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DESIGN OF DIRECTIONAL RESIDUALS FOR OPTIMAL TESTABILITY

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Abstract. Residuals in fault detection and diagnosis are usually designed with directional or structured properties to facilitate fault isolation. With directional residuals, best isolation is achieved if the residual directions are orthogonal. In the presence of noise, the residuals are subjected to statistical testing. Testing conditions are ideal if the Fisher information matrix of the residuals is diagonal. In this paper, we introduce a design technique which, subject to certain restrictions, provides orthogonal directions and diagonal information matrix simultaneously.

Keywords. Fault detection and diagnosis; Fault isolation; Directional residuals; Statistical testing; Fisher information matrix

1. INTRODUCTION

A broad class of model-based fault detection and diagnosis methods is built on the concept of analytical redundancy (Willsky, 1976). Measurements of plant outputs are compared to predictions based on a plant model and measured inputs. Discrepancies are expressed as residuals which are ideally zero if no faults are present. These residuals are enhanced by algebraic manipulations to facilitate the isolation of faults. One of the enhancement techniques involves residuals which always point in a specific direction in response to a particular fault. Design techniques for directional residuals have been proposed by Massoumnia (1986) and White and Speyer (1987) in the framework of diagnostic observers and by Gertler nad Monajemy (1995) using dynamic consistency relations; see also (Gertler, 1998, 2000).

If noise is present in the plant, the residuals are random and need to be subjected to statistical testing (Basseville and Nikiforov, 1993). The testing of residuals designed for arbitrary response directions has been discussed in (Gertler, 1998). Basseville (1997) has given a thorough analysis of the interaction between directional design and testability. As she has pointed out, the interaction is represented by the Fisher information matrix of the residual vector, which characterizes the signal-to-noise ratio in the residual. To create ideal conditions for isolation and testing, the residuals should be so designed that the fault response directions are orthogonal and the information matrix is diagonal (or close to diagonal).

Our objective in this paper is to propose a design technique that satisfies the above requirements. Our interest is in the diagnosis of additive faults in linear time-invariant discrete dynamic systems. Such systems are characterized by discrete transfer functions or equivalent state-space models. However, as shown by Basseville (1997) for the off-line approach to the diagnosis of dynamic systems, directional isolation and testing may be posed as a static problem. Therefore, we will investigate linear static systems in this paper. It will then be shown in a later paper that the design for dynamic systems, even under on-line testing conditions, can be decomposed so that the static results apply.

2. PROBLEM STATEMENT

We will use the usual assumptions that the system model is known completely and that the noise is (independent) Gaussian with zero mean and known covariance.

Consider a linear static system

$$\mathbf{y} = \mathbf{K}\,\mathbf{u} + \mathbf{L}\,\mathbf{f} + \mathbf{M}\,\mathbf{v} \tag{1}$$

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where $\mathbf{y}$ is a vector of m measured outputs, $\mathbf{u}$ is a vector of k known (measured or manipulated) inputs, $\mathbf{f}$ is a vector of p unknown deterministic faults and $\mathbf{v}$ is a vector of q noises, normally distributed with zero mean and known covariance $\mathbf{S}$. Assume at this point that $p \leq m$ and $q=m$. The first assumption is necessary for orthogonal response directions; the second will be relaxed later in the paper.

A primary residual vector can be computed as

$$\mathbf{e} = \mathbf{y} - \mathbf{K}\,\mathbf{u} = \mathbf{L}\,\mathbf{f} + \mathbf{M}\,\mathbf{v} \tag{2}$$

then a transformed residual obtained as

$$\mathbf{r} = \mathbf{W}\,\mathbf{e} = \mathbf{W}\,\mathbf{L}\,\mathbf{f} + \mathbf{W}\,\mathbf{M}\,\mathbf{v} \tag{3}$$

The primary residual is naturally directional, the columns of the $\mathbf{L}$ matrix being the response directions, but these columns are usually not orthogonal. The transformed residual is also directional, with the columns of $\mathbf{W}\,\mathbf{L}$ as the response directions; with the proper choice of $\mathbf{W}$, any desired directions can be obtained.

