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A FUZZY AND ROUGH SETS INTEGRATED APPROACH TO FAULT DIAGNOSIS

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Abstract: This paper proposes a fuzzy and rough sets integrated approach to fault diagnosis. The basic concepts of the rough set theory are firstly introduced, and then it describes how the rough sets theory is combined with fuzzy logic to form a new fault diagnostic scheme. An application example, marine diesel engine fault detection and diagnosis system, is presented, and the simulation results with on board real data demonstrate the effectiveness of the proposed approach. Copyright © 2002 IFAC

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1. INTRODUCTION

In recent years, there is an increasing demand for modern industry systems to become safer and more reliable (Patton, *et al*., 1995; Frank, 1990). The key issue is how to detect and diagnose faults automatically to avoid systems shut down. In order to satisfy the requirement of modern industry, many fault diagnosis methods have been developed. A fault diagnosis system should include the capacity of detecting, isolating and identifying faults. Recently, artificial intelligence (AI) approaches, such as expert systems, artificial neural networks, fuzzy sets and so on, is widely used to detect and diagnose faults. Usually, most information is imprecise, incomplete and uncertain. In order to draw conclusion, one must be able to handle such uncertain information. Normally, there are two kinds of imperfect knowledge: vagueness and indiscernibility. Fuzzy sets theory, which was introduced by Zadeh, has already demonstrated its usefulness in dealing with vagueness. To deal with indiscernibility, rough sets theory was first proposed in Pawlak (1982). From then on, rough sets theory has been well developed and applied in many fields, such as cement kiln control (Sandness, 1986) and decision analysis (Pawlak, *et al*. 1994), etc. The combination of rough sets and fuzzy sets theory can deal with the uncertainty of the diagnosis problem more effectively. In this paper, the rough set theory is used to analyze the decision table composed of condition attributes and decision attributes. The knowledge is represented by a group of fuzzy rules, which can be obtained from the historical information of the diagnosis system. In order to reduce abundant fuzzy rules and attributes or inconsistent information, rough sets theory is used to find a minimal reduct and form a group of final fault diagnostic rules.

2. AN OUTLINE OF ROUGH SETS THEORY

Basic properties of rough sets are related to the knowledge about the universe of discourse expressed by the indiscernibility relation. The two key concepts of rough sets theory are reduct and classification. Reduct means the subset of attributes that determines the equivalence classes as the set of all attributes, and classification means a family of subsets of the universe.

2.1 Information system and decision table

Formally, an information system (IS) is used to represent uncertain knowledge (Shen, *et al.*, 2000).
$IS = (U, \Omega, V_q, f_q)$

U --a nonempty, finite set called the universe.

Ω --a nonempty, finite set of attributes. $\Omega = C \cup D$, in which C is a finite set of condition attributes and D is a finite set of decision attributes.

V_q --for each $q \in \Omega$, V_q is called the domain of q.

f_q --an information function $f_q : U \rightarrow V_q$.

The decision table is a knowledge representation system. The columns are labeled by attributes (include condition attributes and decision attributes), and each row describes one elementary set. Example of a decision table is given in Table 1, $U = \{1,2,3,4,5\}$, $\Omega = \{a,b,c,d,e\}$, $V_q = \{0,1,2\}$, condition attributes C ={a, b, c}, decision attributes D={d, e}.

Table 1 a decision table

U	a	b	c	D	e
1	2	1	1	1	0
2	0	1	2	2	1
3	1	1	1	0	0
4	0	1	1	2	2
5	2	2	1	1	0

2.2 Indiscernibility relation

For every subset of attributes $B \subset A$, an indiscernibility relation *Ind(B)* is defined in the following way: two objects, x_i and x_j, are indiscernible by the set of attributes B in A, if $b(x_i) = b(x_j)$ for every $b \in B$.

$$Ind(B) = \{(x_i, x_j) \in U^2 \mid for \forall b \in B, b(x_i) = b(x_j)\} \quad (1)$$

Where *Ind(B)* is an equivalence relation and

$$Ind(B) = \bigcap_{b \in B} Ind(b) \quad (2)$$

The equivalence class of *Ind(B)* is called the B-indiscernibility relation, for it represents the smallest discernible group of objects for set B. Objects x_i, x_j satisfying relation *Ind(B)* are indiscernible by attributes from B. Furthermore, for any element $x_i \in U$, the equivalence class of x_i in relation *Ind(B)* is represented as $[x_i]_{Ind(B)}$.

