15th Triennial World Congress, Barcelona, Spain

ADAPTIVE FLIGHT CONTROL DESIGN FOR NONLINEAR MISSILE

Antonios Tsourdos *,[1] **Brian A. White** *

* *Department of Aerospace, Power and Sensors,*
Royal Military College of Science,
Cranfield University,
Shrivenham,SN6 8LA, England,UK

Abstract: An autopilot combining an indirect adaptive controller with approximate feedback linearisation is proposed in order to achieve asymptotic tracking. Adaptation is introduced to enhance closed-loop robustness, while approximate feedback linearisation is used to overcome the problem of unstable zero dynamics. Computer simulations show that this approach offers a possible autopilot design for non-linear missiles with uncertain parameters.

Keywords: non-linear control, feedback linearisation, flight control, missile system, adaptive control

1. INTRODUCTION

The performance of aerospace systems such as aircraft, spacecraft and missiles is highly dependent on the capabilities of the guidance, navigation and control systems. To achieve improved performance in such aerospace system systems, it is important that more sophisticated control systems be developed and implemented. In particular, as the performance envelope is expanded, the control schemes must become adaptive and nonlinear, to provide performance over a greater range, in the face of uncertain or changing operating conditions.

The tracking performance of a missile is also dependent on the location within the flight envelope and varies with factors such as Mach number and dynamic pressure. Several approaches, including adaptive control Lin and Cloutier (1991), nonlinear control White et al. (1998), and gain scheduling Shamma and Cloutier (1993) have been used to alleviate these tracking problems. While gain scheduling is conceptually simple and has been proven successful, it has virtually no guarantee of stability in the transitional periods between operating points and relies on the fact that the scheduling variables should only change slowly. Furthermore, there is a heavy design overhead due to the large number of linear controllers which must be derived and, as the performance demands of modern-day missile systems become more stringent, alternatives to linear control are of increasing practical significance.

Feedback linearisation is a popular method used in nonlinear control applications, and there have been several flight control demonstrations Snell (1992). Dynamic model inversion is the feedback linearisation method employed to design the missile autopilot. This method is very effective in applications to aircrafts and missiles. The main drawback of dynamic model inversion is the need for high-fidelity nonlinear force and moments models that must be invertible in real time, which implies a detailed knowledge of the plant dynamics, and the approach tends to be computationally intensive. In general, dynamic model inversion

[1] A.Tsourdos@rmcs.cranfield.ac.uk

is sensitive to modelling errors. The application of robust and/or adaptive control can alleviate this sensitivity and therefore the need for detailed knowledge of nonlinearities.

In this paper an adaptive nonlinear control design technique is applied to the autopilot for the missile model which is aerodynamically controlled. Missile motion is modelled to be nonlinear with unknown parameters. Based on the model, we adopt a design procedure similar to Sastry and Bodson (1989), basically an adaptive feedback linearisation method. In this scheme, unknown parameters are estimated and based on these estimates, control parameters are updated. Computer simulations show that this approach is very promising to apply the autopilot design for the missiles which are highly nonlinear in aerodynamics with unknown parameters.

The missile model can be represented in the general nonlinear state space:

$$\dot{x}(t) = f(x,\theta) + g(x,\theta)u$$
$$y(t) = h(x,\theta) \qquad (1)$$

Typically the control law is based on a vector $\hat{\theta}$ which is an on-line estimate of the true parameter vector θ. The update laws for these adjusted parameters are determined as a part of the design and shall be such that the closed loop system stability is preserved. The convergence of these parameters estimates to their true value θ is a necessary condition in indirect schemes of adaptive control.

An indirect adaptive controller consists of a parameter identification scheme and a controller whose gains are calculated on-line based on estimates of the plant model parameters. The structure of the plant assumed a priori, but the coefficients or parameters involved are estimated based on the available input/output information. Figure (1) show the schematic diagram for indirect adaptive control schemes. The identification block estimates the plant parameters from the control signal and the output measurement. The estimated parameters are then used to update the controller gains according to one of the several control methodologies.

