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# OBSERVER BASED NONUNIFORM SAMPLING PREDICTIVE CONTROLLER FOR A SOLAR PLANT

**R. N. Silva** * **L. M. Rato** ** **J. M. Lemos** ***

* *DEE-FCT/UNL, 2825-114 Monte da Caparica, Portugal, rns@mail.fct.unl.pt*
** *INESC-ID/Univ. Évora, Portugal, email: lmr@di.uevora.pt*
*** *INESC-ID/IST, R. Alves Redol, 9, Lisboa*

Abstract: The work presented in this paper exploits the transport characteristic of a solar plant where the transport velocity (a flow) is the manipulated variable, i.e. the control input. The solar field is modelled by a partial differential equation. A non-uniform sampling in time is performed in order to obtain a discrete linear model. Due to the transport dynamics of the plant the resulting transfer function has a finite impulse response and the optimal control derived from a black-box approach of such a systems yield pure feed-forward compensators. The main contribution of this paper is the use of a state-space description of the plant in conjunction with the nonuniform sampling that allows to introduce the feedback mechanism through the state observer. The controller results from the optimization of a multi-step quadratic cost function. Experimental results performed with the solar plant are shown.


Keywords: Time-scaling, adaptive control, predictive control, solar energy.

## 1. INTRODUCTION

This paper concerns the control of a solar energy collector field and extends the work presented in (Silva, 1999; Silva and Lemos, 2001). This work is characterized by the use of non-uniform sampling in time to linearize the partial differential equation (PDE) in the discretization procedure. In the work referenced above an I/O description of the plant was being used resulting in finite impulse response (FIR) transfer function. This meant that the optimal predictive controller is given by a feedforward block with the inputs: setpoint value, accessible disturbances and past control actions, i.e. there is no dependency on the plant output. Instead here, the internal dynamics are taken into account and the control law includes feedback terms by means of a state observer. Plus, the introduction of the internal state estimation allows the controller to damp the internal oscillations that arise from the cancellation of anti-ressonant modes.

Although the authors, among other researchers, have participate on the development of several controllers for this field using adaptive predictive control techniques (Coito et al., 1997; Silva et al., 1998; Silva et al., 1997; Rato et al., 1997; Pickhardt and Silva, 1998), i.e. without no a priori knowledge about the plant, in this work, the goal was to step up performance by the development of an adaptive predictive controller including, during design, the relevant physical characteristics of the plant. The ACUREX field used in these experiments is described in the available literature (Camacho et al., 1992; Camacho et al., 1988).

In this plant the main sources of disturbances are measured and its dynamic behaviour depends strongly on its geometry (available). This allows us to derive an accurate model which is described by a partial differential equation (PDE). The plant is characterized by:

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Fig. 1. Solar collectors (one of twenty rows).

non linearity caused by the dependency of bandwith and static gain with flow (the manipulated variable); fast accessible disturbances (solar radiation with sudden clouds); time varying dynamics with the daily and annually cycles, and pluvial cycles that modify the reflectivity of the mirrors; and sudden plant changes when groups of collectors are entering/exiting solar track.

Since the controller is to be implemented in a digital computer, the dynamic dependency on flow can be overcome by time-scaling, replacing the elements of time (sampling period) by elements of volume. This results in a discrete linear model with variable sampling period dependent on the imposed flow. Plants, such as rolling mills, conveyor belts or fluids in pipes can be transformed with time-scaling (Åström and Wittenmark, 1984). Plants of this class may be controlled with advantage using the method reported here.

The loss of generality of the results presented here from the Adaptive Predictive Control point of view is compensated by the possibility of extending them for the class of systems modelled by similar equations (PDEs), e.g. heat-exchangers, road traffic, river pollution, etc.

The paper is organized as follows. In section 2 the plant and the discretization of its the non linear model is described. Section 3 describes the controller design. In section 4 some experimental results on the real plant are shown and some conclusions are drawn in section 5.