The notation *U/A* denotes elementary sets of the universe U in the space A. Consider the subset B={a, b} in Table 1, then

$$U/Ind(\{e\}) = \{\{1,3,5\},\{2\},\{4\}\}$$

$$U/Ind(B) = \{\{1\},\{2,4\},\{3\},\{5\}\}$$

2.3 Lower and upper approximations

The rough sets approach to data analysis hinges on two basic concepts, namely the lower and the upper approximations of a set, referring to: 1) the elements that surely belong to the set; 2) the elements that possibly belong to the set (Walczak, *et al.*, 1999). X denotes the subset of elements of the universe U ($X \subseteq U$) and $B \subseteq \Omega$, then the lower approximation of X in B, denoted as $\underline{BX}$, is defined as the union of all these elementary sets which are contained in X.

$$\underline{BX} = \{x_i \in U \mid [x_i]_{Ind(B)} \subset X\} \quad (3)$$

The upper approximation of the set X, denoted as $\overline{BX}$, is the union of these elementary sets, which have a non-empty intersection with X:

$$\overline{BX} = \{x_i \in U \mid [x_i]_{Ind(B)} \cap X \neq \Phi\} \quad (4)$$

BNX, the boundary of X in U, is the set of elements that can be classified neither in X nor in $\overline{X}$ on the basis of the values of attributes from B.

$$BNX = \overline{BX} - \underline{BX} \quad (5)$$

For Table 1, Let B={a, b}, X={3, 4}, then the lower and upper approximations can be derived:
$\underline{BX} = \{3\}$, $\overline{BX} = \{2,3,4\}$, $BNX = \{2,4\}$.

2.4 Accuracy of approximation

There are two kinds of measures to describe the quality of approximation. The first measure is named the accuracy of approximation of Ω by B:

$$\alpha_B(\Omega) = \frac{\Sigma card(\underline{BX_i})}{\Sigma card(\overline{BX_i})} \quad (6)$$

Which expresses the possible correct decisions when classifying objects employing the attribute B.

The second measure is called the quality of approximation of Ω by B:

$$\gamma_B(\Omega) = \frac{\Sigma card(\underline{BX_i})}{\Sigma card(U)} \quad (7)$$

Which expresses the percentage of objects, which can be correctly classified into Ω by B.

For Table 1, Let B={a, b}, X={3, 4}, then the accuracy of approximation can be derived:
$\alpha_B(\Omega) = 1/3$, $\gamma_B(\Omega) = 1/5$.

2.5 Reduct

For each attribute a_i, if $Ind(A) = Ind(A - a_i)$, the attribute a_i is called superfluous. Otherwise, the attribute a_i is indispensable in A. The reduct is the essential part of an IS, the core is the common part of all reducts.

2.6 The discernibility matrix and function

The elements of a discernibility matrix are defined as follows (Skowron, et al. 1992):

$$(C_{ij}) = \{b \in B \mid b(x_i) \neq b(x_j)\} \text{ for } i, j = 1,2,...n \quad (8)$$

For the decision table as shown in Table 1, the discernibility matrix is shown in Table 2. The discernibility matrix can be used to find the reduct and core. To do this, one has to construct the so-called discernibility function *f*(*B*), which is defined as:

Table 2 The discernibility matrix for Table 1

U	1	2	3	4	5
1					
2	a,c,d,e				
3	A	a,c,d,e			
4	a,d,e	c,e	a,d,e		
5	B	a,b,c,d,e	a,b	a,b,d,e	

$$f(B) = \Pi \sum_{(x,y)\in U^2} \delta(x,y)$$

$$= \bigcap\{\bigcup(c_{ij}) : 1 \leqslant j \leqslant i \leqslant n^2, c_{ij} \neq 0\} \quad (9)$$

