mathematical programming techniques for long-term planning have been extensively studied and implemented, but much less work has been devoted to short-term operation scheduling, which in fact reproduces the operational decision-making process. The short-term scheduling requires the explicit modelling of discrete decisions. The approach to solve this problem is manifold. A general one is to use a mixed integer linear programming formulation (Pritsker *et al.*, 1969). It comprises a collection of variables under constraints, and an objective function to be either maximised or minimised in the process of assigning values to the variables. The objective function may encode a single scheduling goal, or it may attempt to satisfy a collection of multiple objectives (e.g., minimisation of both order tardiness and amount of changeover activities). Among the MILP solution methods, it can be found branch-and-bound, enumeration, and dynamic programming. A complete survey in mixed integer programming and techniques for several application problems is presented in (Wolsey, 1998). The great concern of a real-world MILP formulation is related to the combinatorial explosion. In practice, it is often impossible to find solutions in a reasonable computational time. An analytical investigation of the combinatorial nature and computational complexity of problems in process systems can be found in (Ahmed and Sahinidis, 2000). According to Applequist *et al.* (1997), the number of integer variables required to represent a practical problem in a MILP feature can be quite large, thus the computational expense should be concerned. Subrahmanyam *et al.* (1995) demonstrates that one approach to avoid the combinatorial explosion introduced by integer variables is based on decomposition strategies.

1.1 Problem Definition.

This work focuses on the short-term scheduling of activities in a specific pipeline system. It connects a harbour to an inland refinery.

Figure 1 illustrates the pipeline physical structure.

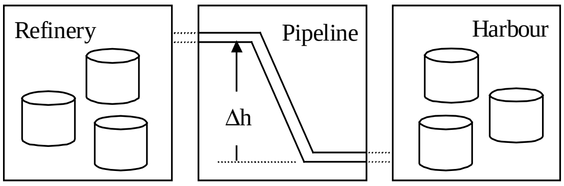

Fig. 1. Pipeline physical structure overview.

The pipeline is 93.5 km length, it can store a total volume of 7,314 m^3, and it connects the refinery tank farm to the harbour tank farm going along regions with 900-meter-altitude difference (Δh). The pipe conveys multiple types of commodities. It is possible to have flow either from the refinery to the harbour or from the harbour to the refinery. There is no physical separation between different products as they move in the pipe. Consequently, there is a contamination area between products: the interface. In order to avoid a specific contamination, a plug product can be used between elements. This procedure increases the operating cost. The tank farm infrastructure, an up-to-date storage scenario, the pipeline flow rate details, and the demand requirements are known a priori. The scheduling process must take into account product availability, tankage constraints, pumping sequencing, flow rate determination, and a wide variety of operational requirements. The task is to predict the pipeline operation during a limited time horizon (T), providing low cost operational procedures.

2. METHODOLOGY

The methodology employed in this work is the mixed integer linear programming with uniform time discretization. The computational complexity is concerned, and the problem is splited in small entities. The division is based on the three key elements of scheduling: *assignment* of resources, *sequencing* of activities and determination of resource *timing* utilisation by these activities (Reklaitis, 1992). The idea is to share the basic scheduling elements among an integrated architecture (Figure 2), providing a framework that aims to reduce the computational expense.

The integrated architecture is based upon a MILP main model (*Main Model*), two auxiliary MILP models (*Tank Bound* and *Plug Bound*), and a computational procedure (*Time Bound*), all of them sharing a *Data Base*. To summarise, the tank bound is responsible for the assignment of resources, the plug bound determines the sequencing of activities, and both time bound and main model are used to process the timing feature. Care was taken in order to provide a consistency among the scheduling features interchanged by different optimisation blocks.

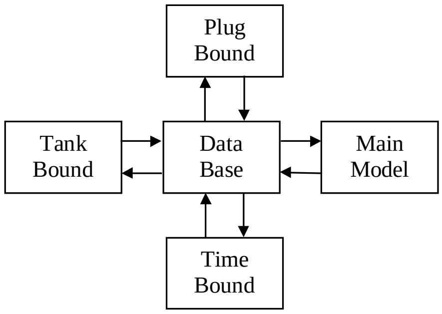

Fig. 2. Integrated architecture overview.

The tank bound task involves the appropriate selection of some resources (tanks) for a given activity (pumping the demanded product). Its main inputs are demand requirements, product availability, and tankage constraints. As an output, it specifies the tanks to be used in operational procedures.

The main input to the plug bound is the compatibility matrix, which informs the plug necessity between demanded products. Based on this information, this auxiliary model determines the pumping sequence that minimises the use of plugs.

The auxiliary routine time bound uses the tank bound and the plug bound information to determine temporal constraints, which are applied on the main model. The main model task is the choice of specific starting and stopping times of each pumping activity.

The final scheduling is attained by first addressing the assignment problem, followed by the sequencing task, and, at last, by determining the timing over a limited period. Figure 3 illustrates the integrated-architecture-solving-precedence. A fundamental issue of this approach is that the output of one optimisation block (determined variables) can be used as parameters to the subsequent block. An in-depth description of the integrated architecture can be found in (Magatão, 2001).

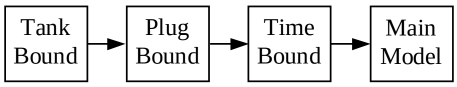

Fig. 3. Integrated architecture solving precedence.

2.1 Optimisation Model: Mathematical Formulation.

The modelling process takes into account the following conditions: (i) Pipeline can fill or empty only one tank at a time; (ii) Tanks being emptied can not be filled; (iii) A tank always stores the same product; (iv) The tank farm infrastructure limits must be respected; (v) The product flow rate range must be respected; (vi) The product demand has to be within an operational range; (vii) Every product must be pumped uninterruptedly; (viii) It is possible to use a plug product between incompatible elements, but plug inclusions increase the operating cost; (ix) The plug volume is significantly smaller than any demanded batch, so that its pumping time is neglected; (x) Changeover times are neglected; (xi) Tank changeovers should be minimised; (xii) Use of plugs should be minimised; (xiii) In order to pump the entire demand, it is required a minimum time horizon (T_{min}). In such a horizon, every product is pumped at its maximum flow rate; (xiv) To pump products from refinery to harbour is called *flow* procedure. To pump from harbour to refinery is called *reflow* procedure; (xv) Complete pumping operation covers either a flow followed by a reflow procedure or a reflow followed by a flow procedure. The user specifies the ordering flow/reflow or reflow/flow; (xvi) The pipeline stores 7,314 m^3 and it always operates completely filled. There is a time between sending a product and receiving it. Therefore, after sending either the last flow sequenced product or last reflow sequenced product, it is necessary to pump an extra product amount to maintain the pipe filled. Between flow/reflow or reflow/flow operations, pumping the extra product amount is called *gap* procedure. After sending all demanded products, filling the pipe with an extra product amount is called *end* procedure; (xvii) The system starts pumping at the initial time (t=1). In case $T>T_{min}$ the pumping procedure can be finished before T, but the pipeline must remain pressurised. This procedure also increases the operating cost.

The mathematical approach, as stated, is based on MILP with uniform time discretization. Expressions (1) to (137) present the integrated architecture formulation, exploiting some of its features. Space restrictions preclude a detailed formulation description. Such an information can be obtained in (Magatão, 2001).

Tank Bound. Objective Function: minimise tank changeovers.

$$minimise = TB\_P + TB\_R + \sum_p (1 - LP_p) + \sum_p (1 - LR_p) \qquad (1)$$

Subject to constraints: The minimum number of tank changeovers occurs when each required product is pumped from just one tank.

$$\sum_p \sum_{j \in PT_p} TBT^p_{p,j} - npr = TB\_P \qquad (2)$$

$$\sum_p \sum_{i \in RT_p} TBT^r_{p,i} - npp = TB\_R \qquad (3)$$

The required product volume has to respect operational limits.

$$ER^{\min}_{p,i} \le Vf^r_{p,i} \le ER^{\max}_{p,i} \quad \forall p; i \in RT_p; \qquad (4)$$

$$EP^{\min}_{p,j} \le Vf^p_{p,j} \le EP^{\max}_{p,j} \quad \forall p; j \in PT_p; \qquad (5)$$

$$QR^{\min}_p \le QR_p \le QR^{\max}_p \quad \forall p; \qquad (6)$$

$$QP^{\min}_p \le QP_p \le QP^{\max}_p \quad \forall p; \qquad (7)$$

$$QR^{\max}_p \le \sum_{i \in RT_p} (ER^{\max}_{p,i} - Vi^r_{p,i}) \quad \forall p; \qquad (8)$$

$$QP^{\max}_p \le \sum_{j \in PT_p} (EP^{\max}_{p,j} - Vi^p_{p,j}) \quad \forall p; \qquad (9)$$

$$QR^{\max}_p \le \sum_{j \in PT_p} (Vi^p_{p,j} - EP^{\min}_{p,j}) \quad \forall p; \qquad (10)$$

$$QP^{\max}_p \le \sum_{i \in RT_p} (Vi^r_{p,i} - ER^{\min}_{p,i}) \quad \forall p; \qquad (11)$$