The covariance of the primary residual is

$$\mathbf{S}_e = E\{\mathbf{M}\,\mathbf{v}\,\mathbf{v}'\,\mathbf{M}'\} = \mathbf{M}\,\mathbf{S}\,\mathbf{M}' \tag{4}$$

while the covariance of the transformed residual is

$$\mathbf{S}_r = \mathbf{W}\,\mathbf{S}_e\,\mathbf{W}' = \mathbf{W}\,\mathbf{M}\,\mathbf{S}\,\mathbf{M}'\mathbf{W}' \tag{5}$$

Consider the spectral decomposition of $\mathbf{S}_r$:

$$\mathbf{S}_r = \mathbf{Q}_r\,\mathbf{\Sigma}_r\,\mathbf{Q}_r' \tag{6}$$

where the columns of the eigenvector matrix $\mathbf{Q}_r$ are the principal directions of the distribution and the elements of the diagonal eigenvalue matrix $\mathbf{\Sigma}_r$ are the variances in the principal directions. By the choice of $\mathbf{W}$, both the directions and the variances can be manipulated.

What we want to achieve is response directions which are orthogonal and which coincide with the principal directions of the covariance matrix. Simple residual transformation allows for the manipulation of the response directions or the covariance matrix separately, but there is interaction between the two; while "fixing" one, we "spoil" the other. Further insight may be gained by observing the behavior, under the transformation, of the Fisher information matrix (Basseville, 1997). The Fisher information for the primary residuals is

$$\mathbf{F}_e = \mathbf{L}'\,\mathbf{S}_e^{-1}\,\mathbf{L} \tag{7}$$

while for the transformed residuals it is

$$\mathbf{F}_r = \mathbf{L}'\mathbf{W}'\mathbf{S}_r^{-1}\,\mathbf{W}\,\mathbf{L} = \mathbf{L}'\mathbf{W}'(\mathbf{W}\mathbf{S}_e\mathbf{W}')^{-1}\,\mathbf{W}\,\mathbf{L} \tag{8}$$

For testing, a diagonal Fisher matrix would be ideal. If the response directions and the principal directions did coincide then $\mathbf{WL}=\mathbf{Q}_r\mathbf{\Delta}$, where $\mathbf{\Delta}$ is a diagonal matrix, and

$$\mathbf{F}_r = \mathbf{L}'\mathbf{W}'\,(\mathbf{Q}_r\,\mathbf{\Sigma}_r\,\mathbf{Q}_r')^{-1}\,\mathbf{W}\,\mathbf{L} = \mathbf{\Delta}\,\mathbf{\Sigma}_r\,\mathbf{\Delta} \tag{9}$$

which, of course, would be diagonal. However, trying to achieve this by a square (and invertible) transformation would fail, since, as seen from (8), any such $\mathbf{W}$ would cancel out, leaving $\mathbf{F}_r=\mathbf{F}_e$.

3. SIMULTANEOUS DIAGONALIZATION

Let us summarize the design task here. Given (2) and (3), with $q=m$, find a transformation $\mathbf{W}$ so that $\mathbf{WL}=\mathbf{Q}_r\mathbf{\Delta}$, where $\mathbf{Q}_r$ comes from (5) and (6) and $\mathbf{\Delta}$ is any diagonal matrix. Since an additional transformation may always diagonalize $\mathbf{S}_r$, an equivalent but more straightforward design requirement calls for finding $\mathbf{W}$ so that

$$\mathbf{S}_r = \mathbf{W}\,\mathbf{S}_e\,\mathbf{W}' = \mathbf{I} \tag{10}$$

and

$$\mathbf{WL}=\mathbf{D} \tag{11}$$

where $\mathbf{D}$ is diagonal.

3.1 Full-size diagonalization

First consider the case $p=m$. We will utilize the following theorem;

Theorem 1 (Stark and Woods, 2001). Let $\mathbf{P}$ and $\mathbf{R}$ be $m.m$ real symmetric matrices and $\mathbf{P}$ positive definite. Then there exists an $m.m$ matrix $\mathbf{V}$ which achieves

$$\mathbf{V}'\,\mathbf{P}\,\mathbf{V} = \mathbf{I} \tag{12a}$$

$$\mathbf{V}'\,\mathbf{R}\,\mathbf{V} = \mathbf{\Lambda} = \mathrm{Diag}\,(\lambda_1 \ldots \lambda_m) \tag{12b}$$

where $\mathbf{V}$ and $\mathbf{\Lambda}$ are the solutions of the generalized eigenvalue problem

$$\mathbf{R}\,\mathbf{v}_i = \lambda_1\,\mathbf{P}\,\mathbf{v}_i \qquad i = 1 \ldots m \tag{13}$$

That is, the columns of $\mathbf{V}$ are the eigenvectors and the diagonal elements of $\mathbf{\Lambda}$ the eigenvalues of the matrix $\mathbf{P}^{-1}\mathbf{R}$. The proof is given in (Stark and Woods, 2001).