For the discernibility matrix as shown in Table 2, the discernibility function is:

$$f(A) = abc + abe \quad (10)$$

2.7 Classification

Let $F = \{X_1, X_2,..., X_n\}, X_i \subset U$ be a family of subsets of the universe *U*. If the subsets in *F* do not overlap, i.e., $X_i \bigcap X_j = \Phi$, and the entity of them contains all elementary sets, i.e., $\bigcup X_i = U$ for $i = 1,\ldots,n$, then *F* is called a classification of *U*, while X_i are called classes.

3. INTEGRATED AI DIAGNOSIS APPROACH

3.1 Integrated AI diagnosis system structure

The basic structure of the proposed integrated AI approach to fault diagnosis is shown in Fig.1. It has two major parts; one is a family of diagnostic rules, which are derived from knowledge base, these rules form a fuzzy-rule base for fault diagnosis. Another one is data processing, since the data are incomplete, fuzzy logic and rough set are used to process the original data.

3.2 The fuzziness and roughness

Traditional quantity spaces require exact limits for the ranges that characterize qualitative value. Inexact behaviors and uncertain measurements are once again sources of problems, especially in the diagnosis where relatively small deviations from normal behavior have to be recognized, classified, and explained. Usually, it is impossible to determine the exact value of a variable. Thus, enumeration of possible values for unknown variables is more naturally accomplished using ranges rather than precise numbers.

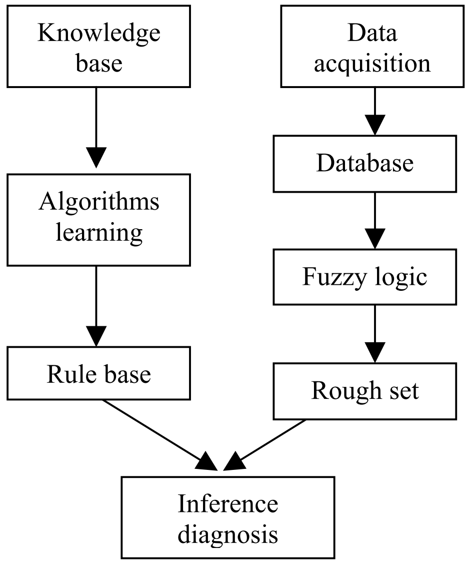

Fig. 1. The structure of integrated AI diagnosis

For faults diagnosis, the input data of the system must be converted into a fuzzy set membership function by fuzzification. There are a variety of choices for membership functions, such as triangle, Gaussian and exponential shape functions.

A family of fuzzy diagnostic rules can be derived from the knowledge base and produced by learning algorithms or fault mechanism analysis. Those rules form a rule base. Normally, a fuzzy diagnostic rule is describe as follows:

IF effects of the system,
THEN causes of faults.

In this paper, the rule base is considered as an information system, shown in Table 3. As the set of condition attributes of decision table, characteristic parameter space $C = \{c_1, c_2, c_3...c_m\}$ denotes the set of characteristic parameters of diagnosis system, where c_i denotes the i-th kinds of characteristic parameter of the diagnosis system. As the set of decision attributes of decision table, fault candidate space $D = \{d_1, d_2, d_3...d_n\}$ denotes the set of fault elements, where d_i denotes the i-th kind of fault source. Condition space $E = \{e_0, e_1, e_2...e_k\}$ denotes the set of condition attribute value, as a set of fuzzy-qualitative values, which describes the degree of system parameter's deviations from normal state, where e_i , denotes the i-th level of characteristic parameter's deviations from normal state. Such as, e_0 denotes this characteristic parameter's value in the range of normal state, e_1 denotes this parameter's value in the range of relatively small deviations of normal behavior, and usually, e_i denotes more deviations than e_{i-1} . Fault classification space $F=\{f_0, f_1, f_2 ... f_j\}$ denotes the set of the degree of fault behavior, as a set of fuzzy-qualitative values, where f_i denotes the i-th level of faults. Such as, f_0 denotes no faults, f_1 denotes that a slight fault is occurred, and similar to *E*, when the subscript i becomes larger, the fault becomes more serious. Normally, all the faults can be divided into three levels or five levels according to the practical knowledge of the system. When the more detailed information of fault diagnosis is given, the more levels can be divided.