## 2. PLANT DESCRIPTION

The ACUREX field of the Plataforma Solar de Almería (PSA) in Southern Spain, consists of 480 distributed solar collectors. They are arranged in 10 loops along an east-west axis (fig. 1). The collector has a reflective cylindrical parabolic surface in order to concentrate the incident solar radiation on a pipe located on the surface focal line.

A heat transfer fluid (oil) is pumped from the bottom of a storage tank through the collectors, where it collects solar energy, and from the output of the field, again to the top of the tank. By manipulating the oil flow, with the pump, it is possible to control the output temperature of the oil.

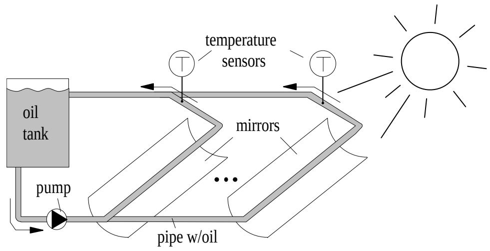


Fig. 2. Process schematic.

The controlled variable is computed from the average values of an array of 10 temperature sensors located at the output of each loop.

Due to safety reasons the oil flow is limited between 2.0 and 10.0 liters per second. The heated oil from the collector field stored in the tank can be used e.g. for the production of electrical energy or for the operation of a desalination plant.

The field is equipped with a tracking system by which the mirrors can rotate parallel to the axis of the receiving tube in order to follow the sun in height throughout the day. There is a temperature sensor located at the input of the field, measuring the temperature of the oil entering the active part (mirrors). It is also available a 2 d.o.f. solar radiation sensor that is able to follow the sun, measuring the total incident radiation. With an algorithm using the actual day and time it is possible to compute the corrected radiation (i.e. the effective radiation heating the oil) from that measure.

### 2.1 *PLANT NONLINEAR MODEL*

The focus of the modelling work has been put into the transport effect since this is the most relevant part for the dynamics. Other thermal mechanisms, such has e.g. thermal diffusion, have been negleted in the model presented next (Pickhardt and Silva, 1998). The model is nonlinear, yet, it is given by a simple formula with only two parameters that relates the output temperature, $T_{out}$, with the input temperature, $T_{in}$, and the solar radiation, $R$:

$$T_{out}(t) = \Upsilon\, T_{in}(t-\tau) + \Gamma \int_{t-\tau}^{t} R(\sigma) d\sigma \qquad (1)$$

The parameter $\Gamma = (D\eta_o)/(\rho A S_f)$ (where $D$ is the mirrors width, $\eta_o$ is the optical efficiency, $A$ is the transversal pipe section area, $S_f$ is the specific thermal capacity of the oil and $\rho$ is the oil density approximated by a constant) has been estimated using real plant data and the same was made for the parameter $\Upsilon$ that takes into account the losses inside the collector.

The input-output travelling time, $\tau$, is obtained from

$$\int_{t-\tau}^{t} F(\sigma) d\sigma = V \qquad (2)$$

where $F(.)$ is the volumetric flow inside the collector and $V$ is the total collector volume.

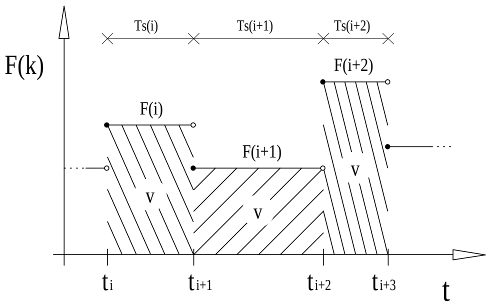


Fig. 3. Relation between the flow and sampling time.

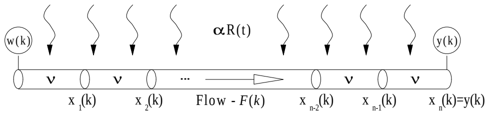


Fig. 4. State temperatures.