We may apply Theorem 1 to a 'cubic' system, where $p=q=m$, in two ways:

a. $\mathbf{P}=\mathbf{S}_e$, $\mathbf{R}=\mathbf{LL}'$, $\mathbf{W}=\mathbf{V}'$. Then $\mathbf{S}_r=\mathbf{I}$, the distribution is spherical; the principal directions are undefined, any orthogonal set qualifies. But now $\mathbf{WLL}'\mathbf{W}'=\mathbf{\Lambda}$, which does not imply that the columns of $\mathbf{WL}$ are orthogonal.

b. $\mathbf{R}=\mathbf{S}_e$, $\mathbf{P}=\mathbf{LL'}$, $\mathbf{W}=\mathbf{V'}$. Then $\mathbf{S}_r=\mathbf{\Lambda}$, the distribution is elliptical, with the coordinate axes as principal directions. Now $\mathbf{WLL'W'}=\mathbf{I}$, implying $\mathbf{L'W'WL}=\mathbf{I}$, that is, the columns of $\mathbf{WL}$ are orthogonal, but in general they do not coincide with the principal directions.

3.2 Reduced-size diagonalization

We will solve the simultaneous diagonalization problem by transforming the m-dimensional residual $\mathbf{e}$ into a smaller dimensional space. This will also limit the number of faults for which optimal isolation is possible to $p<m$.

Perform first a full-size diagonalization with $\mathbf{P}=\mathbf{S}_e$ and $\mathbf{R}=\mathbf{LL'}$. This yields

$$\mathbf{V'}\,\mathbf{S}_e\,\mathbf{V}=\mathbf{I} \tag{14}$$

$$\mathbf{V'}\,\mathbf{L}\,\mathbf{L'}\,\mathbf{V}=\mathbf{\Lambda} \tag{15}$$

with a $\mathbf{V}$, $\mathbf{\Lambda}$ pair which are $m.m$. Now seek the transformation as

$$\mathbf{W}=\mathbf{Z'V'} \tag{16}$$

where $\mathbf{Z'}$ is $p.m$. Then condition (10) becomes

$$\mathbf{S}_r=\mathbf{Z'}\,\mathbf{V'}\,\mathbf{S}_e\,\mathbf{V}\,\mathbf{Z}=\mathbf{I} \tag{17}$$

from which, with (14), the new condition is

$$\mathbf{Z'}\,\mathbf{Z}=\mathbf{I} \tag{18}$$

Further, condition (11) becomes

$$\mathbf{T}\,\mathbf{Z}=\mathbf{D} \tag{19}$$

$$\text{where}\quad \mathbf{T}=\mathbf{L'V} \tag{20}$$

which is known at this point. The task is now to find $\mathbf{Z}$ so that (18) and (19) are satisfied.

Consider now (15) and observe that Rank$\mathbf{LL'}=p<m$, thus m-p eigenvalues in $\mathbf{\Lambda}$ are zero. By decomposing $\mathbf{T}$, (15) may be written as

$$\mathbf{TT'}=\begin{bmatrix}\mathbf{T}_1'\\ \mathbf{T}_2'\end{bmatrix}[\mathbf{T}_1\quad \mathbf{T}_2]=\begin{bmatrix}\mathbf{\Lambda}_1 & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix} \tag{21}$$

where $\mathbf{T}_1'\mathbf{T}_1=\mathbf{\Lambda}_1$ and $\mathbf{T}_2=\mathbf{0}$. Decompose $\mathbf{Z}$ as

$$\mathbf{Z}=\begin{array}{ll}\begin{bmatrix}\mathbf{Z}_1\\ \mathbf{Z}_2\end{bmatrix} & \begin{array}{l}p\\ m\text{-}p\end{array}\end{array} \tag{22}$$