Table 3 A fuzzy set based rough sets decision table

U	Condition Attributes			Decision attributes		
	c1	c2	C3…	d1	D2	d3…
Rule_1	e_i	e_j	$e_k \ldots$	f_i	f_j	$f_k \ldots$
Rule_2	…	…	…	…	…	…
Rule_3	…	…	…	…	…	…
……	…	…	…	…	…	…
Rule_n	…	…	…	…	…	…

When the fuzzy-rules set is build, there must be some redundant and information-poor attributes, which are contained in fuzzy rules, particularly for complex system. In order to remove redundancy information, a preprocessing step using rough sets theory is necessary. Rough set theory reduces redundant and information-poor attributes without losing any information that is needed for rules induction. Furthermore, the reduction increases the speed of fault diagnosis. In addition, this approach is fast and efficient, while it maintains the underlying semantics of data. According to the final reduct, the most important attributes can be chosen from original attributes.

3.3 Method of inference diagnosis

When the system is in normal condition, there is no obvious fault and all the characteristic parameters are varied around the normal state. When there is at least one characteristic parameter deviating from normal state and beyond the acceptable range, the system performance will degrade, and it means that faults have occurred (Zhou, *et al.*, 2000). From the knowledge base and decision table such as Table 3, some theorems can be drawn as follows:

Theorem 1. The effects that the deviations of condition attribute c_i from normal state take on the deviation of all the decision attributes. The effects is defined as follows:

Definition 1: Let *D, F, C, E* form a knowledge base, with the condition c_j , the $\mu_{ijk}=\mu(d_i \in f_k | c_j)$ denotes the measure vector as followed:

$$(\mu_{ij0}, \mu_{ij1}, \ldots, \mu_{iju\text{-}1})$$

$$\sum_{k=0}^{u\text{-}1} \mu_{ijk}=1, \left(for\ i=1,2, \ldots n, \quad j=1,2, \ldots m\right) \qquad (11)$$

So the group of matrixes

$$\mu_i=\begin{bmatrix} \mu_{i10} & \mu_{i11} & \cdots & \mu_{i1(u-1)} \\ \mu_{i20} & \mu_{i21} & \cdots & \mu_{i2(u-1)} \\ \vdots & \vdots & \vdots & \vdots \\ \mu_{im0} & \mu_{im1} & \cdots & \mu_{im(u-1)} \end{bmatrix} \qquad (i=1,2,...,n) \qquad (12)$$

This group of matrixes named the matrix of single condition for fault diagnosis.

Theorem 2: The effects that all the condition attributes take on the single decision attribute. For single candidate fault, the m kinds of characteristic parameters take different effect on the same fault. And the same parameter takes different effect on different candidate faults. The effects is defined below:

Definition 2: Let *D, F, C, E* form a knowledge base, for the single candidate fault d_i $(i=1,2,...,n)$, the weight parameters $w_{ij}(j=1,2,...,m)$ reflect all the parameter's effects on the candidate fault.

$$\hat{w}_i=\left(w_{i1}, w_{i2}, ..., w_{im}\right) \quad i=1,2,...,n \qquad (13)$$

is the weighting vector.

$$w=(w_{ij})_{n \times m}, 0 \leqslant w_{ij} \leqslant 1, \sum_{j=1}^{m} w_{ij}=1 \qquad (14)$$

is the weighting matrix.