2.2 *TIME-SCALED DISCRETIZATION*

Consider again equations (1) and (2) and use elementary volumes instead of elementary time intervals (sampling period) on a discretization procedure. Thus, let us divide de collector volume, $V$, in $n$ smaller equal volumes, $v$, and consider a zero order hold (ZOH) for the flow command since the controller will be implemented in a digital computer. Choose the sampling period, $T_S$, for each discrete time instant, $k$, such that the product of the flow value by the sampling period results in the elementary volume (fig. 3), i.e.

$$v = \frac{V}{n}$$

$$F(k) \times T_s(k) = v \tag{3}$$

Then, equation (2) implies that

$$\sum_{i=1}^{n} T_s(k-i) = \tau_k \tag{4}$$

where $\tau_k$ stands for the I/O transport delay (through volume $V$), in seconds, for the fluid element that is present, at time instant $k$, at the output. The discrete time radiation signal can be computed from the continuous one with a forward average at time $k$ as

$$R(k) = \frac{1}{T_s(k)} \int_{t_k}^{t_{k+1}} R(\sigma) d\sigma \tag{5}$$

Equation (1) represents the I/O model for the complete pipe with volume $V$. This equation can be applied for each of the $n$ volumes $v$ with the variable sampling period (3) yielding, in the discrete time,

$$x_j(k+1) = \beta x_{j-1}(k) + \Gamma R(k) T_s(k)$$

where $\beta$ is a loss parameter such that $\beta^n = \Upsilon$; and $x_i(k)$ is the oil temperature at the output of the subvolume $i$ (see fig. 4) at the (discrete) time instant $k$ . Using (3) this can be written as

$$x_j(k+1) = \beta x_{j-1}(k) + \alpha \frac{R(k)}{F(k)} \quad j = 1 \cdots n \tag{6}$$

$$y(k) = x_n(k) \text{ and } x_0(k) = w(k)$$

. where $\alpha = \Gamma v$ is the solar gain for a subvolume $v$.

Gathering these equations in a state-space frame yields

$$X(k+1) = \underbrace{\begin{bmatrix} 0 \cdots 0 & 0 \\ & 0 \\ \beta I_{n-1} & \vdots \\ & 0 \end{bmatrix}}_{A} X(k)$$

$$+ \underbrace{\begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}}_{B} \underbrace{(\alpha \frac{R(k)}{F(k)})}_{u(k)} + \underbrace{\begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}}_{D} w(k) \tag{7}$$

$$y(k) = \underbrace{[0 \cdots 0 \; 1]}_{C} X(k)$$

where $X(k) = [x_1(k) \cdots x_n(k)]^T$ is the state vector.

The characteristic polynomial of (7) is given by $Q(\lambda) = \lambda^n$, i.e. $n$ eigenvalues placed at the origin resulting from the transport dynamics.

The controlability (for $u$ and $w$ inputs) and observability matrices are given by:

$$\mathcal{C}_B = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 1 & \beta & 0 & \vdots \\ \vdots & \vdots & \ddots & 0 \\ 1 & \beta & \cdots & \beta^{n-1} \end{bmatrix}$$

$$\mathcal{C}_D = \begin{bmatrix} \beta & 0 & \cdots & 0 \\ 0 & \beta^2 & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \cdots & 0 & \beta^n \end{bmatrix}$$

$$\mathcal{O} = \begin{bmatrix} 0 & \cdots & 0 & 1 \\ \vdots & 0 & \beta & 0 \\ 0 & .\cdot^{\cdot} & 0 & \vdots \\ \beta^{n-1} & 0 & \cdots & 0 \end{bmatrix}$$

and thus the system is fully controllable and fully observable.

## 3. CONTROLLER DESIGN

The controller is obtained from the minimization of a quadratic cost in order to compute a state-feedback gain.