Demand requirements must be satisfied.

$$QR_p = \sum_{j \in PT_p} (Vi^p_{p,j} - Vf^p_{p,j}) \quad \forall p; \qquad (12)$$

$$QP_p = \sum_{i \in RT_p} (Vi^r_{p,i} - Vf^r_{p,i}) \quad \forall p; \qquad (13)$$

Expressions to establish the emptying of tanks:

$$Vi^r_{p,i} - Vf^r_{p,i} \ge 0 \quad \forall p; i \in RT_p; \qquad (14)$$

$$Vi^p_{p,j} - Vf^p_{p,j} \ge 0 \quad \forall p; j \in PT_p; \qquad (15)$$

It is admitted an operational transition in case the final tank volume differs from its initial time volume. Therefore, binary variables $TBT^r_{p,i}$ and $TBT^p_{p,j}$ must assume the unitary value when, respectively, expressions (14) and (15) were greater than zero. The *Big-M* technique (Shah *et al.*, 1993) is used to model these conditions. In equation (16), it is important to notice that $K > Vf^p_{p,j} - Vi^p_{p,j} \quad \forall p, j \in PT_p$ and $K > Vf^r_{p,i} - Vi^r_{p,i}$ $\forall p, i \in RT_p$.

$$K = \sum_p \sum_{i \in RT_p} ER^{\max}_{p,i} + \sum_p \sum_{j \in RT_p} EP^{\max}_{p,j} \qquad (16)$$

$$Vf^p_{p,j} - Vi^p_{p,j} \ge -K \cdot TBT^p_{p,j} \quad \forall p; j \in PT_p; \qquad (17)$$

$$Vf^r_{p,i} - Vi^r_{p,i} \ge -K \cdot TBT^r_{p,i} \quad \forall p; i \in RT_p; \qquad (18)$$

Due to operational facilities, at the end of flow or reflow operation, the last sequenced product also fills the pipeline. The tank bound verifies product availability, indicating elements that can be used to finish flow/reflow operations.

$$\sum_{j \in PT_p} (Vi^p_{p,j} - EP^{\min}_{p,j}) - QR_p - VD \ge -K \cdot (1 - LP_p) \quad \forall p; \qquad (19)$$

$$\sum_{i \in RT_p} (Vi^r_{p,i} - ER^{\min}_{p,i}) - QP_p - VD \ge -K \cdot (1 - LR_p) \quad \forall p; \qquad (20)$$

Plug Bound. Objective Function: minimises the use of plugs.

$$minimise = SB\_R + SB\_P + SB\_SW + SB\_PND \qquad (21)$$

Subject to constraints: Two products sequentially pumped generate an operational transition. The total number of operational transitions is related to the number of demanded products.

$$\sum_p \sum_{pa} \sum_{s=1}^{npr-1} TR\_P_{p,pa,s} = npr - 1 \quad \forall p \ne pa; \qquad (22)$$

$$\sum_p \sum_{pa} \sum_{s=1}^{npp-1} TR\_R_{p,pa,s} = npp - 1 \quad \forall p \ne pa; \qquad (23)$$

A pumping transition between incompatible products demands the use of a plug.

$$\sum_p \sum_{pa} \sum_{s=1}^{npr-1} TR\_P_{p,pa,s} \cdot I_{p,pa} = SB\_P \quad \forall p \ne pa; \qquad (24)$$

$$\sum_p \sum_{pa} \sum_{s=1}^{npp-1} TR\_R_{p,pa,s} \cdot I_{p,pa} = SB\_R \quad \forall p \ne pa; \qquad (25)$$

Logical arrangement to guarantee the sequencing of all demanded products:

$$\sum_{pa}\sum_{s=1}^{npr-1} TR\_P_{p,pa,s} \le 1 \quad \forall p; \tag{26}$$

$$\sum_{pa}\sum_{s=1}^{npp-1} TR\_R_{p,pa,s} \le 1 \quad \forall p; \tag{27}$$

$$\sum_{p}\sum_{s=1}^{npr-1} TR\_P_{p,pa,s} \le 1 \quad \forall pa; \tag{28}$$

$$\sum_{p}\sum_{s=1}^{npp-1} TR\_R_{p,pa,s} \le 1 \quad \forall pa; \tag{29}$$

$$\sum_{p}\sum_{pa} TR\_P_{p,pa,s} \le 1 \quad 1 \le s < npr; \tag{30}$$

$$\sum_{p}\sum_{pa} TR\_R_{p,pa,s} \le 1 \quad 1 \le s < npp; \tag{31}$$

The operational transition between either flow/reflow or reflow/flow is considered (switch transition), and it can be expressed as an implication form:

$$(TR\_P_{pa,pb,sf})\, and\, (TR\_R_{pc,pd,si}) \Rightarrow TR\_PR_{pa,pb,pc,pd} \\ \forall pa \ne pb, pc \ne pd; sf = npr-1, si = 1; P\_R = 1; \tag{32}$$

$$(TR\_R_{pa,pb,sf})\, and\, (TR\_P_{pc,pd,si}) \Rightarrow TR\_RP_{pa,pb,pc,pd} \\ \forall pa \ne pb, pc \ne pd; sf = npp-1, si = 1; R\_P = 1; \tag{33}$$

Considering the implication: (A) and (B) $\Rightarrow$ C, where A and B are binary variables, expression (34) demonstrates the implication in an equivalent mathematical programming formulation (LINDO, 1999).

$$C \le A; \quad C \le B; \quad C \ge A + B - 1 \tag{34}$$

Thus, implication (32) and (33) can be expressed as:

$$TR\_PR_{pa,pb,pc,pd} \le TR\_P_{pa,pb,sf}; \\ TR\_PR_{pa,pb,pc,pd} \le TR\_R_{pc,pd,si}; \\ TR\_PR_{pa,pb,pc,pd} \ge TR\_P_{pa,pb,sf} + TR\_R_{pc,pd,si} - 1 \\ \forall pa \ne pb, pc \ne pd; sf = npr-1, si = 1; P\_R = 1; \tag{35}$$

$$TR\_RP_{pa,pb,pc,pd} \le TR\_R_{pa,pb,sf}; \\ TR\_RP_{pa,pb,pc,pd} \le TR\_P_{pc,pd,si}; \\ TR\_RP_{pa,pb,pc,pd} \ge TR\_R_{pa,pb,sf} + TR\_P_{pc,pd,si} - 1 \\ \forall pa \ne pb, pc \ne pd; sf = npp-1, si = 1; R\_P = 1; \tag{36}$$

The switch transition must occur, and it is possible to use a plug.

$$1 = \sum_{pa}\sum_{pb}\sum_{pc}\sum_{pd} TR\_PR_{pa,pb,pc,pd} \cdot P\_R + \\ + \sum_{pa}\sum_{pb}\sum_{pc}\sum_{pd} TR\_RP_{pa,pb,pc,pd} \cdot R\_P \tag{37}$$

$$SB\_SW = \sum_{pa}\sum_{pb}\sum_{pc}\sum_{pd} TR\_PR_{pa,pb,pc,pd} \cdot I_{pb,pc} \cdot P\_R + \\ + \sum_{pa}\sum_{pb}\sum_{pc}\sum_{pd} TR\_RP_{pa,pb,pc,pd} \cdot I_{pb,pc} \cdot R\_P \tag{38}$$

Logical arrangement to obtain the sequencing of all demanded products:

$$\sum_{pa} TR\_P_{p,pa,s} = AUX\_P_{p,s} \quad \forall p; 1 \le s < npr; \tag{39}$$

$$\sum_{pa} TR\_R_{p,pa,s} = AUX\_R_{p,s} \quad \forall p; 1 \le s < npp; \tag{40}$$

$$1 - \sum_{pa}\sum_{sa=1}^{npr-1} TR\_P_{p,pa,sa} = AUX\_P_{p,s} \quad \forall p; s = npr; \tag{41}$$

$$1 - \sum_{pa}\sum_{sa=1}^{npp-1} TR\_R_{p,pa,sa} = AUX\_R_{p,s} \quad \forall p; s = npp; \tag{42}$$

$$\sum_{p} s \cdot AUX\_P_{p,s} = OP_s \quad 1 \le s \le npr; \tag{43}$$

$$\sum_{p} s \cdot AUX\_R_{p,s} = OR_s \quad 1 \le s \le npp; \tag{44}$$

The pipe operates filled. There is an operational transition between the first sequenced product and the element that is pressurised in the pipeline. It is also possible to use a plug in such transition.

$$SB\_PND = I_{pa,p} \cdot P\_R \cdot \sum_{p} AUX\_P_{p,s} + \\ + I_{pa,p} \cdot R\_P \cdot \sum_{p} AUX\_R_{p,s} \quad pa = PND; s = 1; \tag{45}$$

The product ordering considers tankage constraints, which were previously determined by the tank bound.

$$\sum_{p} TR\_P_{p,pa,s} \le LP_{pa} \quad \forall p \ne pa; s = npr-1; \tag{46}$$

$$\sum_{p} TR\_R_{p,pa,s} \le LR_{pa} \quad \forall p \ne pa; s = npp-1; \tag{47}$$

Time Bound. This auxiliary computational procedure determines parameters that are dependent on both the product ordering (obtained by the plug bound) and the usage of tanks (determined by the tank bound).

