15th Triennial World Congress, Barcelona, Spain

PARAMETER IDENTIFICATION OF A CAR SUSPENSION SYSTEM USING NON-INTRUSIVE SIGNALS

Alberto Herreros * **Enrique Baeyens** * **José R. Perán** *
and Andrés Melgar **

* *Instituto de las Tecnologías Avanzadas de la Producción*
** *Dept. Ingeniería Energética y Fluido-Mecánica*
ETSII. University of Valladolid
Paseo del Cauce, s/n
Valladolid, SPAIN 47011
Fax +34-983-423358
{albher,enrbae,peran,andmel}@eis.uva.es

Abstract: The suspension system of a car is of vital importance for the safety of its occupants. Therefore, it is very important to develop reliable tests for inspecting the condition of its components. A simple model to identify the parameters of a car suspension system is proposed in this paper. It is proven that these parameters are identifiable by using only non-intrusive signals. Unfortunately, the application of conventional identification methods produces suspension parameters without physical meaning. The reason is the loss of consistency of the estimators due to the presence of unknown noise and unmodeled dynamics. In order to avoid this effect, the distance between the magnitude of the true and the predicted power spectrum density of the output signal is chosen as the objective to be minimized on a bounded search space with physical meaning. The optimization problem is solved using the MRCD genetic algorithm. Promising results have been obtained for several real-world cases.

Keywords: Car Suspension Systems, System Identification, Testing Machines, Parameter Estimation, Genetic Algorithms.

1. INTRODUCTION

The suspension system is one of the main components for providing safety and comfort for the occupants of a car. A safe vehicle must be able to stop and maneuver over a wide range of road conditions. The suspension system is responsible for keeping good contact between the tires and the road. There are many different types of suspension, but all of them share two essential components: springs and shock absorbers (dampers).

Springs isolate the driver from road imperfections by allowing the tire to move over a bump without drastically disturbing the chassis. Springs are durable items and are easily inspected. Spring problems are generally easy to identify.

The shock absorber controls spring motion by damping energy from the spring. Shock absorber also control the reaction of the body to road undulations. Shock absorber condition is difficult to evaluate. The Car Care Council recommends inspection at 25,000 and every 6,000 miles thereafter. Frequently, a shock absorber stops working without any visible indication.

There are two different types of suspension testing used today: low frequency, which tests the suspension's resistance to chassis motion, and high frequency, which measures the suspension's resistance to excessive wheel hop as well as chassis movement. The disadvantage of this test is that the suspension is only checked in a small part of its operating range.

High frequency testers vibrate the suspension through the entire range of oscillations that it can react to. High frequency testers have a plate where the car is placed. The plate vibrates through a range of speeds and sensors measure how much the tire is pressing on the plate at all times. During the high frequency measurement, the body does not move, therefore the other three shocks do not affect the results. The tester is simulating a bumpy road and measuring how well the suspension keeps the tire in contact with the tester. The result is given in a percent adhesion. The higher the percentage the more the suspension maintained good contact with the tester. The adhesion number that is used is the lowest obtained in the test sequence.

The European Shock Absorber Manufacturers' Association (EuSAMA) has establised a set of guidelines for vehicle suspension evaluation and has defined a type of tester based on the road adhesion measurement. Many companies produce machines that meet EuSAMA requirements. The main disadvantage of EuSAMA suspension testers is that adhesion is an indicator of the overall suspension condition, but a poor adhesion number is not specific enough to recommend shock replacement.

Therefore, more specific tests for suspension condition need to be developed. These tests should evaluate not only the overall condition but the state of each element of the suspension. In this paper, system identification techniques are proposed to develop improved tests for analyzing suspension condition.

In order to demonstrate the feasibility of system identification methods, a simple car suspension model is proposed and it is proven that their physical parameters are identifiable by using only the external or non-intrusive signals measured by a standard high frequency tester.