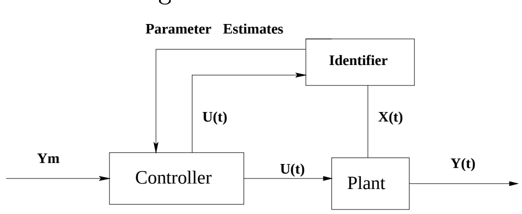

Fig. 1. Schematic diagram for indirect adaptive control schemes

The parameter identifier is used in the outer loop design and continuously adjusts the parameters estimates based on observation error. The certainty equivalence principle suggests that these parameter estimates that are converging to their true values may be employed to asymptotically achieve the desired objective as a parameter estimates converge to their true value. The adaptive scheme developed for the lateral missile flight control system is presented in the following sections.

Other adaptive schemes such as direct adaptive control schemes are discussed in details in Sastry and Bodson (1989). In schemes of that form, parameters do not need to converge to their true value but they are required to stay bounded and converge to some constant. Typically, if the system is persistently exciting then all the parameters will converge to their true values.

2. MISSILE MODEL

The missile model used in this study derives from a non-linear model produced by Horton of Matra-British Aerospace Horton (1992). It describes a 5 DOF model in parametric format with severe cross-coupling and non-linear behaviour. This study will look at the reduced problem of a 2 DOF controller for the pitch and yaw planes without roll coupling. The angular and translational equations of motion of the missile airframe are given by:

$$\dot{r} = \frac{1}{2} I_{yz}^{-1} \rho V_o S d (\frac{1}{2} d C_{nr} r + C_{nv} v + V_o C_{n\zeta} \zeta)$$
$$\dot{v} = \frac{1}{2m} \rho V_o S (C_{yv} v + V_o C_{y\zeta} \zeta) - Ur \qquad (2)$$

where the variables are defined in Figure 2. Equations 2) describe the dynamics of the body

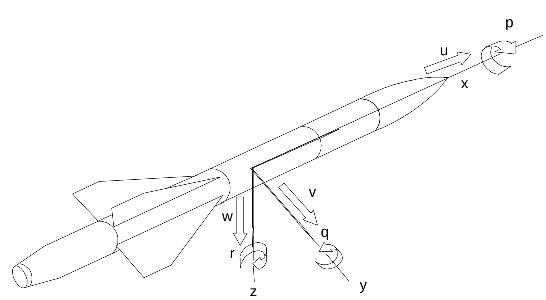

Fig. 2. Airframe axes

rates and velocities under the influence of external forces (e.g. C_{zw}) and moments (e.g. C_{mq}), acting on the frame. These forces and moments are derived from wind tunnel measurements and by using polynomial approximation algorithms $C_{yv}, C_{y\zeta}, C_{nr}, C_{nv}$ and $C_{n\zeta}$ Horton (1992) can be represented by polynomials which can be fitted to the set of curves taken from a look-up tables for different flight conditions. A detailed description of the model can be found in Horton (1992).

The aerodynamic forces and moments acting on the airframe are non-linear functions of Mach number, longitudinal and lateral velocities, control surface deflection, aerodynamic roll angle and body rates. Control of the missile will be accomplished in this paper by controlling an augmented version of lateral acceleration. The dynamic equation for lateral acceleration can be derived White et al. (1998) and is given by:

$$\begin{aligned}
\alpha &= \dot{v} + Ur \\
\alpha &= V^o(C_{yv}v + V_o C_{y\zeta}\zeta) \\
&= V^o[(C_{yv_0} + C_{yv_M}M + C_{yv_\sigma} \mid \sigma \mid)v \\
&\quad + V_o(C_{y\zeta_0} + C_{y\zeta_M}M + C_{y\zeta_\sigma} \mid \sigma \mid)\zeta] \\
&= V^o[(\bar{C}_{yv_0}v + \bar{C}_{yv_\sigma} \mid v \mid)v \ + V_o\bar{C}_{y\zeta_0}\zeta + V_o\bar{C}_{y\zeta_\sigma} \mid v \mid \zeta] \\
&= \phi(v) + \psi(v,\zeta) \qquad (3)
\end{aligned}$$

where the Mach number M, and the total velocity V_o are slowly varying.

3. NON-LINEAR STATE-SPACE MODEL FOR LATERAL DYNAMICS

The equations (2) describing the angular and translational dynamics of the non-linear system, have also been recast in polynomial format, to give:

$$\begin{aligned}
\dot{v} &= f_v(v,r) + g_v(v,r)\zeta \\
\dot{r} &= f_r(v,r) + g_r(v,r)\zeta \qquad (4)
\end{aligned}$$

which can be written in matrix format. As the pitch and yaw equations are not coupled in this example, and as the missile is symmetric in both planes, only one plane (the yaw plane) need be treated as shown in equation (5).