Gap procedure lower/upper time interval:

$$TGap^{\min} = \left\langle \frac{VD}{PP_{pa}^{\max}} \right\rangle \cdot P\_R + \left\langle \frac{VD}{PR_{p}^{\max}} \right\rangle \cdot R\_P \\ pa = OP_{sa}, p = OR_s; sa = npr; s = npp; \tag{48}$$

$$TGap^{\max} = \left\langle \frac{VD}{PP_{pa}^{\min}} \right\rangle \cdot P\_R + \left\langle \frac{VD}{PR_{p}^{\min}} \right\rangle \cdot R\_P \\ pa = OP_{sa}, p = OR_s; sa = npr; s = npp; \tag{49}$$

End procedure lower/upper time interval:

$$TT^{\min} = \left\langle \frac{VD}{PP_{pa}^{\max}} \right\rangle \cdot R\_P + \left\langle \frac{VD}{PR_{p}^{\max}} \right\rangle \cdot P\_R \\ pa = OP_{sa}, p = OR_s; sa = npr; s = npp; \tag{50}$$

$$TT^{\max} = \left\langle \frac{VD}{PP_{pa}^{\min}} \right\rangle \cdot R\_P + \left\langle \frac{VD}{PR_{p}^{\min}} \right\rangle \cdot P\_R \\ pa = OP_{sa}, p = OR_s; sa = npr; s = npp; \tag{51}$$

Time limits to the end pumping of batches (the symbol $\langle\ \rangle$ indicates that is necessary to round off the division to the next integer value):

$$TFBI_{p,j}^{p} = \left\{ \sum_{\substack{k=1 \\ pv=OP_k}}^{k=s} \left\langle \frac{QR_{pv}}{PP_{pv}^{\max}} \right\rangle + R\_P \cdot \left[\sum_{pa} \left\langle \frac{QP_{pa}}{PR_{pa}^{\max}} \right\rangle + TGap^{\min} \right] \right\} \cdot TBT_{p,j}^{p} \\ p = OP_s; 1 \le s \le npr; j \in PT_p; \tag{52}$$

$$TFBS_{p,j}^{p} = \left\{ \sum_{\substack{k=1 \\ pv=OP_k}}^{k=s} \left\langle \frac{QR_{pv}}{PP_{pv}^{\min}} \right\rangle + R\_P \cdot \left[\sum_{pa} \left\langle \frac{QP_{pa}}{PR_{pa}^{\min}} \right\rangle + TGap^{\max} \right] \right\} \cdot TBT_{p,j}^{p} \\ p = OP_s; 1 \le s \le npr; j \in PT_p; \tag{53}$$

$$TFBI_{p,i}^{r} = \left\{ \sum_{\substack{k=1 \\ pv=OR_k}}^{k=s} \left\langle \frac{QP_{pv}}{PR_{pv}^{\max}} \right\rangle + P\_R \cdot \left[\sum_{pa} \left\langle \frac{QR_{pa}}{PP_{pa}^{\max}} \right\rangle + TGap^{\min} \right] \right\} \cdot TBT_{p,i}^{r} \\ p = OR_s; 1 \le s \le npp; i \in RT_p; \tag{54}$$

$$TFBS_{p,i}^{r} = \left\{ \sum_{\substack{k=1 \\ pv=OR_k}}^{k=s} \left\langle \frac{QP_{pv}}{PR_{pv}^{\min}} \right\rangle + P\_R \cdot \left[\sum_{pa} \left\langle \frac{QR_{pa}}{PP_{pa}^{\min}} \right\rangle + TGap^{\max} \right] \right\} \cdot TBT_{p,i}^{r} \\ p = OR_s; 1 \le s \le npp; i \in RT_p; \tag{55}$$

Time limits to the start pumping of batches:

$$TIBI_{p,j}^{p} = TFBI_{p,j}^{p} - \left\langle \frac{QR_p}{PP_p^{\max}} \right\rangle \quad p = OP_s; 1 \le s \le npr; j \in PT_p; \tag{56}$$

$$TIBS_{p,j}^{p} = TFBS_{p,j}^{p} - \left\langle \frac{QR_p}{PP_p^{\max}} \right\rangle \quad p = OP_s; 1 \le s \le npr; j \in PT_p; \tag{57}$$

$$TIBI_{p,i}^{r} = TFBI_{p,i}^{r} - \left\langle \frac{QP_p}{PR_p^{\max}} \right\rangle \quad p = OR_s; 1 \le s \le npp; i \in RT_p; \tag{58}$$

$$TIBS_{p,i}^{r} = TFBS_{p,i}^{r} - \left\langle \frac{QP_p}{PR_p^{\max}} \right\rangle \quad p = OR_s; 1 \le s \le npp; i \in RT_p; \tag{59}$$

Lower time limit of gap/end procedure start:

$$INIG = \left\lceil \frac{\sum_{j \in PT_{pa}} TFBI_{pa,j}^{p}}{\sum_{j \in PT_{pa}} TBT_{pa,j}^{p}} \right\rceil \cdot P\_R + \left\lceil \frac{\sum_{i \in RT_p} TFBI_{p,i}^{r}}{\sum_{i \in RT_p} TBT_{p,i}^{r}} \right\rceil \cdot R\_P \tag{60}$$

$$pa = OP_{sa}, p = OR_s; sa = npr; s = npp;$$

$$INIT = \left\lceil \frac{\sum_{j \in PT_{pa}} TFBI_{pa,j}^{p}}{\sum_{j \in PT_{pa}} TBT_{pa,j}^{p}} \right\rceil \cdot R\_P + \left\lceil \frac{\sum_{i \in RT_p} TFBI_{p,i}^{r}}{\sum_{i \in RT_p} TBT_{p,i}^{r}} \right\rceil \cdot P\_R \tag{61}$$

$$pa = OP_{sa}, p = OR_s; sa = npr; s = npp;$$

Upper time limit of gap/end procedure completion:

$$FIMG = \left\lceil \frac{\sum_{j \in PT_{pa}} TIBS_{pa,j}^{p}}{\sum_{j \in PT_{pa}} TBT_{pa,j}^{p}} \right\rceil \cdot R\_P + \left\lceil \frac{\sum_{i \in RT_p} TIBS_{p,i}^{r}}{\sum_{i \in RT_p} TBT_{p,i}^{r}} \right\rceil \cdot P\_R \tag{62}$$

$$pa = OP_s, p = OR_s; s = 1;$$

$$FIMT = T \tag{63}$$

Upper time limit of end procedure start:

$$TL = T - \left\langle \frac{VD}{PP_{pa}^{\max}} \right\rangle \cdot R\_P - \left\langle \frac{VD}{PR_p^{\max}} \right\rangle \cdot P\_R \tag{64}$$

$$pa = OP_{sa}, p = OR_s; sa = npr; s = npp;$$

Main Model. Objective function: minimises the operating cost:

$$\begin{aligned}
&minimize\ \ Cost\ = \\
&= CR_{pump} \cdot \sum_{p} \sum_{i \in RT_p} (TFB_{p,i}^{r} - TIB_{p,i}^{r}) + \\
&+ CP_{pump} \cdot \sum_{p} \sum_{j \in PT_p} (TFB_{p,j}^{p} - TIB_{p,j}^{p}) + \\
&+ C_{plug} \cdot \sum_{p} \sum_{\substack{pa \\ p \ne pa}} \sum_{i \in RT_p} \sum_{m \in RT_{pa}} \sum_{t=2}^{T\text{-}1} SR_{p,pa,i,m,t} + \\
&+ C_{plug} \cdot \sum_{p} \sum_{\substack{pa \\ p \ne pa}} \sum_{j \in PT_p} \sum_{n \in PT_{pa}} \sum_{t=2}^{T\text{-}1} SP_{p,pa,j,n,t} + \\
&+ C_{tank} \cdot \sum_{p} \sum_{\substack{pa \\ p \ne pa}} \sum_{i \in RT_p} \sum_{m \in RT_{pa}} \sum_{t=2}^{T\text{-}1} TR_{p,pa,i,m,t} + \\
&+ C_{tank} \cdot \sum_{p} \sum_{\substack{pa \\ p \ne pa}} \sum_{j \in PT_p} \sum_{n \in PT_{pa}} \sum_{t=2}^{T\text{-}1} TP_{p,pa,j,n,t} + \\
&+ C_{tank} \cdot \sum_{p} \sum_{i \in RT_p} \sum_{\substack{k \in RT_p \\ i \ne k}} \sum_{t=2}^{T\text{-}1} TR_{p,i,k,t} + \\
&+ C_{tank} \cdot \sum_{p} \sum_{j \in PT_p} \sum_{\substack{l \in PT_p \\ j \ne l}} \sum_{t=2}^{T\text{-}1} TP_{p,j,l,t} + \\
&+ \sum_{t=1}^{T-1} Ce_t \cdot (PP_t + PR_t) + CS \cdot TS + \\
&+ C_{plug} \cdot (SB_SW + SB_PND) + \\
&+ TGAP \cdot (CP_{pump} \cdot P_R + CR_{pump} \cdot R_P) + CEGAP + \\
&+ TT \cdot (CP_{pump} \cdot R_P + CR_{pump} \cdot P_R) + CET
\end{aligned} \tag{65}$$