The proposed model is a "white box" model because its parameters are the physical car suspension parameters, see (Ljung, 1987; Söderström and Stoica, 1989; Nelles, 2001; Johansson, 2001). It is proved that these physical parameters can be obtained from the coefficients of the input/output transfer function, therefore conventional system identification methods (Ljung, 1987; Ljung, 1988) could be applied to obtain a fixed structure transfer function and later recover the physical parameters. However, the application of conventional identification methods produces good models from the input/output signals point of view but the equivalent suspension parameters have no physical meaning. The reason is the loss of consistency of the estimates due to the unknown noise and unmodelled dynamics.

In order to avoid this, the identification problem has been formulated as the minimization of the error between the magnitudes of the power spectrum densities for the real and predicted output signals. The use of the power spectrum density instead of the temporal signals is motivated by the high frequency content of the input/ouput signals and turns out to be a very good alternative for this problem. Unfortunately, the system identification problem formulated in terms of the power spectrum density is nonconvex in the parameters and local optimization methods may not be appropriate.

The global solution of a nonconvex parametric minimization problem over a bounded search space can be obtained by using stochastic algorithms. The MRCD genetic algorithm has been used to solve the proposed problem and gives a set of good models as solution.

The rest of this paper is organized as follows: Section 2 describes the suspension model, and proves that the physical parameters are identifiable. Section 3 analyzes the suspension model for a set of average nominal values of its parameters. Section 4 formulates the system identification problem as a minimization problem with respect to the poles of the proposed model. Section 5 briefly describes the genetic algorithm used in the solution of the minimization problem. Finally, several real-world study cases are presented in Section 6.

2. STATE SPACE AND TRANSFER FUNCTION EQUATIONS OF THE MODEL

The car suspension system for each wheel is modeled as shown in Figure 1. The total mass m of the vehicle is considered to be divided into sprung mass m_1 and unsprung mass $m_2 = m - m_1$. These terms refer to the component motion relative to the road. Basically, the sprung mass is the body and the unsprung mass is the wheel. The tire is modeled as a spring k_2 and the suspension as the parallel combination of a spring k_1 and a damper C. The objective of this article is to estimate these five parameters with a simple non-intrusive test to check the state of the suspension system. The non-intrusive signals used to identify these parameters are the vertical position changes, input signal $d(t)$, and the car weight changes, output signal $F(t)$. These signals are directly obtained with a high frequency suspension testing machine.

Let p_1 and p_2 be the positions of the sprung and unsprung mass, respectively. The linear differential equations relating the input and output signals, $d(t)$ and $F(t)$, and the positions, p_1 and p_2, are:

$$\begin{cases} m_1\ddot{p}_1 &= k_1(p_2-p_1)+C(\dot{p}_2-\dot{p}_1) \\ m_2\ddot{p}_2 &= k_1(p_1-p_2)+C(\dot{p}_1-\dot{p}_2) \\ & \quad +k_2(d-p_2) \\ F &= k_2(d-p_2) \end{cases} \quad (1)$$

The equations (1) can be converted into a state space equation, $\begin{bmatrix} \dot{x}=Ax+Bd \\ F=Cx+Dd \end{bmatrix}$ using the state variables $x_1=p_1$, $x_2=\dot{p}_1$, $x_3=p_2$, and $x_4=\dot{p}_2$, where

$$A=\begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_1}{m_1} & -\frac{C}{m_1} & \frac{k_1}{m_1} & \frac{C}{m_1} \\ 0 & 0 & 0 & 1 \\ \frac{k_1}{m_2} & \frac{C}{m_2} & -\frac{k_1+k_2}{m_2} & -\frac{C}{m_2} \end{bmatrix}, \quad (2)$$

$$B=\begin{bmatrix} 0 \ 0 \ 0 \ \frac{k_2}{m_2} \end{bmatrix}^T,$$

$$C=\begin{bmatrix} 0 \ 0 \ -k_2 \ 0 \end{bmatrix}, \quad D=k_2.$$

Next, it will be shown that the five parameters of the suspension model are identifiable from the external input and output signals $d(t)$ and $F(t)$.

Theorem 1. Let the total mass of the car $m = m_1+m_2$ be known, then the five physical parameters m_1, m_2, k_1, k_2 and C of the suspension model of Figure 1 are identifiable from the external signals $d(t)$ and $F(t)$.