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} f_{x_1}(x) \\ f_{x_2}(x) \end{bmatrix} + \begin{bmatrix} g_{x_1}(x) \\ g_{x_2}(x) \end{bmatrix} u \qquad (5)$$

where:

$$x = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T = \begin{bmatrix} v & r \end{bmatrix}^T$$

$$u = \zeta$$

4. APPROXIMATE INPUT–OUTPUT LINEARISATION

The state–space form of the non-linear system of the home missile can now be written in a compact parametric format, as:

$$\begin{aligned}
\dot{x}_1 &= a_1x_1 + a_2x_1^2 + a_3x_2 + (a_4x_1 + a_5)u_1 \\
\dot{x}_2 &= b_1x_1^3 + b_2x_1^2 + b_3x_1 + b_4x_1x_2 + b_5x_2 \\
&\quad + (b_6x_1 + b_7)u_1 \qquad (6)
\end{aligned}$$

or in matrix form:

$$\dot{x} = f(x) + g(x)u$$

$$y = h(x) = (a_1x_1 + a_2x_1)\,x_1 \qquad (7)$$

This equation is now in standard form and input-output linearisation techniques can be applied to it. In order to retain the system order with no zero dynamics, an approximate input–output linearisation technique is applied to the missile model. It is based at the second approximation method involving the modification of the function g presented in Hauser et al. (1992).

Using this approximation technique, terms are discarded in order to retain an approximate system with an equivalent order and relative degree. In other words the g vector field is modified. This is achieved by neglecting the term $\psi(x_1, u_1)$ shown in (8) as it will not affect the stability of the closed loop dynamics.

Let $\xi_1 = \phi_1 = h(x) = x_1$. Then:

$$\begin{aligned}
\dot{\xi}_1 &= \xi_2 + \psi(x, u_1) \\
\dot{\xi}_2 &= \alpha(x) + \beta(x)u = \nu(x,u) \qquad (8)
\end{aligned}$$

Hence the output y possesses a relative degree γ of 2.

The relative degree of the system is now $\gamma = 2$, and has the same order as the original system. Therefore there are no internal dynamics. Since the relative degree is equal with the order of the system, fully linearisation of the non-linear system can now be achieved.

The effect of neglecting the term $\psi(x, u_1)$ in equation (8) is to eliminate a non-linear zero in the system within the model description. It had be shown in White et al. (1998) this will not affect the performance of the control design in a significant manner as the zero can be approximated by:

$$z \approx -\frac{\psi_1(x)}{\beta_1(x)} \qquad (9)$$

Examination of equation (9) shows that z is always positive if the fin moment arm is greater than the static margin. This is true in all well designed missiles as the fin moment arm gives the small fin force sufficient turning moment to overcome the lift induced moment from the static margin. Hence the non-linear zero will always be in the stable left half s plane. It will tend to enhance the stability of the closed loop system, rather than detract from it; hence if the linearisation takes place without taking the zero into account, the resulting system should be more stable. Equation (8) represent a direct relationship between the outputs h and the inputs u Wang (1994). The required static state feedback for decoupled closed loop input/output behaviour is given by:

$$u = \beta^{-1}\,[\nu - \alpha] \qquad (10)$$

where β is the characteristic or decoupling polynomial of the system which has nonsingular solutions for the operating envelope. The linearised closed loop system is now given by:

$$\ddot{y} = \nu \qquad (11)$$

Where nu is the new linearised system input Wang (1994). Now choose the new control input to be:

$$\nu = \ddot{y}_d - k_1 \dot{e} - k_2 e \qquad (12)$$

where $e \equiv y - y_d$. The close–loop system is thus characterised by:

$$\ddot{e} + k_1 \dot{e} + k_2 e = 0 \qquad (13)$$

where k_1 and k_2 are chosen using classical pole-placement such that all roots of $s^2 + k_1 s + k_2 = 0$ are in the open left–half plane, which ensures $\lim_{t \to \infty} e(t) = 0$ Wang (1994).