With this, (19) becomes

$$\mathbf{T}_1\,\mathbf{Z}_1=\mathbf{D} \tag{23}$$

from which

$$\mathbf{Z}_1=\mathbf{T}_1^{-1}\,\mathbf{D} \tag{24}$$

Let us turn now to (18). With (23) and (24),

$$\mathbf{D}\,(\mathbf{T}_1\mathbf{T}_1')^{-1}\,\mathbf{D}+\mathbf{Z}_2'\,\mathbf{Z}_2=\mathbf{I} \tag{25}$$

Eq. (25) needs to be solved numerically. Because of its symmetry, it represents $p(p+1)/2$ scalar conditions. The unknowns are the $p(m\text{-}p)$ elements of $\mathbf{Z}_2$ and p elements of $\mathbf{D}$. This leads to the condition

$$p \le (2m+1)/3 \tag{26}$$

The algorithm consists of the following steps:

1. Full-size diagonalization (14)-(15)
2. Computing $\mathbf{T}$ from (20)
3. Solving (25)
4. Computing $\mathbf{Z}_1$ from (24)
5. Computing $\mathbf{W}$ from (16).

The critical step is the numerical solution of (25). There may be multiple solutions, some of them complex. Real solutions cannot always be found and, at this time, no explicit existence conditions are known either.

Note that if a solution can be found, it results in a perfect diagonal Fisher information matrix in the reduced residual space, namely

$$\mathbf{F}_r=\mathbf{D}^2 \tag{27}$$

3.3 Diagonalization without full-rank noise

Up to this point, the diagonalization algorithm has required that the rank of noise be equal to the number of outputs, i.e. $q=m$. According to Theorem (12a), $\mathbf{S}_e$ is also required to have full rank. These conditions are actually the same. We will now relax this restriction.

The idea is that, since the rank of the noise is less than the number of the outputs, we can divide the residual space into two subspaces: one containing the noises and a subset of faults, the other noise free. A similar approach was outlined in (Levy et al, 1996).

Divide fault vector $\mathbf{f}$ into two vectors:

$$\mathbf{f}=\begin{array}{ll}\begin{bmatrix}\mathbf{f}_1\\ \mathbf{f}_2\end{bmatrix} & \begin{array}{l}p_1\\ p_2\end{array}\end{array} \tag{28}$$

where $p_1+p_2=p$ and

$$p_2=m-q \tag{29}$$

Now let

$$\mathbf{r}_1=\mathbf{W}_1\,\mathbf{e} \tag{30}$$

be a $q<m$ dimensional residual, responding to the q noises and the p_1 faults in $\mathbf{f}_1$ but decoupled from $\mathbf{f}_2$. Let

$$\mathbf{r}_2 = \mathbf{W}_2\, \mathbf{e} \tag{31}$$

be a m-q dimensional residual that is decoupled from the noise (but responds to all faults). The task is to find the transform matrices $\mathbf{W}_1$ and $\mathbf{W}_2$.

With (2) and (28), (30) and (31) become

$$\mathbf{r}_1 = \mathbf{W}_1\, [\mathbf{L}_1\, \mathbf{f}_1 + \mathbf{L}_2\, \mathbf{f}_2 + \mathbf{M}\mathbf{v}] \tag{32}$$

$$\mathbf{r}_2 = \mathbf{W}_2\, [\mathbf{L}_1\, \mathbf{f}_1 + \mathbf{L}_2\, \mathbf{f}_2 + \mathbf{M}\mathbf{v}] \tag{33}$$

Decoupling $\mathbf{r}_1$ from $\mathbf{f}_2$ requires that $\mathbf{W}_1$ be orthogonal to $\mathbf{L}_2$. The remaining design freedom may be used to shape the noise-response in $\mathbf{r}_1$; a possible choice is $\mathbf{W}_1\, \mathbf{M}=\mathbf{I}$. Thus $\mathbf{W}_1$ is defined as

$$\mathbf{W}_1\, [\, \mathbf{L}_2 \quad \mathbf{M}\,] = [\, \mathbf{0} \quad \mathbf{I}\,] \tag{34}$$

Similarly, decoupling $\mathbf{r}_2$ from the noise requires that $\mathbf{W}_2$ be orthogonal to $\mathbf{M}$. The remaining design freedom may be used to make the response of $\mathbf{r}_2$ to the faults in $\mathbf{f}_2$ orthogonal to one another; a possible choice is $\mathbf{W}_2\, \mathbf{L}_2=\mathbf{I}$. Thus $\mathbf{W}_2$ is defined as

$$\mathbf{W}_2\, [\, \mathbf{L}_2 \quad \mathbf{M}\,] = [\, \mathbf{I} \quad \mathbf{0}\,] \tag{35}$$