Proof: The relation between a space state equation and its equivalent matrix transfer function is given by

$$G(s)=C(Is-A)^{-1}B. \quad (3)$$

The analytical solution of the equation (3) is given by

$$G(s)=C\frac{\text{adj}(Is-A)}{|Is-A|}B, \quad (4)$$

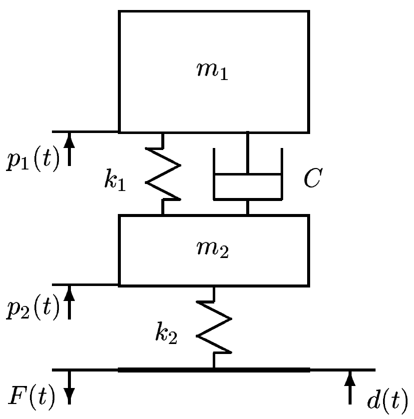

Figure 1. Graphic model representation.

where adj$(\cdot)$ is the adjoint matrix. After some algebra, the result obtained is,

$$G(s)=\frac{s^2(b_2s^2+b_1s+b_0)}{s^4+a_3s^3+a_2s^2+a_1s+a_0} \quad (5)$$

where

$$\begin{aligned} b_2 &= k_2;\ b_1=\frac{k_2C(m_1+m_2)}{m_1m_2} \\ b_0 &= \frac{k_2k_1(m_1+m_2)}{m_1m_2} \\ a_3 &= \frac{C(m_2+m_1)}{m_1m_2} \\ a_2 &= \frac{m_1(k_1+k_2)+m_2k_1}{m_1m_2} \\ a_1 &= \frac{k_2C}{m_1m_2};\ a_0=\frac{k_1k_2}{m_1m_2}, \end{aligned} \quad (6)$$

Assuming that the signal $d(t)$ is sufficiently exciting, the coefficients of the fixed structure transfer function $G(s)$ are identifiable.

If the total mass $m=m_1+m_2$ is known, then the masses m_1 and m_2 can be expressed as $m_1=m\alpha$, $m_2=m(1-\alpha)$, and substituting in 6,

$$\begin{aligned} a_3 &= \frac{C}{m\alpha(1-\alpha)};\ a_2=\frac{k_1+k_2(1-\alpha)}{m\alpha(1-\alpha)}; \\ a_1 &= \frac{k_2C}{m^2\alpha(1-\alpha)};\ a_0=\frac{k_1k_2}{m^2\alpha(1-\alpha)}. \end{aligned} \quad (7)$$

Therefore, the physical parameters can be obtained from the coefficients of the denominator of the transfer function $G(s)$,

$$\begin{aligned} \alpha &= \frac{a_1^2}{a_3(a_1a_2-a_0a_3)} \\ k_2 &= \frac{a_1}{a_3}m \\ k_1 &= \frac{a_0a_3}{a_1}m\alpha(1-\alpha) \\ C &= a_3m\alpha(1-\alpha). \end{aligned} \quad (8)$$

□

Note that the numerator coefficients, that have not been used in the computation of the physical parameters, can be used to obtain a measure of the quality of the parameter estimates.

3. SYSTEM ANALYSIS

In this section, the suspension system modeled by equations (5) and (6) is analyzed. From the literature (Dixon, 1996; Bastow, 1987), the following average nominal values have been selected for the physical parameters.

$$\begin{aligned} & m_1=300 \ \text{Kg}, \quad m_2=25 \ \text{Kg}, \\ & k_1=2\cdot10^4 \ \text{N/m}, \quad k_2=2\cdot10^5 \ \text{N/m}, \\ & C=1\cdot10^3 \ \text{N·s/m}, \end{aligned} \quad (9)$$

The input/output transfer function has two zeros in the origin, a pair of conjugate complex zeros and two pairs of conjugate complex poles. The pair of complex conjugate zeros ($w \approx 20$ rad/s) is related to the damper parameter C. The high frequency pair of conjugate poles ($w \approx 100$ rad/s) is related to the spring of the wheel k_2, and the low frequency pair ($w \approx 10$ rad/s) to the spring of the suspension k_1.