It can be said that now the tracking control problem for the non-linear system has been solved using the control law in equation (10) and (12). Indeed, since equation (13) has the same order as the non-linear system, there is no part of the system dynamics which is rendered "unobservable" in the approximate input–output linearisation. Since there are no zero dynamics in the linearised system, the stability of the linearised system can be guaranteed and the tracking problem has been solved.

5. TRAJECTORY CONTROLLER DESIGN

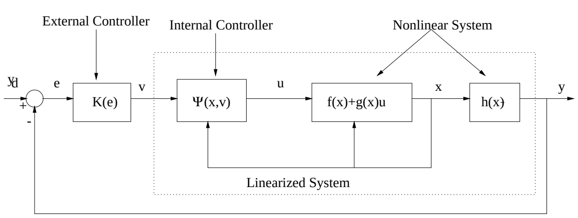

Fig. 3. Trajectory control design

Figure 3 shows the non-linear controller structure. A fast linear actuator with natural frequency of 250 rad/sec has been included in the non-linear system. The error dynamics are constructed using the a_d signal and the feedback of the actual states - velocity, rate and acceleration.

The error coefficients in (13) are chosen to satisfy a Hurwitz polynomial. For the second order error equation in each channel, $k_1 = 2\zeta w_n$ and $k_2 = w_n^2$, where $w_n = 60(rad/sec)$ and $\zeta = 0.65$. The speed of response is significantly faster than the open loop missile response and so should exercise the dynamics of the non-linear missile sufficiently for meaningful conclusions to be drawn.

The results of a 100 m/sec^2 demand in acceleration is shown in Figures 4, 5 for the 100 m/sec^2 case. The figures show almost identical step responses with some variation in peaks and steady state values for the body rate, the actuator movement and the lateral velocity. The difference between the lateral acceleration and the augmented acceleration shows that there is a good match between the two and that steady state values are very close. This illustrates the small effect that the fin force has on the missile acceleration and justifies the use of the augmented acceleration. The results also show that the actuator does not significantly affect the design. The non-linear approach is also shown to be reasonably accurate, as the predicted and actual performance are very close.

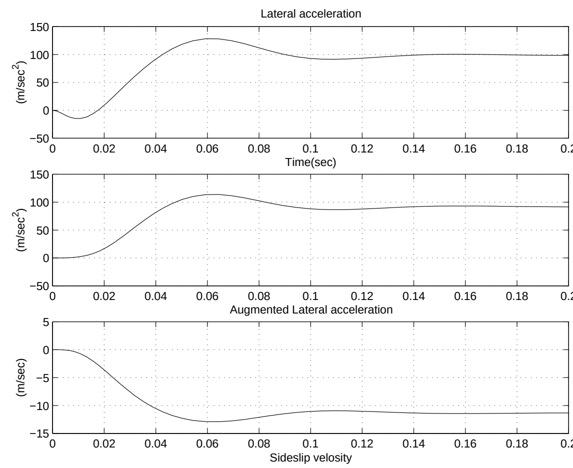

Fig. 4. Acceleration and lateral velocity for $a_d = 100$

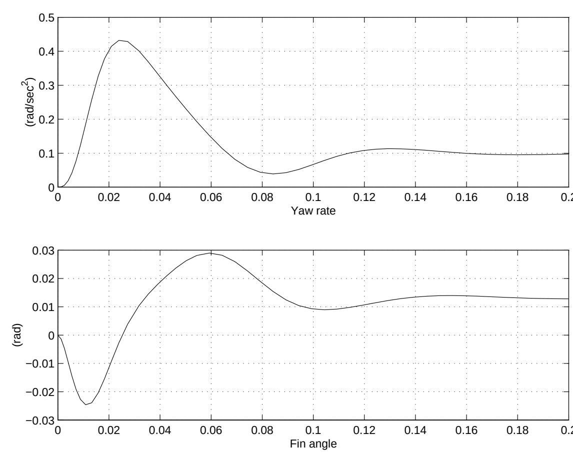

Fig. 5. Rate and fin angle for $a_d = 100$

6. ADAPTIVE NONLINEAR CONTROL

The design presented in the previous section was for the nominal missile model. However neither the Mach number nor the mass of the missile remain constant. As the flight conditions vary both Mach number and mass vary. We showed that the different aerodynamic coefficients are multi-linear functions of Mach number and mass. Hence, as these two variables vary, th aerodynamics coefficients vary. Since the feedback control law was design only for the nominal values of the aerodynamic coefficients, any variations of these will cause inexact cancellation of the system's nonlinearities, i.e. inexact decoupling. Consider a SISO nonlinear system of the form (5) under parametric uncertainty:

$$
\begin{aligned}
\dot{x}(t) &= f(x,\theta) + g(x,\theta)u \\
y(t) &= h(x,\theta)
\end{aligned} \tag{14}
$$