Equation (65) demonstrates that the plug inclusion ($SP_{p,pa,t}$, $SR_{p,pa,t}$) and the occurrence of changeovers ($TP_{p,j,l,t}$, $TR_{p,i,k,t}$, $TP_{p,pa,j,n,t}$, $TR_{p,pa,i,m,t}$) increase the operating cost. As a result, the optimisation solution method seeks scheduling answers that minimise both the number of plug inclusions and the number of changeover occurrences. There is also a cost associated with the time period that a product is pumped in the pipeline ($TFB_{p,j}^{p}$, $TFB_{p,i}^{r}$, $TIB_{p,j}^{p}$, $TIB_{p,i}^{r}$, $TGAP$, TT). This time is related to the flow rate by an inverse ratio: if the flow rate increases, the product pumping time decreases and the operating cost decreases. On the other hand, if the flow rate increases, the electric pumping cost increases and so the operating cost. Consequently, there are contrasting objectives, and the optimisation model must determine the ideal flow rate policy during a limited time horizon (*T*).

Subject to constraints: Each product is pumped only once throughout the scheduling horizon.

$$\sum_{t=2}^{TL-1} IB_{p,i,t}^{r} \le 1 \quad \forall p, i \in RT_p; \tag{66}$$

$$\sum_{t=2}^{TL-1} IB_{p,j,t}^{p} \le 1 \quad \forall p, j \in PT_p; \tag{67}$$

At least one batch starts being pumped at the initial time.

$$\sum_{i \in RT_p} (IB_{p,i,t}^{r} \cdot TBT_{p,i}^{r}) = 1 \quad p = OR_s; s = 1; t = 1; R\_P = 1; \tag{68}$$

$$\sum_{j \in PT_p} (IB_{p,j,t}^{p} \cdot TBT_{p,j}^{p}) = 1 \quad p = OP_s; s = 1; t = 1; P\_R = 1; \tag{69}$$

Each time unit can have one pumping start variable set to one.

$$\sum_{p} \sum_{i \in RT_p} IB_{p,i,t}^{r} + \sum_{p} \sum_{j \in PT_p} IB_{p,j,t}^{p} \le 1 \quad 2 \le t \le TL - 1; \tag{70}$$

Each time unit can have one pumping finish variable set to one.

$$\sum_{p} \sum_{i \in RT_p} FB_{p,i,t}^{r} + \sum_{p} \sum_{j \in PT_p} FB_{p,j,t}^{p} \le 1 \quad 2 \le t \le TL; \tag{71}$$

The pumping finish variable is set to one only if the product starts being pumped in the scheduling horizon.

$$\sum_{t=1}^{TL-1} IB_{p,i,t}^{r} - \sum_{t=2}^{TL} FB_{p,i,t}^{r} = 0 \quad \forall p, i \in RT_p; \tag{72}$$

$$\sum_{t=1}^{TL-1} IB_{p,j,t}^{p} - \sum_{t=2}^{TL} FB_{p,j,t}^{p} = 0 \quad \forall p, j \in PT_p; \tag{73}$$

Equations for determining the pumping initial time:

$$\sum_{t=1}^{TL-1} t \cdot IB_{p,i,t}^{r} = TIB_{p,i}^{r} \quad \forall p, i \in RT_p; \tag{74}$$

$$\sum_{t=1}^{TL-1} t \cdot IB_{p,j,t}^{p} = TIB_{p,j}^{p} \quad \forall p, j \in PT_p; \tag{75}$$

Equations for determining the pumping final time:

$$\sum_{t=2}^{TL} t \cdot FB_{p,i,t}^{r} = TFB_{p,i}^{r} \quad \forall p, i \in RT_p; \tag{76}$$

$$\sum_{t=2}^{TL} t \cdot FB_{p,j,t}^{p} = TFB_{p,j}^{p} \quad \forall p, j \in PT_p; \tag{77}$$

Expressions to establish temporal constraints:

$$TFB^{r}_{p,i} \geq TIB^{r}_{p,i} \quad \forall p, i \in RT_p; \tag{78}$$

$$TFB^{p}_{p,j} \geq TIB^{p}_{p,j} \quad \forall p, j \in PT_p; \tag{79}$$

The demanded volume divided by the maximum product flow rate determines the minimum product pumping time.

$$\sum_{i \in RT_p} (TFB^{r}_{p,i} - TIB^{r}_{p,i}) \geq \left\langle \frac{QP_p}{PR^{\max}_p} \right\rangle \quad \forall p; \tag{80}$$

$$\sum_{j \in RT_p} (TFB^{p}_{p,j} - TIB^{p}_{p,j}) \geq \left\langle \frac{QR_p}{PP^{\max}_p} \right\rangle \quad \forall p; \tag{81}$$

Continuous flow constraints: there must not be a time interruption between the pumping finish of one product and the pumping start of the preceding one.

$$TK = \sum_{p} \sum_{i \in RT_p} (TFB^{r}_{p,i} - TIB^{r}_{p,i}) + \sum_{pa} \sum_{j \in PT_{pa}} (TFB^{p}_{pa,j} - TIB^{p}_{pa,j}) + TGAP \tag{82}$$

$$TFB^{r}_{p,i} - TIB^{p}_{pa,j} \leq TK \cdot TBT^{r}_{p,i} \cdot TBT^{p}_{pa,j} \quad \forall p, pa; i \in RT_p; j \in PT_{pa}; P\_R = 1; \tag{83}$$

$$TFB^{p}_{pa,j} - TIB^{r}_{p,i} \leq TK \cdot TBT^{r}_{p,i} \cdot TBT^{p}_{pa,j} \quad \forall p, pa; i \in RT_p; j \in PT_{pa}; R\_P = 1; \tag{84}$$

The gap time has to be considered between either flow or reflow operations.

$$[TIB^{p}_{pa,j} - TFB^{r}_{p,i} - TGAP] \cdot TBT^{r}_{p,i} \cdot TBT^{p}_{pa,j} \geq 0 \quad \forall p, pa; i \in RT_p; j \in PT_{pa}; R\_P = 1; \tag{85}$$

$$[TIB^{r}_{p,i} - TFB^{p}_{pa,j} - TGAP] \cdot TBT^{r}_{p,i} \cdot TBT^{p}_{pa,j} \geq 0 \quad \forall p, pa; i \in RT_p; j \in PT_{pa}; P\_R = 1; \tag{86}$$

Pumping transition between tanks of the same product can be expressed as an implication form.

$$(IB^{r}_{p,i,t}) and (FB^{r}_{p,k,t}) \Rightarrow TR_{p,i,k,t} \quad \forall p; i,k \in RT_p; i \neq k; 2 \leq t \leq T-1; \tag{87}$$

$$(IB^{p}_{p,j,t}) and (FB^{p}_{p,l,t}) \Rightarrow TP_{p,j,l,t} \quad \forall p; j,l \in PT_p; j \neq l; 2 \leq t \leq T-1; \tag{88}$$

Using expression (34), the (87) and (88) can be expressed as:

$$TR_{p,i,k,t} \leq IB^{r}_{p,i,t}; TR_{p,i,k,t} \leq FB^{r}_{p,k,t}; TR_{p,i,k,t} \geq IB^{r}_{p,i,t} + FB^{r}_{p,k,t} - 1 \quad \forall p; i,k \in RT_p; i \neq k; 2 \leq t \leq T-1; \tag{89}$$

$$TP_{p,j,l,t} \leq IB^{p}_{p,j,t}; TP_{p,j,l,t} \leq FB^{p}_{p,l,t}; TP_{p,j,l,t} \geq IB^{p}_{p,j,t} + FB^{p}_{p,l,t} - 1 \quad \forall p; j,l \in PT_p; j \neq l; 2 \leq t \leq T-1; \tag{90}$$

The pumping transition between products can be expressed as an implication form.

$$(IB^{r}_{pa,m,t}) and (FB^{r}_{p,i,t}) \Rightarrow TR_{p,pa,i,m,t} \quad \forall p, pa; i \in RT_p, m \in RT_{pa}; p \neq pa; 2 \leq t \leq T-1; \tag{91}$$

$$(IB^{p}_{pa,n,t}) and (FB^{p}_{p,j,t}) \Rightarrow TP_{p,pa,j,n,t} \quad \forall p, pa; j \in PT_p, n \in PT_{pa}; p \neq pa; 2 \leq t \leq T-1; \tag{92}$$