The input signal chosen for the suspension parameter identification, $d(t)$, is a high frequency vibration signal obtained with an experimental suspension testing machine. This signal comprises the entire range of oscillations that the suspension system can react to. An important drawback of this input signal is that the parameter k_1, responsible the low frequency behavior of the system, is poorly estimated.

4. IDENTIFICATION PROBLEM FORMULATION

The identifiability result of Theorem 1 suggests the application of conventional parametric system identification techniques, see (Ljung, 1987; Ljung, 1988), to estimate a_i and b_i, and later obtain the physical parameters by using the equations (7). Unfortunately, the results are very sensitive to noise and neglected dynamic. This has been checked by solving several simulated examples excited with the real input signal $d(t)$. The physical parameters tend to the true values when no disturbances are present, but otherwise the results are deceptive. The module of the real output signal $F(t)$ is quite similar to the simulated one, but there is a large phase error. Also the resultant poles and zeros of the fixed structure estimated transfer function (5) are far from their expected values and the suspension parameters have unrealistic physical meaning. Motivated by these reasons and also by the high frequency content of the input signal the system identification problem is formulated as follows:

"Find a fixed structure transfer function $G(s)$ (5) that minimizes the error between the estimated and real output power spectrum density,

$$\min_{G(s)} \|\text{psd}(F_{real}(t)) - \text{psd}(F_{est}(t))\|_2 \quad (10)$$

where psd$(\cdot)$ is the signal power spectrum density".

From the two pairs of conjugated complex poles of the transfer function $G(s)$, the coefficients a_i (5), the suspension parameters (8) and the coefficients b_i (6) can be obtained. Thus, the optimization problem (10) can be formulated as follows: "Find two pairs of conjugated poles whose associated fixed structure transfer function $G(s, q)$ minimizes the error between the estimated and real output power spectrum density,

$$\min_{q \in Q} \|\text{psd}(F_{real}(t)) - \text{psd}(F_{est}(t))\|_2 \quad (11)$$

where

$$Q = \{q \,|\, q = \begin{bmatrix} q_1 & q_2 & q_3 & q_4 \end{bmatrix}^T ; \\ q_2 = \bar{q}_1, q_4 = \bar{q}_3;\ q_1, q_3 \in \mathbb{C}\}\text{"} \quad (12)$$

Usually the search space for the complex poles q is a region close to the poles obtained for the average nominal parameter values.

Another alternative is to formulate the optimization problem in terms of the physical parameters $\{\alpha, k_1, k_2, C\}$, but the output signal is not very sensitive to the parameter k_1 and the search is more difficult in this case.

The resultant optimization problem (11) is nonconvex in the search variables q and very difficult to solve by conventional optimization algorithms. Therefore, a genetic algorithm, MRCD, has been applied to solve it.

5. THE MRCD GENETIC ALGORITHM

Genetic algorithms (GAs) are random heuristic search methods where an initial set of possible solutions (the so-called population) is modified in successive steps to converge towards the optimal solution, see (Goldberg, 1989; Mitchell, 1999).

The MRCD algorithm is a multiobjective genetic algorithm (Herreros, 2000; Herreros *et al.*, 1999; Herreros *et al.*, 2000) built to design robust controllers that optimize several objectives at the same time.

The main advantage of using a multiobjective genetic algorithm for solving the optimization problem (11) is to obtain the Pareto optimal set in one step, see (Van Veldhuizen, 1999). However, the optimization problem (10) has a unique objective. In order to apply the MRCD algorithm, a new objective has been introduced. One option is to repeat the same objective, but in this case the Pareto optimal set is a single point and the advantage of the multiobjective methods is not exploited. The selected option has been to use the same objective but measured in decibels. This second objective reduces the importance of the larger error peaks in the power spectrum density. In the noise free case there is no difference with respect to the first option and the Pareto optimal set is a single point. However, in the presence of noise and unmodelled dynamics, a true Pareto set is obtained. The variance of this Pareto optimal set is a measure of the confidence in the identified parameters.

6. REAL STUDY CASES

In this section, the results for three real-world study cases are presented. The same car, a Volvo 460-GLE, has been tested on an experimental high frequency testing machine with three different tire pressures (2.2, 2.6 and 3.2 bar).