Further, assume $f(x)$ and $g(x)$ have the form:

$$
\begin{aligned}
f(x,\theta) &= \sum_{i=1}^{n} \theta_i^f f_i(x) \\
g(x,\theta) &= \sum_{i=1}^{m} \theta_i^g g_i(x)
\end{aligned} \tag{15}
$$

with θ^f and θ^g vectors of unknown parameters and $f_i(x)$ and $g_i(x)$ known functions. The estimates of these functions are given by:

$$
\begin{aligned}
\hat{f}(x,\theta) &= \sum_{i=1}^{n} \hat{\theta}_i^f f_i(x) \\
\hat{g}(x,\theta) &= \sum_{i=1}^{m} \hat{\theta}_i^g g_i(x)
\end{aligned} \tag{16}
$$

where $\hat{\theta}_j$ are the estimates of the unknown aerodynamic parameters θ_j and are multi-linear functions of Mach number and mass. Now let's replace the control law (10) by:

$$
u_{ad} = \frac{1}{L_g \widehat{L_f^{\gamma-1}} h} \left[-\widehat{L_f^{\gamma} h} + \nu_{ad} \right] \tag{17}
$$

with:

$$
\nu_{ad} = y_d^{(\gamma)} + k_{\gamma-1}(y_d^{(\gamma-1)} - \hat{\xi}_\gamma) + \cdots + k_o(y_d - \hat{\xi}_1) \tag{18}
$$

where k_i are chosen as before and $\hat{\xi}_{i-1} = L_f^i h$ are replaced by their estimates $\widehat{L_f^i h}$:

$$
\begin{aligned}
\hat{\xi}_i = \widehat{L_f^i h} &\doteq L_{\hat{f}^{i-1}} \hat{h} \\
L_g \widehat{L_f^{\gamma-1}} h &\doteq L_{\hat{g}} L_{\hat{f}}^{\gamma-1} \hat{h}
\end{aligned} \tag{19}
$$

As in Sastry and Bodson (1989), since these estimates are not linear in the unknown parameters θ_i, we define each of the parameters products to be a new parameter. For example:

$$
L_g \widehat{L_f^{\gamma-1}} h = \sum_{i=1}^{n} \sum_{i=1}^{m} \theta_i^f \theta_j^g L_g L_f^{\gamma-1} h \tag{20}
$$

and we let $\Theta \in \mathcal{R}^p$ be the large p-dimensional vector of all multi-linear parameter products $\theta_i^f, \theta_j^g, \theta_i^f \theta_j^g, \ldots$. The vector containing all the estimates is denoted by $\hat{\Theta} \in \mathcal{R}^p$ with $\Phi \doteq \Theta - \hat{\Theta}$ representing the parameter error.

Due to the indirect nature of our approach, this overparametrization does not increase the complexity of the closed loop system since a parameter identifier is to be used to estimate the unknown parameters θ_j. The parameter vector Θ is, however, constructed here in order to show the stability of the resulting adaptive system. Using the control law(17) in (8) yields:

$$
\begin{aligned}
\dot{\hat{\xi}}_\gamma &= L_f^\gamma h + \left[L_g L_f^{\gamma-1} h - L_g \widehat{L_f^{\gamma-1}} h \right] u_{ad} - \widehat{L_f^\gamma h} + \nu_{ad} \\
&= \left[L_f^\gamma - \widehat{L_f^\gamma} \right] + \left[L_g L_f^{\gamma-1} h - L_g \widehat{L_f^{\gamma-1}} h \right] u_{ad} + \nu_{ad}
\end{aligned} \tag{21}
$$

