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SUPPRESSION OF LOAD OSCILLATIONS IN PRECISION SERVOMECHANISMS SENSING ONLY MOTOR POSITION

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Abstract: An analysis of the two-mass model of a servomechanism is first presented in this paper. The analysis stands as a basis for the design of a P/PI controller where suppression of load oscillations, rather than fast setpoint tracking of the motor position is pursued. Further improvements of the load behavior can be achieved by a notch filter placed outside the velocity loop. Simulation results are given to assess the effectiveness of the proposed approach.

Keywords: Servomechanisms, Elastic joint, PID control, Notch filter.

1. INTRODUCTION

Positioning servomechanisms are used in a large number of applications in robotics, machine tools, packaging, printing and textile machines, entertainment products, computer peripherals and devices, space and defense pointing, motion systems and other applications. The large majority of servos use permanent magnets motors connected to the load by a transmission chain (or gearbox), and a single position sensor, either an encoder or a resolver, mounted on the motor shaft. This is by far the most common solution adopted in articulated robotic manipulators.

By feeding back the motor position only, it is relatively easy to obtain satisfactory control of the motor position and velocity, using either P/PI or PID control, or more advanced compensation techniques (Lee and Tomizuka, 1996; Ohnishi, 1989; Umeno *et al.*, 1993; Umeno and Hori, 1991; Yao *et al.*, 1997). However, this does not guarantee a satisfactory control of the load position and velocity, for demanding motion control applications, in particular during slow motion. For instance, oscillations of the tip of a manipulator arm or of the tool of a milling machine may arise. A large variety of sources of oscillation can be identified: motor (Ferretti *et al.*, 1998; Holtz and Springob, 1996; Jahns and Soong, 1996) and gearbox (Godler *et al.*, 1994) pulsating torque disturbances, torsional elasticity of the transmission chain, sensor noise (Hanselman, 1991), friction (Armstrong-Hélouvry *et al.*, 1994), backlash, and others. Among them, special attention should be paid to the transmission elasticity, which generally causes the lowest ("first") resonance frequency of the positioning system.

In the review work (Ohnishi *et al.*, 1996) the disturbance observer technique, already presented in (Umeno *et al.*, 1993) and (Umeno and Hori, 1991), is used to control a rigid servo affected by an unknown load torque, while the resonance ratio control is used when an elastic transmission is considered. The disturbance observer is also used in (Lee and Tomizuka, 1996; Yao *et al.*, 1997), where torsional flexibility of the joint is not directly accounted for. A review of several works dealing with suppression of torsional oscillations is reported in (Vukosavic and Stojic, 1998), where the contributions are divided in three categories: 1) methods that exploit measurements on both the motor and the load sides; 2) methods based on the motor measurements only and observation of the state of the system; 3) notch filters plus conventional loopshaping techniques. The first category include most of the works where nonlinear control theory is used: singular perturbation and integral manifold theory is used in (Marino and Spong, 1986) for a single joint case and in (Spong, 1987) for the extension to a complete articulated manipulator; feedback linearization is used in (Forrest-Barlach and Babcock, 1986); nonlinear observer theory in (Tomei, 1990); adaptive control theory in (Lozano and Brogliato, 1992). The second group includes more industry-oriented works, such as (Lorenz and Patten, 1991) and (Schmidt and Lorenz, 1992), as well as the early work (Nicosia and Tomei, 1991), where a PD controller on the motor coordinate, enforced with a compensation of the gravitational effect, is considered. In (Ferretti *et al.*, 2001) different strategies for model based control of the system are compared. Finally, the analysis in (Vukosavic and Stojic, 1998) mostly belongs to the third group.

As a matter of fact, load vibrations can be reduced by a proper design of the classical P/PI controller (P on the position, PI on the velocity) , based on a careful analysis of the properties of the two-mass model of the elastic system. As a consequence of the relative position of poles and zeros of the process transfer function, it is straightforward to show, using the root locus analysis, that the common practice of increasing the bandwidth of the velocity loop (high gain of the PI regulator) fosters the oscillation of the load. It is then suggested a way to choose the loop gain, trading off between higher values, for a fast velocity response, and lower values, for more damped oscillations of the load. Further improvement of the damping of the closed loop dominant poles can also be obtained with a proper tuning of a notch filter that, however, has to be placed outside the velocity loop.

The properties of the two-mass model are discussed in Section 2. The effects on load oscillations of the design of a classical P/PI control closed on the motor position are studied in Section 3, and applied to an illustrative example (a prototype servo) in Section 4. The use of a notch filter outside the velocity loop is discussed in Section 5, while concluding remarks and future directions are given in Section 6.