The search space Q for the poles of the optimization problem (11) are rectangles in the complex plane, centered in the poles obtained for the average nominal values (9).

The results obtained by applying the identification procedure explained here are shown in Figure 2 and Table 1. On the left side of Figure 2 the true and estimated output signals $F(t)$ are depicted while on the right side, the estimated physical parameters for all the elements in the computed Pareto front are plotted. These parameters are associated to one wheel measures. The mean and variance of these estimated physical parameters are given in Table 1. The estimated parameters m_1, m_2 and C are very similar in the three experiments, because their values are almost independent of the tire pressure. The value of k_2 increases with the tire pressure as expected. However, the value of k_1 is also affected by it. The reason may be that the suspension system is excited by a high frequency input signal that is not adequate to correctly estimate the parameters k_1 and k_2. This is also corroborated by the large variance values of these parameters.

7. CONCLUSION

The suspension system of a car is very important for the safety and comfort of the occupants, but it is difficult to check its state with non-intrusive tests.

In this article, a simple parametric model and a system identification procedure have been proposed to obtain the state of the components of the suspension system. The proposed model is identifiable from non-intrusive input/output signals.

The parameters of the suspension model could be estimated by conventional system identification methods. However, the presence of noise, neglected dynamics and the high frequency content of the input signals motivate the substitution of the temporal error by a frequency error measured by the difference in magnitude of the power spectrum density of the output signal. The resultant optimization problem has been solved using MRCD, a multiobjective genetic algorithm.

This identification procedure has been extensively tested with simulated and real-world cases obtaining very promising results. The results presented here are preliminary and more research needs to be carried out. The GAs are a good alternative for solving nonconvex optimization problems, however they are still rather time consuming and more efficient methods need to be developed in order to obtain a good suspension test that could eventually be implemented in a machine.

8. REFERENCES

Bastow, D. (1987). *Car Suspension and Handling.* Pentech Press. London, UK.

Dixon, J. C. (1996). *Tires, Suspension and Handling.* Society of Automotive Engineers, Inc. Warrendale, PA.

Goldberg, D.E. (1989). *Genetic Algorithms in Search, Optimization, and Machine Learning.* Addison-Wesley.

Herreros, A. (2000). Design of Robust Controllers using Genetic Algorithms (in Spanish). PhD thesis. University of Valladolid (Spain).

Herreros, A., E. Baeyens and J.R. Perán (1999). Design of Multiobjective Robust Controllers Using Genetic Algorithms. In: *Proceedings of the 1999 Genetic and Evolutionary Computation Conference. Workshop Program* (Annie S. Wu, Ed.). Orlando, Florida. pp. 131–132.

Herreros, A., E. Baeyens and J.R. Perán (2000). Design of PID Controllers Using Multiobjective Genetic Algorithms. In: *Proceedings of the 2000 IFAC Workshop on Digital Control. Past, present and Future of PID Control.*. Terrassa, Spain. pp. 309–315.

Johansson, R. (2001). *System Modelling and Identification.* Prentice Hall.

Ljung, L. (1987). *System Identification. Theory for the User.* first ed.. Prentice Hall.

Ljung, L. (1988). *System Identification Toolbox.* The MathWorks, Inc. .

Mitchell, M. (1999). *An Introduction to Genetic Algorithm.* fifth ed.. MIT press.

Nelles, O. (2001). *Nonlinear System Identification.* first ed.. Springer.

Söderström, T. and P. Stoica (1989). *System Identification.* first ed.. Prentice Hall.

Van Veldhuizen, D. (1999). Multiobjetive Evolutionary Algorithms: Classification, Analyses, and New Innovations. PhD thesis. Air University. AFIT/DS/ENG/99-01.

Table 1. Estimated Car Suspension Parameter Values (mean/variance).