Since $[\mathbf{L}_2 \quad \mathbf{M}]$ is an $m.m$ matrix which normally has full rank, $\mathbf{W}_1$ and $\mathbf{W}_2$ can be solved as

$$\mathbf{W}_1 = [\mathbf{0} \quad \mathbf{I}]\, [\mathbf{L}_2 \quad \mathbf{M}]^{-1} \tag{36}$$

$$\mathbf{W}_2 = [\mathbf{I} \quad \mathbf{0}]\, [\mathbf{L}_2 \quad \mathbf{M}]^{-1} \tag{37}$$

(A rank-defect in $[\mathbf{L}_2 \quad \mathbf{M}]$ would indicate co-linearity between or within $\mathbf{L}_2$ and $\mathbf{M}$, which could be taken into account in the design.)

With (36) and (37), the new residuals become

$$\mathbf{r}_1 = \mathbf{W}_1\, \mathbf{L}_1\, \mathbf{f}_1 + \mathbf{v} \tag{38}$$

$$\mathbf{r}_2 = \mathbf{W}_2\, \mathbf{L}_1\, \mathbf{f}_1 + \mathbf{f}_2 \tag{39}$$

Notice that $\mathbf{r}_1$ is now a q dimensional residual subspace with q noises and p_1 faults To this subsystem, the simultaneous diagonalization algorithm with full noise-rank, introduced earlier in this paper, may be applied, leading to a reduced dimension residual vector

$$\mathbf{r}_1^* = \mathbf{W}_1^*\, \mathbf{r}_1 \tag{40}$$

This algorithm places a limit on p_1 which, by the application of (26), is

$$p_1 \leq (2q+1)/3 \tag{41}$$

In this way, faults contained in $\mathbf{f}_1$ can be detected and isolated from $\mathbf{r}_1$*. The residual $\mathbf{r}_2$ is noise free, which allows for testing with high fault sensitivity. However, $\mathbf{r}_2$ is affected by all faults while providing orthogonal responses only to the elements of $\mathbf{f}_2$. Therefore, the two residuals are to be tested sequentially: if $\mathbf{r}_1$* indicates no fault (that is, no fault in $\mathbf{f}_1$) then one may proceed to $\mathbf{r}_2$ to detect and isolate faults in $\mathbf{f}_2$. Note that the assignment of any particular fault into $\mathbf{f}_1$ or $\mathbf{f}_2$ is arbitrary thus noise-free testing may be reserved for faults which do require high sensitivity.

4. SIMULATION EXAMPLES

4.1 Reduced-size diagonalization with full-rank noise

We will demonstrate the optimization algorithm with a linear static system with 4 outputs, 3 faults and 4 noises. For this type of system, $\mathbf{Z}$ has a closed form solution.

Assume that the fault direction matrix $\mathbf{L}$ and noise to residual transfer matrix $\mathbf{M}$ are as follows:

$$\mathbf{L} = \begin{matrix} 1 & 4 & 2 \\ 3 & 1 & 1 \\ -2 & 0 & 7 \\ -3 & -2 & 0 \end{matrix}$$

$$\mathbf{M} = \begin{matrix} 0.5 & 1.5 & 2.0 & 0.0 \\ 1.5 & 2.0 & 0.5 & 0.0 \\ 1.5 & 0.0 & -2.0 & 0.5 \\ -1.5 & -0.5 & -2.0 & 0.5 \end{matrix}$$

The noise covariance is $\mathbf{S}=\mathbf{I}$.