Subtracting ν in (12) from both sides gives:

$$
\begin{aligned}
e^2 + k_1 e^1 + k_o e &= \left[L_g L_f^1 h - L_g \widehat{L_f^1} h \right] u_{ad} \\
&\quad + \left[L_f^2 - \widehat{L_f^2} \right] + k_1 \left(L_f - \widehat{L_f} \right) \\
&= \Phi^T w(x, u_{ad}(x))
\end{aligned}
$$

where: $w^T \doteq \left[L_g L_f^1 h_k u_{ad}(x) | \cdots | L_f h_k \right]$. Therefore, in the closed loop, for the approximate system, we have in compact form:

$$
\dot{e} = Ae + W^T(x, u_{ad}(x))\Phi \tag{23}
$$

where A is a Hurwitz matrix and note that if $\phi \doteq \theta - \hat{\theta} \to 0$ as $t \to \infty$, then $\Phi \to 0$ as $t \to \infty$. To estimate the unknown parameters, we consider an observer-based identifier proposed in Taylor et al. (1989), Kudva and Narendra (1973). First we rewrite (14) as:

$$
\begin{aligned}
\dot{x} &= (f_1 \cdots f_n | g_1 u \cdots g_m u) \begin{pmatrix} \theta^f \\ \theta^g \end{pmatrix} \\
&\doteq Z^T(x, u_{ad}(x))\theta
\end{aligned} \tag{24}
$$

Consider the following identifier system:

$$
\begin{aligned}
\dot{\hat{x}} &= \hat{A}(\hat{x} - x) + Z^T(x, u_{ad}(x))\hat{\theta} \\
\dot{\hat{\theta}} &= -Z(x,u)P(\hat{x} - x)
\end{aligned} \tag{25}
$$

where $\hat{A}$ is a Hurwitz matrix, $\hat{x}$ is the observer state, x is the plant state in (14), and $P > 0$ is a solution to the Lyapunov equation $\hat{A}P + P\hat{A} = -\lambda I$ with $\lambda > 0$. We assume that all the states x in (14) are available and hence $\hat{x}$ and $\hat{\theta}$ are given by (25). We also assume that θ is a vector of constant but unknown parameters. Then:

$$
\begin{aligned}
\dot{\hat{e}} &= \hat{A}\hat{e} + Z^T(x,u)\phi \\
\dot{\phi} &= -Z(x,u)P\hat{e}
\end{aligned} \tag{26}
$$

is the observer error system where $\hat{e} \doteq \hat{x} - x$ is the observer state error and $\phi = \hat{\theta} - \theta$ is the parameter error.

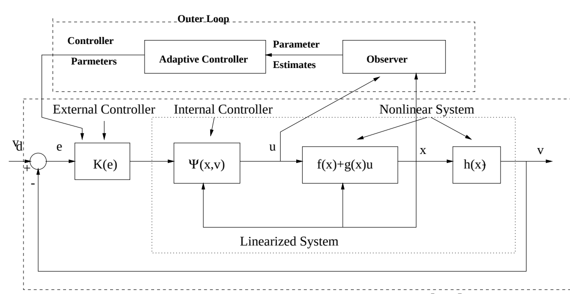

Fig. 6. Adaptive Tracking Control Design

Properties of the observer-based identifier in (26) are given in Sastry and Bodson (1989)and are:

(1) $\phi \in \mathcal{L}_\infty$
(2) with $\hat{e}(0) = 0, \phi(t) \leq \phi(0) \quad \forall t \geq 0$
(3) $\hat{e} \in \mathcal{L}_\infty \cap \mathcal{L}_2$
(4) if $Z^T(x, u_{ad})$ is bounded then $\hat{e} \in \mathcal{L}_\infty$ and $\hat{e} \to 0$ as $t \to \infty$
(5) $\hat{e}$ and ϕ converge exponentially to zero if $Z(x, u)$ is sufficiently rich (i.e. $\exists \delta_1, \delta_2, \sigma > 0$ such that $\forall t : \delta_1 I \leq \int_t^{t+\sigma} ZZ^T d\tau \leq \delta_a I$

However, since $Z(x, u)$ is a function of state x, the condition 5 can not be verified ahead of time.

The block diagram of our adaptive lateral flight control design for the nonlinear missile is shown in Figure 6 while results of the adaptive scheme are shown in Figure 8. Good tracking performance for variation up to 35% in Mach number and mass is achieved.