When candidate fault d_i occurs, there must be some characteristic parameters, which are beyond the normal state. Then $d_i \in f_j$ denotes that fault-i belongs to the j-th faults level. Furthermore, the formula $cof_{ij}=cof(d_i \in f_j)$ denotes the confidence of the $d_i \in f_j$. If *cof* satisfy the conditions:

$$0 \leqslant cof_{ij} \leqslant 1 \qquad (15)$$

$$\sum_j cof_{ij}=1 \qquad (16)$$

$$cof_{i,j \cup k}=cof_{ij}+cof_{ik} \qquad (17)$$

Where $i=1,2,...,n;\ j=0,1,...u-1$ and $c_j \cap c_k=\Phi$, then *cof* is the measure on *F*.

$$cof_{ik}=\sum_{j=1}^{m} w_{ij} \mu_{ijk} \qquad (18)$$

$$cof_i=\left(cof_{i0}, cof_{i1}, ..., cof_{i(u-1)}\right) \qquad (19)$$

$$cof_i=w_i \times \mu_i \qquad (20)$$

$$cof=\left(cof_1, cof_2, ..., cof_n\right)^{\tau} \qquad (21)$$

The cof is the fault identification matrix. According to this matrix, the confidences of the candidate faults can be measured, as follows:

Given d_i, $F = \{f_0, f_1, f_2 \dots f_{u-1}\}$ and the confidence $\lambda(0.5 \leqslant \lambda \leqslant 1)$, if

$$k_{cof} = \min \{k | \sum_{j=0}^{k} cof_{ij} \geqslant \lambda\} \ (0 \leqslant k \leqslant 1) \quad (22)$$

then $d_i \in f_{k_{cof}}$ with the confidence λ.

4. APPLICATIONS

In this section, the integrated AI diagnosis approach based on fuzzy sets and rough sets theory is applied in a marine diesel engine system (Zhou, *et al.*, 2000).

In this system, there are two important condition attributes: exhaust temperature $T_r(°C)$ and utmost pressure $P_z(10^5 Pa)$, considered as the characteristic parameters of the marine diesel engine. The range of value of exhaust temperature is $T_r \in [320°C, 360°C]$ and the range of value of utmost pressure is $P_z \in [123 \times 10^5 Pa, \quad 137 \times 10^5 Pa]$.

There are 6 kinds of common faults: nozzle enlarged (F1), nozzle blocked (F2), valve seat leaked (F3), injection time late (F4), injection time early (F5) and exhaust pipe blocked (F6).

Fault candidate space consists of these six candidate faults, $D = \{d_1, d_2, d_3, d_4, d_5, d_6\}$. Characteristic parameter space consists of two parameters, $C = \{c_1, c_2\}$. In fault classification space $F = \{f_0, f_1, f_2\}$, f_0 denotes no faults; f_1 denotes the slight faults; f_2 denotes serious faults. Similarly, in condition space $E = \{e_0, e_1, e_2\}$, e_0 denotes this characteristic parameter's value is in the range of normal state, e_1 means in the range of relatively small deviations of normal behavior, and e_2 means beyond the normal state.

For each parameter, its range of possible numeric values are divided into qualitative fuzzy subsets, see Fig.2.

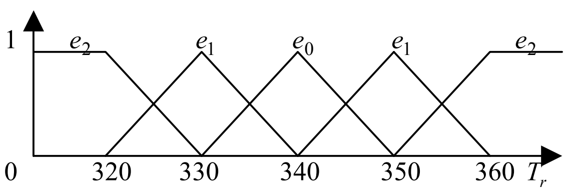

Fig. 2. (a) Fuzzy quantity space of T_r

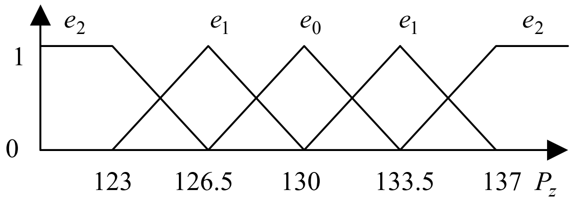

Fig. 2. (b) Fuzzy quantity space of P_z

According to the range of parameter values, the whole space is divided into five subspaces equally. Triangle shape membership functions are selected. Then all the available quantitative parameters are transformed into qualitative fuzzy membership value. After fault mechanism analysis, the 6 kinds of common faults F1~F6 have relationship with the two characteristic parameters, form a family of fuzzy diagnostic rules, as shown in Table 4.