Case	m_1	m_2	k_1	k_2	C
2.2 bar	295.02/0.07	19.58/0.07	24486.68/440.90	231151.30/2723.09	1518.77/51.59
2.6 bar	292.41/1.02	22.19/1.02	38914.34/8434.27	260976.28/6218.48	1505.18/87.26
3.2 bar	291.77/0.49	22.83/0.49	53102.47/3106.96	297136.49/10730.70	1528.23/140.95

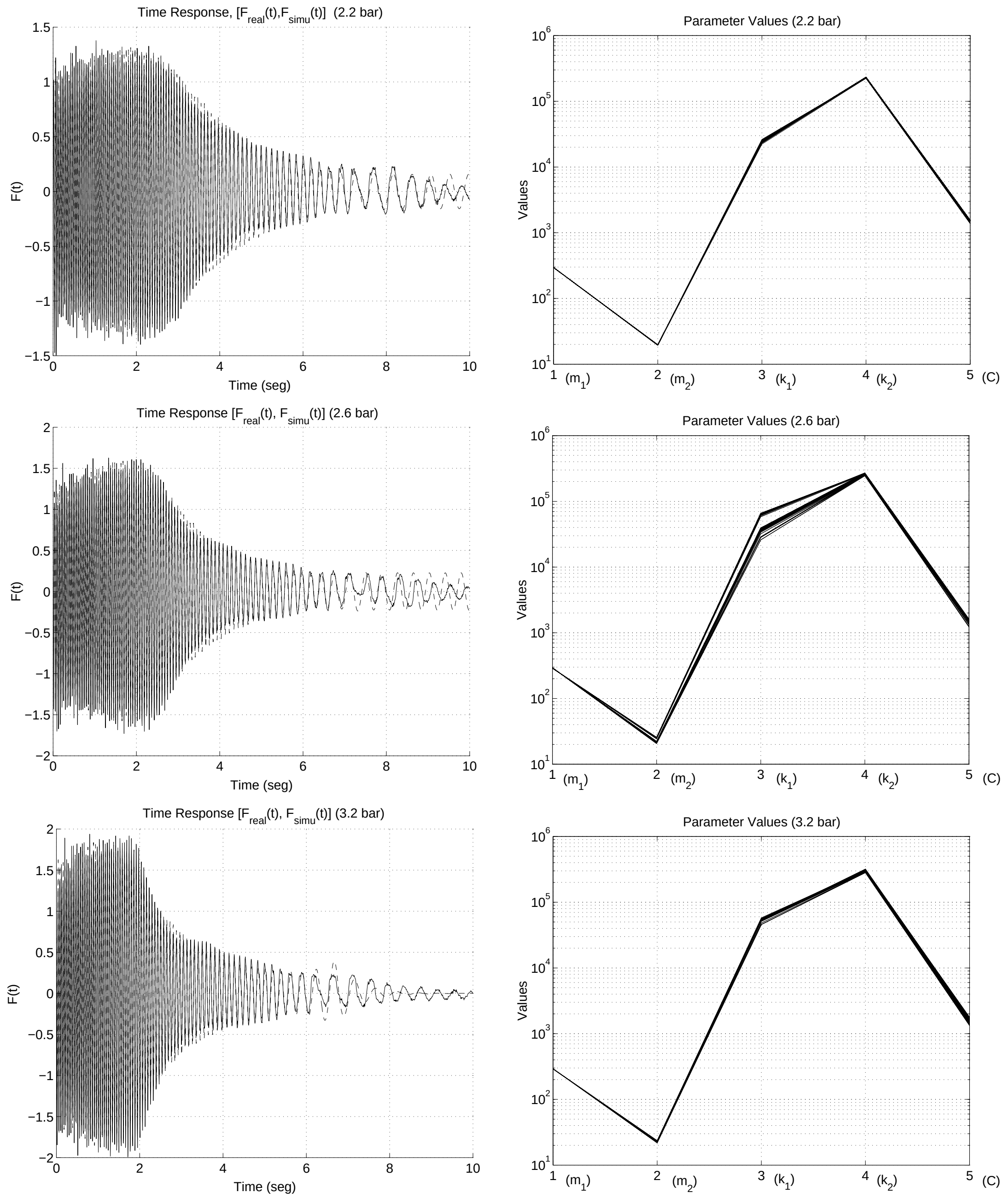

Figure 2. Real and Estimated Time Responses and Estimated Parameter Values.