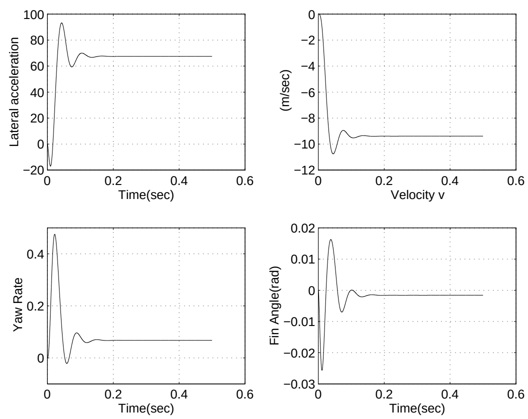

Fig. 7. Non-Adaptive with Uncertainty 35% in Mach number and mass for acceleration demand $a_d = 100$

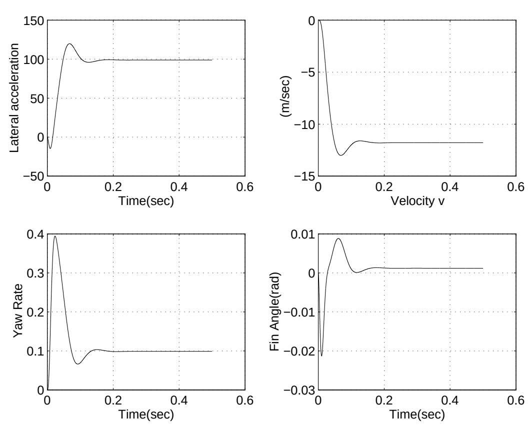

Fig. 8. Adaptive with Uncertainty 35% in Mach number and mass for acceleration demand for $a_d = 100$

7. CONCLUSIONS

Dynamic model inversion is the feedback linearisation method employed to design the missile autopilot. The main drawback of dynamic model inversion is the need for high-fidelity nonlinear force and moments models that must be invertible in real time, which implies a detailed knowledge of the plant dynamics, and the approach tends to be computationally intensive. In general, dynamic model inversion is sensitive to modelling errors. In this paper an adaptive nonlinear control design technique is applied to the autopilot for the missile model which is aerodynamically controlled. Missile motion is modelled to be nonlinear with unknown parameters. In the adaptive scheme used in this paper, unknown parameters are estimated and based on these estimates, control parameters are updated. Computer simulations show that this approach is very promising to apply the autopilot design for the missiles which are highly nonlinear in aerodynamics with unknown parameters.

References

J. Hauser, S.S. Satry, and P. Kokotovic. Nonlinear control via apprriximate input–output linearization: The ball and beam example. *IEEE Transactions on Automatic Control*, 37(3):392–398, 1992.

M.P. Horton. A study of autopilots for the adaptive control of tactical guided missiles. Master's thesis, University of Bath, 1992.

P. Kudva and K.S. Narendra. Synthesis of an adaptive observer using lyapunov's direct method. *Int. J. of Control*, 18:1201–1210, 1973.

C.F. Lin and J.R. Cloutier. High performance, adaptive, robust bank-to-turn missile autopilot. In *AIAA Guidance, Navigation, Control Conference*, pages 123–137, 1991.

S.S. Sastry and M. Bodson. *Adaptive Control, Stability, Covergence and Robustness.* Prentice-Hall Inc., 1989.

J.S. Shamma and J.R. Cloutier. Gain-scheduled missile autopilot design using lpv transformations. *Journal of Guidance, Control and Dynamics*, 16(2):256–263, 1993.

Snell. Nonlinear inversion flight control for a supermaneuverable aircraft. *J. of Guidance, Control and Dynamics*, 15(4):976–984, 1992.

D.G. Taylor, P.V. Kokotovic, R. Marino, and I. Kanellakopoulos. Adaptive regulation of nonlinear systems with unmodeled dynamics. *IEEE Trans. on Automatic Control*, 34:405–412, 1989.

L.X. Wang. Design of adaptive fuzzy controllers for nonlinear systems by input–output linearization. In *Proceedings of the 1994 1st International Joint Conference of NAFIPS IFIS NASA*, pages 89–93, 1994.

B. A. White, A. Tsourdos, and A. Blumel. Lateral acceleration control design of a nonlinear homing missile. In *4th IFAC Nonlinear Control Systems Design Symposium*, pages 708–713, 1998. The Netherlands.