Consider the following cost function ($T < n$):

$$J_T = \sum_{i=1}^{T} \{ \tilde{y}^2(k+i) + \rho u^2(k) \} \tag{8}$$

The future values of the output error are given by

$$\tilde{y}(k+i) = CX(k+i) - r(k+i)$$
$$X(k+i) = A^i X(k) + \mathcal{C}_{B_i}[u(k+i-1)\cdots u(k)]^T$$
$$+\mathcal{C}_{D_i}[w(k+i-1)\cdots w(k)]^T \qquad (9)$$

with

$$A^i = \begin{bmatrix} 0\cdots 0 & 0 \\ & 0 \\ \beta^i I_{n-i} & \vdots \\ & 0 \end{bmatrix} \quad i \le n-1$$

$$\mathcal{C}_{B_i} = \underbrace{\begin{bmatrix} 1 & 0 & \cdots & 0 \\ 1 & \beta & \ddots & \vdots \\ 1 & \beta & \ddots & 0 \\ \vdots & \vdots & \ddots & \beta^{i-1} \\ 1 & \beta & \cdots & \beta^{i-1} \end{bmatrix}}_{\#i} \left.\vphantom{\begin{bmatrix}1\\1\\1\\1\\1\end{bmatrix}}\right\} \#n$$

$$\mathcal{C}_{D_i} = \underbrace{\begin{bmatrix} \beta & 0 & \cdots & 0 \\ 0 & \beta^2 & 0 & 0 \\ \vdots & 0 & \ddots & 0 \\ 0 & \cdots & 0 & \beta^i \\ 0 & \cdots & \cdots & 0 \end{bmatrix}}_{\#i} \left.\vphantom{\begin{bmatrix}1\\1\\1\\1\\1\end{bmatrix}}\right\} \#n$$

If the minimization of (8) is made under the assumption of constant control action and constant input temperature over the control horizon, i.e.,

$$u(k) = \cdots = u(k+i-1)$$
$$w(k) = \cdots = w(k+i-1)$$

then the terms of the r.h.s. of (9) are given by:

$$CA^i X(k) = \beta_i \mathbf{e}_{n-i}$$
$$C\mathcal{C}_{B_i}[u(k+i-1)\cdots u(k)]^T = (1+\beta+\cdots+\beta^{i-1})u(k)$$
$$C\mathcal{C}_{D_i}[w(k+i-1)\cdots w(k)]^T = 0 \text{ or } \beta^n w(k) \text{ if } i=n$$

Assuming also a constant reference value along the control horizon the minimization of (8) yields a control action:

$$u(k) = \frac{(\sum_{i=1}^T \lambda_i) r(k) - \sum_{i=1}^T \lambda_i \beta^i x_{n-i}(k)}{\sum_{i=1}^T \lambda_i^2 + \rho} \qquad (10)$$

where $\lambda_i = 1+\beta+\cdots+\beta^{i-1}$. Then, the flow applied to the field is computed from

$$F(k) = \frac{\alpha R(k)}{u(k)}$$

and

$$T_s(k) = \frac{v}{F(k)}$$

It is remarked that the condition of keeping the control action constant along the horizon arises, not only from

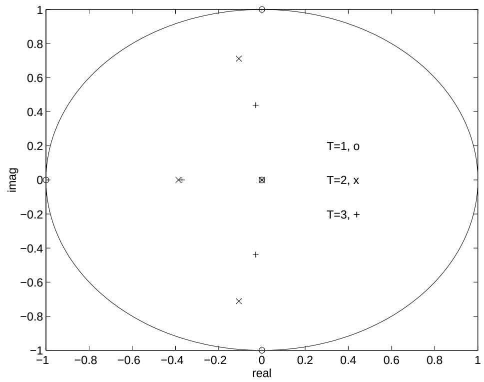


Fig. 5. Controller eigenvalues (n=4).

the simplicity that introduces in the minimization procedure, but also because it should be the best strategy to couple with the existence of anti-resonant modes. Due to the Finite Impulse Response (FIR) structure of the model the best control strategy (assuming constant radiation) should be to apply a step with the adequate amplitude and wait for the new equilibrium point to be set. This strategy, in open-loop would provide a ramp temperature profile inside the collector pipe, with the final temperature equal to the desired one. Now, with a receding horizon strategy only the first control action is applied to the plant and the minimization is repeated over the next time sample.