The covariance matrix of the primary residuals is:

$$\mathbf{S}_e = \begin{matrix} 6.5000 & 4.7500 & -3.2500 & -5.5000 \\ 4.7500 & 6.5000 & 1.2500 & -4.2500 \\ -3.2500 & 1.2500 & 6.5000 & 2.0000 \\ -5.5000 & -4.2500 & 2.0000 & 6.7500 \end{matrix}$$

Simultaneous diagonalization yields:

$$\mathbf{V} = \begin{matrix} 2.4655 & 0.0075 & 0.4022 & -0.1781 \\ -1.8180 & 0.1718 & -0.2225 & -0.4418 \\ 1.5112 & 0.0148 & -0.1919 & 0.1140 \\ 0.4202 & -0.2491 & 0.1750 & -0.5771 \end{matrix}$$

The solution to (25) is then obtained as

$$\mathbf{Z} = \begin{matrix} -0.0369 & 0.0346 & 0.0562 \\ 0.8701 & 0.4645 & 0.1648 \\ -0.3314 & 0.7990 & -0.5017 \\ -0.3630 & 0.3804 & 0.8473 \end{matrix}$$

$$\mathbf{D} = \begin{matrix} 1.4824 & -0.0000 & 0.0000 \\ 0.0000 & 1.4022 & -0.0000 \\ 0.0000 & 0.0000 & 1.1987 \end{matrix}$$

The optimization transform matrix **W** is:

$$\mathbf{W} = \mathbf{Z'}\ \mathbf{V'} = \begin{matrix} -0.1530 & 0.4506 & -0.0206 & -0.0808 \\ 0.3423 & -0.3289 & -0.0508 & -0.1809 \\ -0.2130 & -0.3365 & 0.2802 & -0.5943 \end{matrix}$$

The new fault response matrix is $\mathbf{W}\ \mathbf{L} = \mathbf{D}$ and the new residual covariance matrix is $\mathbf{S}_r = \mathbf{I}$. Finally, the Fisher Information matrix is $\mathbf{F}_r = \mathbf{D}^2$, which is diagonal.

Figure 1 shows the appearance of the primary residuals when, respectively, there is no fault, fault 1 happens and fault 2 happens. Only the subspace of the first two residuals is shown here. Note that the response to fault 1 is along the [1, 3]' direction and to fault 2 is along the [4, 1] direction, just as determined in the fault response matrix **L**. The distribution of the noise is in elliptical shape, which is not ideal for testing.

Figure 2 shows the appearance of the optimized residuals when, respectively, there is no fault, fault 1 happens and fault 2 happens. Only the subspace of the first two residuals is shown here. The size of each fault is the same as in Figure 1. Note that the responses to fault 1 and fault 2 are along the axis directions, just as determined in the fault response matrix **W L**. In addition, the distribution of the noise is in circular shape as well, which is ideal for testing.

4. 2 Diagonalization without full-rank noise

We will demonstrate the method with a 5-output, 4-fault and 3-noise system, with noise covariance **S**=**I**.

Assume that the fault direction matrix **L** and noise to residual transfer matrix **M** are as follows:

$$\mathbf{L} = \begin{matrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 1 & 4 \\ 4 & 1 & 5 & 2 \\ 5 & 3 & 2 & 1 \\ 3 & 4 & 1 & 5 \end{matrix} \qquad \mathbf{M} = \begin{matrix} 1 & 0 & 3 \\ 1 & 2 & 0 \\ 0 & 0 & 0 \\ 3 & 2 & 1 \\ -3 & -1 & 0 \end{matrix}$$

The transform matrices $\mathbf{W}_1$ and $\mathbf{W}_2$ are calculated as:

$$\mathbf{W}_1 = \begin{matrix} 12.800 & 25.400 & 7.800 & -38.400 & -26.000 \\ -15.400 & -30.200 & -9.400 & 46.200 & 31.000 \\ -8.600 & -17.800 & -5.600 & 26.800 & 18.000 \end{matrix}$$

$$\mathbf{W}_2 = \begin{matrix} -2.000 & -4.000 & -1.000 & 6.000 & 4.000 \\ 5.000 & 10.000 & 3.000 & -15.000 & -10.000 \end{matrix}$$