Using expression (34), the (91) and (92) can be formulated as:

$$TR_{p,pa,i,m,t} \leq IB^{r}_{pa,m,t}; TR_{p,pa,i,m,t} \leq FB^{r}_{p,i,t}; TR_{p,pa,i,m,t} \geq IB^{r}_{pa,m,t} + FB^{r}_{p,i,t} - 1 \quad \forall p, pa; i \in RT_p, m \in RT_{pa}; p \neq pa; 2 \leq t \leq T-1; \tag{93}$$

$$TP_{p,pa,j,n,t} \leq IB^{p}_{pa,n,t}; TP_{p,pa,j,n,t} \leq FB^{p}_{p,j,t}; TP_{p,pa,j,n,t} \geq IB^{p}_{pa,n,t} + FB^{p}_{p,j,t} - 1 \quad \forall p, pa; j \in PT_p, n \in PT_{pa}; p \neq pa; 2 \leq t \leq T-1; \tag{94}$$

A pumping transition between incompatible products demands the use of a plug.

$$SR_{p,pa,i,m,t} = TR_{p,pa,i,m,t} \cdot I_{p,pa} \quad \forall p,pa; i \in RT_p; m \in RT_{pa}; 2 \leq t \leq T-1; \tag{95}$$

$$SP_{p,pa,j,n,t} = TP_{p,pa,j,n,t} \cdot I_{p,pa} \quad \forall p,pa; j \in PT_p; n \in PT_{pa}; 2 \leq t \leq T-1; \tag{96}$$

Equations (97) to (100) help to model the time interval that the pipe empties a tank.

$$ON^{r}_{p,i,t} = IB^{r}_{p,i,t} \quad \forall p; i \in RT_p; t = 1; \tag{97}$$

$$ON^{p}_{p,j,t} = IB^{p}_{p,j,t} \quad \forall p; j \in PT_p; t = 1; \tag{98}$$

$$ON^{r}_{p,i,t} = IB^{r}_{p,i,t} - FB^{r}_{p,i,t} + ON^{r}_{p,i,t-1} \quad \forall p; i \in RT_p; 2 \leq t \leq T-1; \tag{99}$$

$$ON^{p}_{p,j,t} = IB^{p}_{p,j,t} - FB^{p}_{p,j,t} + ON^{p}_{p,j,t-1} \quad \forall p; j \in PT_p; 2 \leq t \leq T-1; \tag{100}$$

Overlap between batches is not a valid condition.

$$\sum_{p} \sum_{i \in RT_p} ON^{r}_{p,i,t} + \sum_{p} \sum_{j \in PT_p} ON^{p}_{p,j,t} \leq 1 \quad 2 \leq t \leq T-1; \tag{101}$$

The pipeline flow rate has to be respected.

$$\tau \cdot PR^{\min}_p \cdot ON^{r}_{p,i,t-1} \leq V^{r}_{p,i,t-1} - V^{r}_{p,i,t} \leq \tau \cdot PR^{\max}_p \cdot ON^{r}_{p,i,t-1} \quad \forall p; i \in RT_p; 2 \leq t \leq T; \tag{102}$$

$$\tau \cdot PP^{\min}_p \cdot ON^{p}_{p,j,t-1} \leq V^{p}_{p,j,t-1} - V^{p}_{p,j,t} \leq \tau \cdot PP^{\max}_p \cdot ON^{p}_{p,j,t-1} \quad \forall p; j \in PT_p; 2 \leq t \leq T; \tag{103}$$

The required product volume has to respect operational limits. Expressions (6) to (11), used in the tank bound, are also applied on the main model formulation (considering $Vi^{r}_{p,i} = V^{r}_{p,i,t} \ \forall p; i \in RT_p; t = 1$; $Vi^{p}_{p,j} = V^{p}_{p,j,t} \ \forall p; j \in PT_p; t = 1$). Expressions (104) and (105) establish that tank storage range has to be respected.

$$ER^{\min}_{p,i} \leq V^{r}_{p,i,t} \leq ER^{\max}_{p,i} \quad \forall p; i \in RT_p; t; \tag{104}$$

$$EP^{\min}_{p,j} \leq V^{p}_{p,j,t} \leq EP^{\max}_{p,j} \quad \forall p; j \in PT_p; t; \tag{105}$$

The product pumped must satisfy the demand.

$$QR_p = \sum_{j \in PT_p} (Vi^{p}_{p,j} - V^{p}_{p,j,t}) \quad \forall p; t = T; \tag{106}$$

$$QP_p = \sum_{i \in RT_p} (Vi^{r}_{p,i} - V^{r}_{p,i,t}) \quad \forall p; t = T; \tag{107}$$

Flow rate determination:

$$PR_t = \tau^{-1} \cdot \sum_{p} \sum_{i \in RT_p} (V^{r}_{p,i,t} - V^{r}_{p,i,t+1}) \quad 1 \leq t < T; \tag{108}$$

$$PP_t = \tau^{-1} \cdot \sum_{p} \sum_{j \in PT_p} (V^{p}_{p,j,t} - V^{p}_{p,j,t+1}) \quad 1 \leq t < T; \tag{109}$$

Expressions (110) to (112) help to model the gap time.

$$TGap^{\min} \leq TGAP \leq TGap^{\max} \tag{110}$$

$$ONGP_t - \left(1 - \sum_{p} \sum_{j \in PT_p} ON^{p}_{p,j,t}\right) + \sum_{p} \sum_{i \in RT_p} ON^{r}_{p,i,t} = 0 \quad INIG \leq t < FIMG; P\_R = 1; \tag{111}$$

$$ONGR_t - \left(1 - \sum_{p} \sum_{i \in RT_p} ON^{r}_{p,i,t}\right) + \sum_{p} \sum_{j \in PT_p} ON^{p}_{p,j,t} = 0 \quad INIG \leq t < FIMG; R\_P = 1; \tag{112}$$

During the gap time, the pipeline flow rate must be respected.

$$PP^{\min}_p \cdot ONGP_t \leq PPG_t \leq PP^{\max}_p \cdot ONGP_t \quad p = OP_s; s = npr; INIG \leq t < FIMG; P\_R = 1; \tag{113}$$

$$PR^{\min}_p \cdot ONGR_t \leq PRG_t \leq PR^{\max}_p \cdot ONGR_t \quad p = OR_s; s = npp; INIG \leq t < FIMG; R\_P = 1; \tag{114}$$

Gap time determination:

$$TGAP = \tau \cdot \sum_{t=INIG}^{FIMG} \left(ONGP_t + ONGR_t\right) \tag{115}$$

Either flow/reflow switch or reflow/flow switch occurs when the pipeline is full.

$$VD = \tau \cdot \sum_{t=INIG}^{FIMG} \left(PPG_t \cdot P\_R + PRG_t \cdot R\_P \right) \quad (116)$$

Electric cost during the gap time:

$$CEGAP = \sum_{t=INIG}^{FIMG} \left[Ce_t \cdot \left(PPG_t \cdot P\_R + PRG_t \cdot R\_P \right) \right] \quad (117)$$

Expressions (118) to (123) help to model the end time.

$$TT^{\min} \le TT \le TT^{\max} \quad (118)$$

$$\left(\sum_{j \in PT_{pa}} FB^{p}_{pa,j,t} - \sum_{j \in PT_{pa}} ON^{p}_{pa,j,t} - ITP_t \right) \cdot R\_P + \left(\sum_{i \in RT_p} FB^{r}_{p,i,t} - \sum_{i \in RT_p} ON^{r}_{p,i,t} - ITR_t \right) \cdot P\_R \ge 0 \quad (119)$$
$$pa = OP_{sa}, p = OR_s; sa = npr, s = npp; INIT \le t < FIMT;$$

$$\left(\sum_{t=INIT}^{FIMT-1} ITP_t \right) \cdot R\_P + \left(\sum_{t=INIT}^{FIMT-1} ITR_t \right) \cdot P\_R = 1 \quad (120)$$

$$\left[\left(\sum_{t=INIT+1}^{FIMT} FTP_t \right) - 1 \right] \cdot R\_P + \left[\left(\sum_{t=INIT+1}^{FIMT} FTR_t \right) - 1 \right] \cdot P\_R = 0 \quad (121)$$

$$\left(ITP_t - ONTP_t \right) \cdot R\_P + \left(ITR_t - ONTR_t \right) \cdot P\_R = 0 \quad t = INIT; \quad (122)$$

$$\left(ITR_t - FTR_t + ONTR_{t-1} - ONTR_t \right) \cdot P\_R + \left(ITP_t - FTP_t + ONTP_{t-1} - ONTP_t \right) \cdot R\_P = 0 \quad INIT < t < FIMT; \quad (123)$$

End time duration:

$$TT = \tau \cdot \sum_{t=INIT}^{FIMT-1} \left(ONTP_t \cdot R\_P + ONTR_t \cdot P\_R \right) \quad (124)$$