Since the horizon $T$ is discrete and the sampling period $T_s(k)$ depends on the flow, the value of $T$ establishes, not the time horizon into the future, but the amount of oil inside the collectors that is important to the computation of the control action. This parameter will allow to select the level of "impatience" of the controller. Figure 5 show the distribution of the resulting eigenvalues for $T=1$ to 3 when $n=4$. Note that for $T=1$ the eigenvalues are over the stability limit.

### 3.1 *OBSERVER DESIGN*

In the ACUREX field, the measures of the internal oil temperature are not available. Thus, a state observer has been implemented with observer gain given by

$$L = \left[\frac{1}{n}\ \frac{2}{n}\ \cdots \frac{n}{n}\right]$$

which is equivalent to a progressive linear correction along the pipe. Then, the observer characteristic polynomial is given by

$$P(\lambda) = |\lambda I - A + LC|$$
$$= \lambda^n + \frac{n}{n}\lambda^{n-1} + \frac{n-1}{n}\lambda^{n-2} + \cdots + \frac{1}{n}$$

Figure 6 shows the the distribution of the observer eigenvalues for $n=20$.

It is reminded at this point that the model is based on certain assumptions where all thermal inertia of the oil and metal were left out. In the experimental results to be shown in the next section, the model parameters

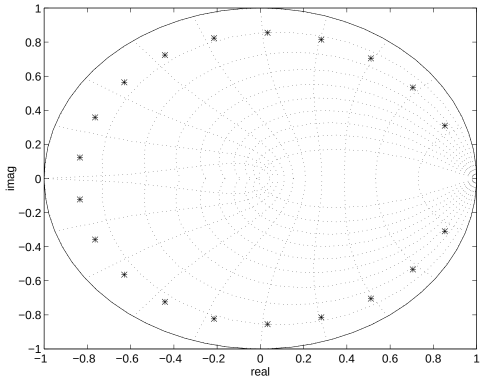


Fig. 6. Observer eigenvalues (n=20).

$\alpha$ and $\beta$ are on-line estimated via Recursive Least Squares (RLS).

## 4. EXPERIMENTAL RESULTS

The following experiments have been made at PSA in June 2001.

From the theory to the real world it is necessary to select the number of collector divisions, $n$. This value will establish, with the range of allowed flows, the sampling period range. The total field volume for the active part is around $1800$ liters. A value of $n = 20$ was chosen in order to get a sampling period between

$$T_{s(min)} \frac{V}{n} \frac{1}{F_{max}} = 9s \quad \text{and} \quad T_{s(max)} \frac{V}{n} \frac{1}{F_{min}} = 45s$$

In previous work with this plant, the authors have knowledge of use, for the (fixed) sampling period, values between $15s$ and $39s$.

**Experiment 1 (28-06-2001)**

This experiment was performed with $T = 8$, and gives emphasis to the ability of the time scaled predictive controller to make set-point temperature changes of $50°$C overcoming the non linearity that results from the sudden change in the flow value. In fig. 7 the output temperature is presented with its reference on the top plot and it is possible to observe the flow signal on the bottom one. Fig. 8 show the solar radiation (top) and the input temperature (bottom).

It can be seen from the plots that there are oscillations in the flow value which are not present at the output. This is due to the antiressonant zeros cancellation by the controller poles.