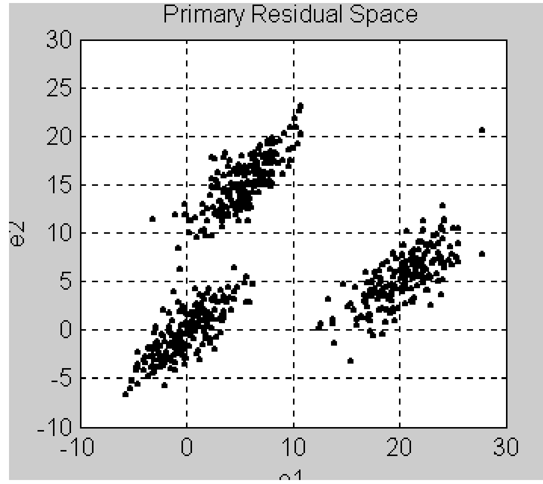

Figure 1. Primary Residual Subspace

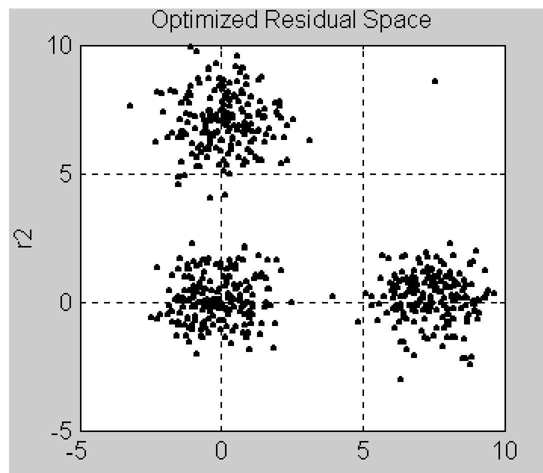

Figure 2. Optimized Residual Subspace

Now in the $\mathbf{r}_1$ subspace, the response to fault $\mathbf{f}_2$ is **0** while the noise to residual transform matrix becomes **I**. The response direction matrix for the faults in $\mathbf{f}_1$ becomes

$$\mathbf{W}_1\ \mathbf{L}_1 = \begin{matrix} -149.800 & -135.000 \\ 180.400 & 162.000 \\ 103.600 & 94.000 \end{matrix}$$

In the $\mathbf{r}_2$ subspace, the noise to residual transform matrix is **0** while the response direction matrix for the faults in $\mathbf{f}_2$ becomes **I** .

Apply the simultaneous diagonalization algorithm with full rank noise to the $\mathbf{r}_1$ space.

The solution of (25) is

$$\mathbf{Z} = \begin{matrix} 0.0013 & 0.0013 \\ -0.6455 & 0.7638 \\ 0.7638 & 0.6455 \end{matrix}$$

$$\mathbf{D} = \begin{matrix} 0.6058 & 0.0000 \\ -0.0000 & 0.6455 \end{matrix}$$

The optimization transform matrix $\mathbf{W}_1^*$ is

$$\mathbf{W}_1^* = \begin{matrix} 0.6799 & 0.6976 & -0.2259 \\ 0.4409 & -0.1428 & 0.8861 \end{matrix}$$

The new fault response matrix is $\mathbf{W}_1^* \mathbf{W}_1 \mathbf{L}_1 = \mathbf{D}$ and the new residual covariance matrix in the $\mathbf{r}_1^*$ subspace is $\mathbf{S}_{r*} = \mathbf{I}$.

5. CONSLUSION

We have studied the generation of directional residuals in the presence of noise. Methods to design residuals, which produce orthogonal responses to as many faults as the number of outputs, are well known. Our objective here has been to design residuals whose fault-response directions are not only orthogonal to one another but also co-linear with the principal directions of the noise-distribution.

The solution rests on a simultaneous diagonalization algorithm, utilizing a generalized eigenvalue technique. This algorithm requires full-rank noise and, in its original form, does not provide orthogonal response directions. We have proposed an extension to the algorithm in which the residuals are transformed into a smaller dimensional space, where orthogonal and co-linear directions can be found for a reduced number of faults. In another extension, we have removed the requirement of full-rank noise, by decomposing the residual space into a noisy and a noise-free subspace, and decoupling the noisy subspace from a subset of faults. The approach presented here does not guarantee solution in every situation; when the exact problem cannot be solved, an approximate solution may be sought.

Though we considered static linear systems, our real interest lies with the on-line diagnosis of linear discrete dynamic systems. For such systems, the response to a fault $f_j(t)$ is specified as

$$\mathbf{r}(t|f_j) = \boldsymbol{\beta}_j \, \gamma_j(z) \, f_j(t) \qquad (42)$$

where $\boldsymbol{\beta}_j$ is a static direction vector, and $\gamma_j(z)$ is a dynamic response which is identical for each element of the residual vector. While dynamic design is concerned with the causality and stability of the dynamic response, it has no influence on the static directional properties. Thus the dynamic directional design may be decomposed into a dynamic and a directional step, and the static results of this paper may be applied to the latter. This will be explored in more detail in forthcoming work.

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