During the end time, the pipeline flow rate must be respected.

$$PP^{\min}_{p} \cdot ONTP_t \le PPT_t \le PP^{\max}_{p} \cdot ONTP_t$$
$$p = OP_s; s = npr; INIT \le t < FIMT; R\_P = 1; \quad (125)$$

$$PR^{\min}_{p} \cdot ONTR_t \le PRT_t \le PR^{\max}_{p} \cdot ONTR_t$$
$$p = OR_s; s = npp; INIT \le t < FIMT; P\_R = 1; \quad (126)$$

After pumping the entire demand, the pipe must be filled up.

$$VD = \tau \cdot \sum_{t=INIT}^{FIMT-1} \left(PPT_t \cdot R\_P + PRT_t \cdot P\_R \right) \quad (127)$$

Electric cost during the end time:

$$CET = \sum_{t=INIT}^{FIMT-1} \left[Ce_t \cdot \left(PPT_t \cdot R\_P + PRT_t \cdot P\_R \right) \right] \quad (128)$$

In case the system finish pumping the demand before *T*, the pipeline must be maintained pressurised. Expressions (129) to (132) help to model this constraint.

$$TIBT = \sum_{t=INIT}^{FIMT-1} \left(ITP_t \cdot t \cdot R\_P + ITR_t \cdot t \cdot P\_R \right) \quad (129)$$

$$TFB^{p}_{p,j,t} - TIBT \le 0$$
$$p = OP_s; s = npr; j \in PT_p; INIT \le t < FIMT; R\_P = 1; \quad (130)$$

$$TFB^{r}_{p,i,t} - TIBT \le 0$$
$$p = OR_s; s = npp; i \in RT_p; INIT \le t < FIMT; P\_R = 1; \quad (131)$$

$$TFBT = \sum_{t=INIT+1}^{FIMT} \left(FTP_t \cdot t \cdot R\_P + FTR_t \cdot t \cdot P\_R \right) \quad (132)$$

Time period that the pipe remains pressurised:

$$TS = T - TFBT \quad (133)$$

The computational auxiliary routine time bound determines temporal constraints that must be respect by the main model. Moreover, setting up binary variable values decisively aids the search process (Wolsey, 1998).

$$IB^{r}_{p,i,t} = 0 \quad \forall p; i \in RT_p; (1 < t < TIBI^{r}_{p,i}) \cup (TIBS^{r}_{p,i} < t < T); \quad (134)$$

$$IB^{p}_{p,j,t} = 0 \quad \forall p; j \in PT_p; (1 < t < TIBI^{p}_{p,j}) \cup (TIBS^{p}_{p,j} < t < T); \quad (135)$$

$$FB^{r}_{p,i,t} = 0 \quad \forall p; i \in RT_p; (1 < t < TFBI^{i}_{p,i}) \cup (TFBS^{r}_{p,i} < t \le T); \quad (136)$$

$$FB^{p}_{p,j,t} = 0 \quad \forall p; j \in PT_p; (1 < t < TFBI^{p}_{p,j}) \cup (TFBS^{p}_{p,j} < t \le T); \quad (137)$$

3. RESULTS

This section considers an example involving the pumping of four products from the harbour to the refinery followed by another four pumped from the refinery to the harbour. Each product has two tanks enabled to sending operations. For simplicity, units are standardised and omitted. The normalisation is based on the pipeline volume. The entire pipe has 7,314 m^3. It is admitted a *NF* (normalisation factor) that equally divides the pipe volume. The product demand is expressed based upon *NF*. As an example, *NF*=4 determines batches of 1,828.5 m^3 (7,314÷4). A normalised demand of two units represents a total demanded volume of 3,657 m^3 (1,828.5x2). The system pumps, at most, one normalised volume per time unit. A normalised flow rate of one at a time *t* indicates that a volume of 1,828.5 m^3 is pumped between times *t* and *t*+1. The time length selection of each discretised time span involves a trade-off between accurate operation and computational effort. The problem data was rounded, so that the time quantum could be increased and, thus, the number of decision variables decreased. Simulation covers since the minimum normalised time horizon (T_{min}=20) up to twenty-five normalised time units (*T*=25). The pumping process starts from the harbour to the refinery (P_R=1); *NF*=4; CP_{pump}, CR_{pump}, C_{plug}, and *CS* were considered unitary.

Table 1 indicates the plug necessity between products. As an example, the sequence product P1 followed by product P2 demands the use of a plug (P1↔plug↔P2), P1 followed by P3 do not demand the use of a plug (P1↔P3).

Table 1. Product versus Plug Necessity

Product	Plug Necessity			
	P1	P2	P3	P4
P1	no	yes	no	yes
P2	yes	no	no	no
P3	no	no	no	yes
P4	yes	no	yes	no

Table 2 is a system information sketch for the problem main features. It presents a priori information about tanks (storage tank label) that can be used in sending operations: the storage capacity, and the up-to-data volume (initial amount). In the simulation scenario, these tanks are not enabled to receive products. The demanded amount represents the standardised product necessity. As an example, the harbour is in need of two normalised units (3,657 m^3) of P3. This batch has to be supplied by the refinery tank farm of P3 (P3_TR1, P3_TR2). The minimum volume to be left per tank (heel) is equal to one normalised unit for all tanks specified in Table 2. It was considered that both the harbour tank farm and the refinery tank farm were able to receive the entire demanded amount. Table 3 details the product flow rate range.

Table 2. System Information – Main Features

	Product	Storage Tank Label	Storage Capacity	Initial Amount	Demanded Amount
HARBOUR	P1	P1_TP1	12	6	1
		P1_TP2	12	6	
	P2	P2_TP1	12	2	1
		P2_TP2	12	2	
	P3	P3_TP1	12	6	2
		P3_TP2	12	6	
	P4	P4_TP1	12	6	1
		P4_TP2	12	6	
REFINERY	P1	P1_TR1	15	6	1
		P1_TR2	15	6	
	P2	P2_TR1	15	6	2
		P2_TR2	15	6	
	P3	P3_TR1	15	3	1
		P3_TR2	15	2	
	P4	P4_TR1	15	6	1
		P4_TR2	15	6	

Table 3. Product Flow Rate Range

Parameter	Product			
	P1	P2	P3	P4
PR_p^{min}	0.5	0.5	0.5	0.5
PR_p^{max}	1	1	1	1
PP_p^{min}	0.5	0.5	0.5	0.25
PP_p^{max}	1	1	1	0.5

Table 4 shows the electric cost at each time unit. Pumping start time is at 6 a.m. (t=1). The cost variation is due to on-peak demand hours. A uniform time discretization of six hours was adopted.

Table 4. Electric Cost Variation

t	1	2	3	4	5	6	7	8	9	10	11	12	13
Ce_t	1	1	5	1	1	1	5	1	1	1	5	1	1
t	14	15	16	17	18	19	20	21	22	23	24	25	26
Ce_t	1	5	1	1	1	5	1	1	1	5	1	1	1

The modelling and optimisation tool *Extended LINGO/PC Release 6.0* (LINDO, 1999) was used to implement and solve the model. LINGO is a commercial tool, which allows formulating linear and non-linear large problems, solving them, and analysing the solution. It has four internal solvers: a direct solver, a linear solver, a non-linear solver, and a branch and bound manager. The Lingo's solvers are all part of the same program, which is directly linked to its modelling language. This allows the data exchange directly through memory, rather than through intermediate files. Direct links also minimise compatibility problems between the modelling language and the solver components.

Table 5 provides information about the integrated architecture simulation. The computational time is in the worst case of ten runs on a platform Pentium III, 933 MHz, 256 MB RAM. It was not applied any optimality margin (Shah *et al.*, 1993), and the search tree was entirely executed. The integrated architecture blocks tank bound, plug bound, and time bound required a computational time lower than one second for all simulation instances. *T* indicates the scheduling horizon, *NV* stands for the total number of variables, *NBV* stands for the total number of binary variables, *NC* stands for the total number of constraints, *Time* indicates the simulation time (seconds), and *Cost* ($) indicates the normalised objective function value - equation (65).

Table 5. Integrated Architecture Simulation: Main Model Data

T	NV	NBV	NC	Time (s)	Cost ($)
20	1,325	82	4,297	17	64
21	1,405	92	4,526	24	62
22	1,483	102	4,749	43	61
23	1,561	112	4,972	134	60
24	1,639	122	5,197	146	61
25	1,717	132	5,422	340	62

Considering a time horizon equal to twenty-three normalised units (T=23), Figure 4 is a Gantt chart about sending operations established by the integrated architecture.

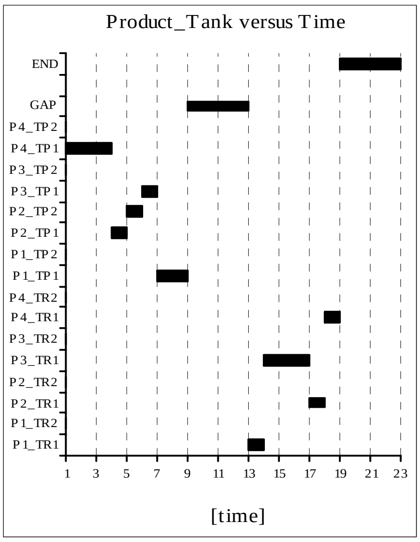

Fig. 4. Sending operations during time horizon.