**Experiment 2 (29-06-2001)**

In this experiment, in order to reduce the effect of the pole-zero cancellation the control horizon has been increased to $T = 16$. In fig. 9 the output temperature is presented with the flow signal. Solar radiation and input temperature can be seen in fig. 10.

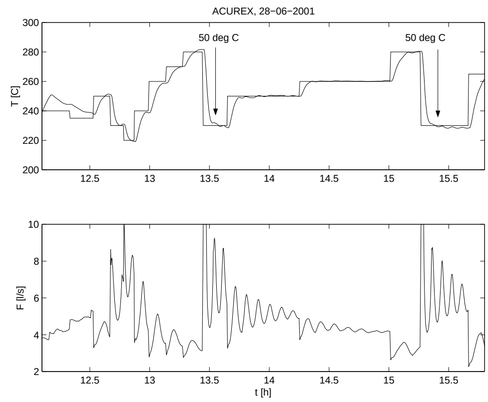


Fig. 7. Exp. 1 Output temperature and reference (top); and field flow (bottom).

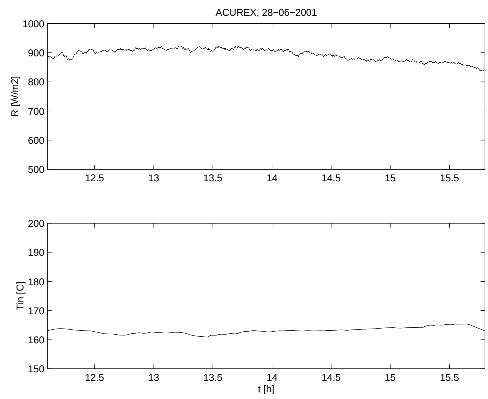


Fig. 8. Exp. 1. Solar radiation (top); and input temperature (bottom).

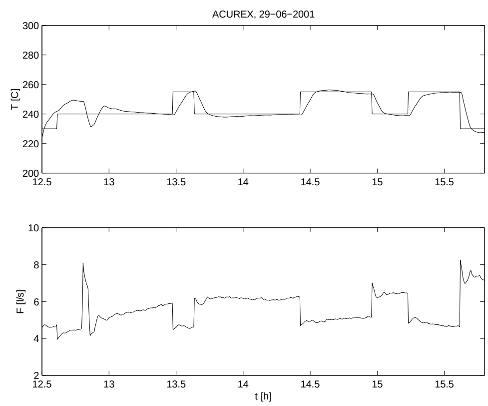


Fig. 9. Exp. 2 Output temperature and reference (top); and field flow (bottom).

## 5. CONCLUSIONS

A time scaled predictive controller has been developed for the special case of a solar power plant. The algorithm takes advantage of the non-linear structure of the plant reflecting the transport mechanisms inside the collectors to obtain, via a time scale non-linear transformation, a linear structure in discrete time. This transformation implies varying the sampling interval according to the value of the manipulated variable, the flow. The algorithm has been applied to the real plant and the results were presented in this paper.

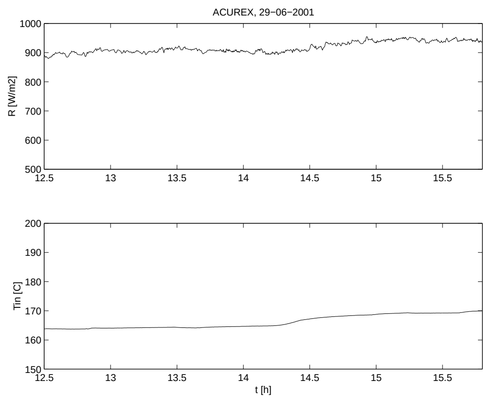


Fig. 10. Exp. 2. Solar radiation (top); and input temperature (bottom).

This concept is not restricted to this particular field (the ACUREX). Any solar field using this concept of manipulating flow to change the residence time inside the collector can overcome the time constants dependence on flow with this kind of transformation.

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