Table 4 the decision table of fault diagnostic rules

U	Condition T_r	P_z	Decision F1	F2	F3	F4	F5	F6
R1	e_2 H	e_2 H	f_2	f_0	f_0	f_0	f_0	f_0
R2	e_2 H	e_1 H	f_0	f_0	f_0	f_0	f_0	f_2
R3	e_2 H	e_2 L	f_2	f_0	f_0	f_2	f_0	f_0
R4	e_2 H	e_1 L	f_0	f_0	f_2	f_0	f_0	f_0
R5	e_2 L	e_1 L	f_0	f_2	f_0	f_0	f_0	f_0
R6	e_1 L	e_2 H	f_0	f_0	f_0	f_0	f_2	f_0

Where H and L denote the direction of deviations of condition attributes, H denotes positive deviation while L denotes negative deviation.

Generally, after built up the decision table of given information system, one should use the rough set theory to reduce the redundancy of the fuzzy fault diagnostic rules, deal with the inconsistency and get the minimization of decision algorithms. These algorithms form a rule base.

In general, the rest procedure is divided into the following three steps:

Step 1: From the qualitative values of parameters and the diagnostic rule base, $\mu_i, (i = 1,2,\dots,n)$, a group of matrixes are obtained.

Step 2: Then, the weight matrix w is calculated with the given information.

Step 3: With $\mu_i, (i = 1,2,\dots,n)$ and w, the fault identification matrix can be obtained, from which the possible faults are diagnosed.

For example, a set of parameters is shown in Table 5, which is measured on board in a ship.

Table 5. A set of practical parameters

	Group 1	Group 2	Group 3
T_r	356.0	340.0	356.0
P_z	137.2	137.2	131.0

For Group 1 (T_r =356.0, P_z =137.2), from Table 4 and Fig. 2, one can draw:

$$\mu_1 = \begin{bmatrix} 0 & 0.4 & 0.6 \\ 0 & 0 & 1 \end{bmatrix} \quad \hat{w}_1 = (0.28, 0.72)$$

$$\mu_6 = \begin{bmatrix} 0 & 0.4 & 0.6 \\ 0 & 0 & 1 \end{bmatrix} \quad \hat{w}_6 = (0.80, 0.20)$$

Where the weight vector is calculated by the rules as follows:

1) If the rules tell that there is a parameter more important than another one, then select the weight of that parameter to be 0.80, and the weight of another parameter to be 0.20.

2) Otherwise, the weight vector can be calculated by the entropy method.

Thus, the matrix of fault identification cof is

$$cof = \begin{bmatrix} 0 & 0.11 & 0.89 \\ 0 & 0.32 & 0.68 \end{bmatrix}$$

Similarly, the diagnosis results for group 2 (T_r = 340.0, P_z =137.2) and group3 (T_r =356.0, P_z =131.0), which are shown in Table 6 (With the confidence $\lambda = 0.7$).

Table 6 The result of fault diagnosis

Condition		Result					
T_r	P_z	F1	F2	F3	F4	F5	F6
356.0	137.2	f_2	f_0	f_0	f_0	f_0	f_2
340.0	137.2	f_0	f_0	f_0	f_0	f_2	f_0
356.0	131.0	f_0	f_0	f_2	f_0	f_0	f_0

5. CONCLUSIONS

In this paper, an integrated AI fault diagnosis approach using fuzzy sets and rough sets theory is presented. An application example, marine diesel engine diagnosis system, has been discussed in order to verify the effectiveness of the proposed approach. The result demonstrates that the combination of fuzzy sets and rough sets theory may play important role in fault diagnosis.

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