Figure 4 demonstrates that the system established the pumping sequence *P4_TP1*, *P2_TP1*, *P2_TP2*, *P3_TP1*, *P1_TP1*, and *GAP* in reflow procedure. The sequence *P1_TR1*, *P3_TR1*, *P2_TR1*, *P4_TR1* and *END* was established in flow procedure. In accordance with Table 1, these pumping sequences minimise the use of plugs. Considering a time horizon equal to twenty-three normalised units (T=23), Figure 5 demonstrates the emptying of tanks determined by the integrated architecture. Figure 5 and Table 2 demonstrate that the predicted operation schedule minimises the tank changeovers.

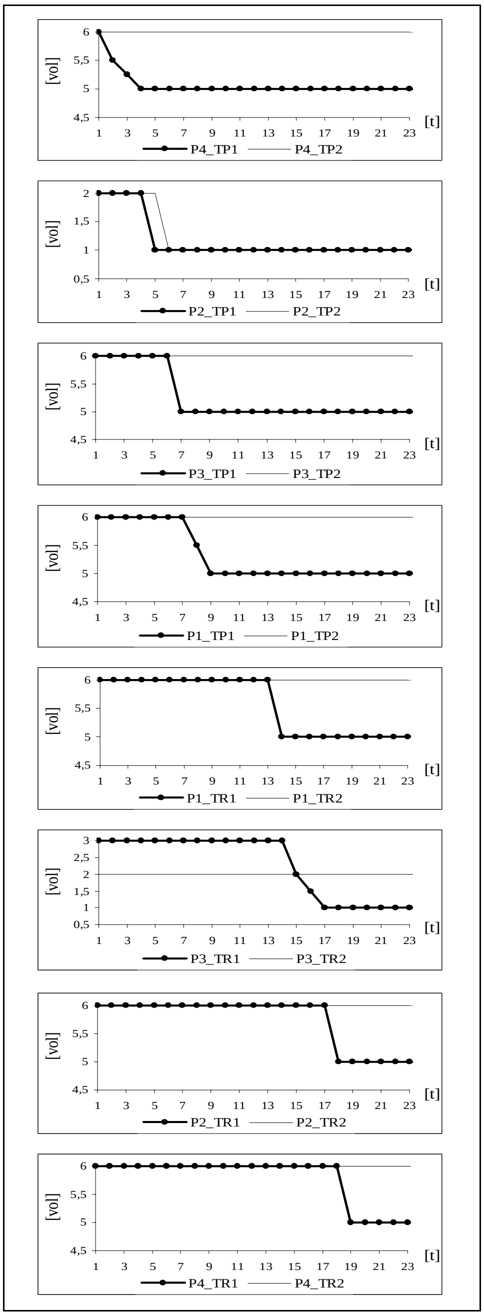

Fig. 5. Tank volume during time horizon.

Based on electric cost variations on the available time horizon, the optimisation model determines the ideal pipe flow rate. Considering a time horizon equal to twenty-three units (T=23), Figure 6 shows the normalised flow rate as a time function.

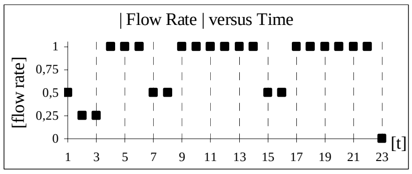

Fig. 6. Normalised flow rate during time horizon.

In order to pump the entire demand pumping chart, it is required a minimum time horizon (T_{min}). In such a horizon, every product is pumped at its maximum flow rate. However, in case $T>T_{min}$ the integrated architecture determines the optimal flow rate policy, according to the available time horizon (T). Figure 7 shows the normalised cost - equation (65) - as a time horizon function. For each time horizon value presented in Figure 7, the integrated architecture is run, and a specific cost is attained (see Table 5).

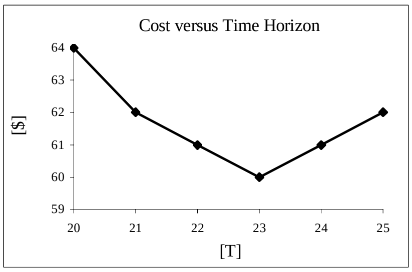

Fig. 7. Normalised cost versus time horizon.

Figure 7 demonstrates the existence of a specific time horizon that yields the minimum operating cost (T=23). The cost versus time horizon function clearly demonstrates that a correct pipeline timing policy provides significant cost saving.

4. CONCLUSIONS

It was presented a mathematical programming approach to the economically important problem of oil distribution through pipelines. It was developed a computer-optimisation system to aid the operational decision-making process. It was considered a study upon a tank farm and a pipeline connecting a refinery to a harbour. The scheduling of activities took into account product availability, tankage constraints, product sequencing constraints, and also satisfied a wide variety of operational requirements. The task was to specify the pipeline operation during a limited time horizon, providing low cost operational procedures. The scheduling approach based on mixed integer linear programming with uniform time discretization was applied on formulating the problem. The computational expense was concerned and an integrated architecture was proposed. This architecture separately solves the three scheduling fundamental components: the assignment of resources, the sequencing of activities, and the timing utilisation of resources by these activities. The large-scale mixed integer linear problem was implemented and solved by using the commercial tool *Extended LINGO/PC Release 6.0*. Currently pipeline operation is based on experience, and no computer algorithm is used; plug product usage and energy consumption are not rigorously taken on account. Simulation examples demonstrated the economic potential involved in the problem of sequencing commodities in a multi-product pipeline.

5 NOTATION

5.1 General parameters.

C_{plug}	Average cost to pump a plug product ($);
C_{tank}	Average cost of a tank changeover ($);
Ce_t	Average electric cost per flow rate unit at a time t ($·h/m^3);
CP_{pump}	Average cost to pump a product. Flow direction: harbour to refinery ($/h);
CR_{pump}	Average cost to pump a product. Flow direction: refinery to harbour ($/h);
CS	Average cost to maintain the pipe pressurised ($/h);
$EP_{p,j}^{\min}$ / $EP_{p,j}^{\max}$	Minimum/maximum storage capacity of p in a tank j - harbour tank farm - (m^3);
$ER_{p,i}^{\min}$ / $ER_{p,i}^{\max}$	Minimum/maximum storage capacity of p in a tank i - refinery tank farm - (m^3);
i,k,m	Refinery tanks;
$I_{p,pa}$	1 if pumping the product p followed by pa requires a plug between then, 0 otherwise. It is a dimensionless parameter;
K	Auxiliary constant (m^3);
j,l,n	Harbour tanks;
npp	Number of products demanded by the harbour;
npr	Number of products demanded by the refinery;
p, pa	Products;
PND	Product that fills the pipeline at the initial time (t=1);
P_R	1 if reflow procedure is followed by flow procedure, 0 otherwise;
$PP_p^{\min}$ / $PP_p^{\max}$	Minimum/maximum flow rate of p. Flow direction: harbour to refinery (m^3/h);
$PR_p^{\min}$ / $PR_p^{\max}$	Minimum/maximum flow rate of p. Flow direction: refinery to harbour (m^3/h);
PT_p	Harbour tanks that storage a product p;
QP_p	Volume of p demanded by the harbour (m^3);
$QP_p^{\min}$ / $QP_p^{\max}$	Minimum/maximum volume of p demanded by the harbour (m^3);
QR_p	Volume of p demanded by the refinery (m^3);
$QR_p^{\min}$ / $QR_p^{\max}$	Minimum/maximum volume of p demanded by the refinery (m^3);
R_P	1 if flow procedure is followed by reflow procedure, 0 otherwise;
RT_p	Refinery tanks that storage a product p;
t	Discrete time t=1..T (time horizon). Unit: h;
T	Time horizon. Unit: h;
$T_{\min}$	Minimum time horizon to complete the entire pumping procedure (h);
$Vi_{p,j}^p$	Product p storage volume in tank j at the initial time (t=1). Harbour tank farm (m^3);
$Vi_{p,i}^r$	Product p storage volume in tank i at the initial time (t=1). Refinery tank farm (m^3);
VD	Pipeline volume (7,314 m^3);
τ	Discrete time duration (h).

5.2 Tank Bound Variables.

LP_p	Binary variable that indicates whether p can be the last sequenced product of reflow procedure (1) or not (0);
LR_p	Binary variable that indicates whether p can be the last sequenced product of flow procedure (1) or not (0);
TB_P	Indicates the number of tank changeovers occurred in the harbour tank farm to supply sending operations. It is a dimensionless variable;
TB_R	Indicates the number of tank changeovers occurred in the refinery tank farm to supply sending operations. It is a dimensionless variable;
$TBT_{p,j}^p$	1 if occurs an operational transition on tank j of p, 0 otherwise. Harbour tank farm. It is a dimensionless binary variable;
$TBT_{p,i}^r$	1 if occurs an operational transition on tank i of p, 0 otherwise. Refinery tank farm. It is a dimensionless binary variable;
$Vf_{p,j}^p$	Product p storage volume in tank j at the final scheduling time (t=T). Harbour tank farm (m^3);
$Vf_{p,i}^r$	Product p storage volume in tank i at the final scheduling time (t=T). Refinery tank farm (m^3).

5.3 Plug Bound Parameters.

LP_p	Binary parameter that indicates whether p can be the last sequenced product of reflow procedure (1) or not (0). The parameter is determined by the tank bound;
LR_p	Binary parameter that indicates whether p can be the last sequenced product of flow procedure (1) or not (0). The parameter is determined by the tank bound.

5.4 Plug Bound Variables.

$AUX\_P_{p,s}$ Auxiliary binary variable used to model OP_s;

$AUX\_R_{p,s}$ Auxiliary binary variable used to model OR_s;

OP_s Integer variable. Indicates the pumping sequence (value ranging from 1 to *npr*) of *p* in the reflow procedure;

OR_s Integer variable. Indicates the pumping sequence (value ranging from 1 to *npp*) of *p* in the flow procedure;

SB_P Number of plugs used in the reflow procedure;

SB_PND Number of plugs used between the product that fills the pipe at the initial time and the first sequenced product;

SB_R Number of plugs used in the flow procedure;

SB_SW Number of plugs used in the switch transition (either flow/reflow or reflow/flow);

$TR\_P_{p,pa,s}$ Binary variable that indicates the transition between *p* and *pa* at the reflow pumping sequence *s*. In case, $TR\_P_{1,2,3}$, the third reflow transition occurs between products *p*=1 and *pa*=2;

$TR\_R_{p,pa,s}$ Binary variable that indicates the transition between *p* and *pa* at the flow pumping sequence *s*;

$TR\_PR_{pa,pb,pc,pd}$ Binary variable that indicates the switch transition between products *pb* and *pc* (reflow procedure);

$TR\_RP_{pa,pb,pc,pd}$ Binary variable that indicates the switch transition between products *pb* and *pc* (flow procedure).

5.5 Time Bound Determined Parameters.

$FIMG$ Upper time limit of *gap* procedure completion (h);

$FIMT$ Upper time limit of *end* procedure completion (h);

$INIG$ Lower time limit of *gap* procedure start (h);

$INIT$ Lower time limit of *end* procedure start (h);

$TFBI^{p}_{p,j}$ Upper time limit to pumping completion of product *p* stored in tank *j*, in case T=T_{min}. Reflow operation (h);

$TFBS^{p}_{p,j}$ Upper time limit to pumping completion of product *p* stored in tank *j*. Reflow operation (h);

$TFBI^{r}_{p,i}$ Upper time limit to pumping completion of product *p* stored in tank *i*, in case T=T_{min}. Flow operation (h);

$TFBS^{r}_{p,i}$ Upper time limit to pumping completion of product *p* stored in tank *i*. Flow operation (h);

$TGap^{\min}$ / $TGap^{\max}$ *Gap* procedure lower/upper time interval;

$TIBI^{p}_{p,j}$ Lower time limit to pumping start of product *p* stored in tank *j*, in case T=T_{min}. Reflow operation (h);

$TIBI^{r}_{p,i}$ Lower time limit to pumping start of product *p* stored in tank *i*, in case T=T_{min}. Flow operation (h);

$TIBS^{p}_{p,j}$ Lower time limit to pumping start of product *p* stored in tank *j*. Reflow operation (h);

$TIBS^{r}_{p,i}$ Lower time limit to pumping start of product *p* stored in tank *i*. Flow operation (h);

TL Upper time limit of *end* procedure start (h);

$TT^{\min}$ / $TT^{\max}$ *End* procedure lower/upper time interval.

5.6 Main Model Variables.

$CEGAP$ Total electric cost to pump a product during the *gap* procedure ($);

CET Total electric cost to pump a product during the *end* procedure ($);

$FB^{p}_{p,j,t}$ 1 if the end pumping of *p* in tank *j* occurs at a time *t*, 0 otherwise. Reflow procedure. It is a dimensionless binary variable;

$FB^{r}_{p,i,t}$ 1 if the end pumping of *p* in tank *i* occurs at a time *t*, 0 otherwise. Flow procedure. It is a dimensionless binary variable;

FTP_t 1 if the *end* procedure was completed at a time *t*, 0 otherwise. It is a dimensionless binary variable used when *P_R*=0;

FTR_t 1 if the *end* procedure was completed at a time *t*, 0 otherwise. It is a dimensionless binary variable used when *R_P*=0;

$IB^{p}_{p,j,t}$ 1 if the start pumping of *p* in tank *j* occurs at a time *t*, 0 otherwise. Reflow procedure. It is a dimensionless binary variable;

$IB^{r}_{p,i,t}$ 1 if the start pumping of *p* in tank *i* occurs at a time *t*, 0 otherwise. Flow procedure. It is a dimensionless binary variable;

ITP_t 1 if the *end* procedure was started at a time *t*, 0 otherwise. It is a dimensionless binary variable used when *P_R*=0;

ITR_t 1 if the *end* procedure was started at a time *t*, 0 otherwise. It is a dimensionless binary variable used when *R_P*=0;

$ON^{p}_{p,j,t}$ 1 during $TIB^{p}_{p,j} \le t < TFB^{p}_{p,j}$, 0 otherwise. Reflow procedure. It is a dimensionless variable;

$ON^{r}_{p,i,t}$ 1 during $TIB^{r}_{p,i} \le t < TFB^{r}_{p,i}$, 0 otherwise. Flow procedure. It is a dimensionless variable;

$ONGP_t$ 1 during the *gap* procedure. It is a dimensionless binary variable used when *P_R*=1;

$ONGR_t$ 1 during the *gap* procedure. It is a dimensionless binary variable used when *R_P*=1;

$ONTP_t$ 1 during the *end* procedure. It is a dimensionless binary variable used when *P_R*=0;

$ONTR_t$ 1 during the *end* procedure. It is a dimensionless binary variable used when *R_P*=0;

PP_t Flow rate at a time *t*. Reflow procedure (m^3/h);

PPG_t Flow rate at a time *t* (during the *gap* procedure). It is used when *P_R*=1 (m^3/h);

PPT_t Flow rate at a time *t* (during the *end* procedure). It is used when *P_R*=0 (m^3/h);

PR_t Flow rate at a time *t*. Flow procedure (m^3/h);

PRG_t Flow rate at a time *t* (during the *gap* procedure). It is used when *R_P*=1 (m^3/h);

PRT_t Flow rate at a time *t* (during the end procedure). It is used when *R_P*=0 (m^3/h);

$SP_{p,pa,t}$ 1 if *p* followed by *pa* at a time *t* requires a plug between then, 0 otherwise. Reflow procedure. It is a dimensionless variable;

$SR_{p,pa,t}$ 1 if *p* followed by *pa* at a time *t* requires a plug between then, 0 otherwise. Flow procedure. It is a dimensionless variable;

$TFB^p_{p,j}$ End pumping time of *p* in tank *j*. Reflow procedure (h);

$TFB^r_{p,i}$ End pumping time of *p* in tank *i*. Flow procedure (h);

$TFBT$ Discretized time of *end* procedure completion (h);

$TGAP$ *Gap* procedure duration (h);

$TIB^p_{p,j}$ Start pumping time of *p* in tank *j*. Reflow procedure (h);

$TIB^r_{p,i}$ Start pumping time of *p* in tank *i*. Flow procedure (h);

$TIBT$ Discretized time of *end* procedure start (h);

TK Total time to complete reflow, flow and gap procedures (h);

$TP_{p,j,l,t}$ 1 if the changeover between tanks *j* and *l* of *p* occurs at a time *t*, 0 otherwise. Reflow procedure. It is a dimensionless variable;

$TR_{p,i,k,t}$ 1 if the changeover between tanks *i* and *k* of *p* occurs at a time *t*. Flow procedure. It is a dimensionless variable;

$TP_{p,pa,j,n,t}$ 1 if the changeover between tank *j* of *p* and tank *n* of *pa* occurs at a time *t*. Reflow procedure. It is a dimensionless variable;

$TR_{p,pa,i,m,t}$ 1 if the changeover between tank *i* of *p* and tank *m* of *pa* occurs at a time *t*. Flow procedure. It is a dimensionless variable;

TS Time period that the pipe remains pressurised (h);

TT *End* procedure duration (h);

$V^p_{p,j,t}$ Product *p* storage volume in tank *j* at a time *t* ($2 \le t \le T$). Harbour tank farm (m^3);

$V^r_{p,i,t}$ Product *p* storage volume in tank *i* at a time *t* ($2 \le t \le T$). Refinery tank farm (m^3).

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