2. MODEL OF AN ELASTIC SERVO MECHANISM

A fast dynamics servo may have several flexible elements and connections, resulting in a dynamic behavior affected by several resonances. From the point of view of the control design, it is essential to model correctly the lowest frequency resonance,

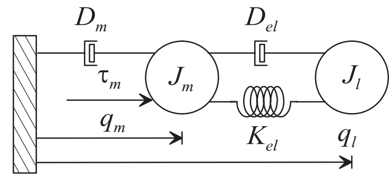

Fig. 1. Elastic servo positioning system

which plays an essential role in the design, as it is shown later on. Under certain conditions that are frequently satisfied, the first resonance frequency is correctly predicted by the well known model of two masses connected through an elastic element (Fig. 1).

The model is described by the following linear equations (Spong, 1987; Vukosavic and Stojic, 1998), if the non linear friction terms are neglected:

$$\tau_m = K_t I \tag{1}$$

$$\tau_m = J_m \ddot{q}_m + D_m \dot{q}_m + \tau_t \tag{2}$$

$$\tau_t = K_{el}\left(q_m - n q_l\right) + D_{el}\left(\dot{q}_m - n\dot{q}_l\right) \tag{3}$$

$$n\tau_t = J_l \ddot{q}_l \tag{4}$$

where τ_m is the motor torque, I is the motor current (actually the amplitude of the sinusoidal current of each stator phase of the brushless motor); K_t is the motor torque constant, q_l and q_m are the load and motor angular positions, respectively; J_l, J_m are the load and rotor inertias, respectively; D_m is the viscous friction coefficient at the motor side; K_{el}, D_{el}, are the joint stiffness and damping factors, respectively; n is the transmission ratio; $n\tau_t$ is the torque delivered by the gear reducer at the load side. Furthermore, thanks to the fast dynamics current amplifiers, the difference between the current command $\bar{I}$ (control variable) and the actual current I can be neglected ($\bar{I} = I$) for the purpose of position control design. Pulsating torque disturbances act as additive, variable frequency disturbances to be rejected by the control system. The transfer function $G_{vm}(s)$ from the motor torque to the motor velocity is then given by:

$$G_{vm}(s) = \frac{J_{lr}s^2 + D_{el}s + K_{el}}{\Delta(s)} \tag{5}$$

with

$$\Delta(s) = J_{lr}J_m s^3 + \left[\left(J_m + J_{lr}\right)D_{el} + J_{lr}D_m\right]s^2 + \\ + \left[\left(J_m + J_{lr}\right)K_{el} + D_m D_{el}\right]s + D_m K_{el}$$

where $J_{lr} = J_l/n^2$ is the load inertia referred to the motor side.

The polynomial $\Delta(s)$ does not have a simple factorization but, for common values of the system parameters, it shows a couple of complex roots whose natural frequency and damping factor can be approximately obtained letting $D_m = 0$:

$$\omega_p \cong \sqrt{\frac{J_{eff} K_{el}}{J_{lr} J_m}} \qquad \xi_p \cong \frac{1}{2} D_{el} \sqrt{\frac{J_{eff}}{J_{lr} J_m K_{el}}}$$

where $J_{eff} = J_m + J_{lr}$. The third pole s_r is in a lower frequency range, as it is strongly related to the rigid behavior of the system ($s_r \cong -D_m/J_{eff}$).

The zeros of the transfer function are complex too, with natural frequency and damping factor given by:

$$\omega_z = \sqrt{\frac{K_{el}}{J_{lr}}} \qquad \xi_z = \frac{1}{2} D_{el} \sqrt{\frac{1}{J_{lr} K_{el}}}$$

It is worth noting that

$$\frac{\omega_p}{\omega_z} = \frac{\xi_p}{\xi_z} = \sqrt{1+\rho} > 1$$

$\rho = J_{lr}/J_m$ being the inertia ratio. Motor inertia and transmission ratio are frequently selected to pursue the inertia matching, namely to obtain $\rho = 1$.

3. CLASSICAL CONTROL AND LOAD OSCILLATIONS

The control system requirements concern setpoint tracking and rejection of load and torque disturbances. Both of them demand large control system bandwidth, namely large feedback loop gain. However, a critical problem, especially during slow motion and at motion starts and stops, is due to load oscillations. For instance, load oscillations may reveal as vibrations of the end-effector of a robot arm during the execution of a slow motion, like in arc welding tasks, while in milling and grinding machines they may cause some slight undulation, and thus poor finishing, of working surfaces. Since the analysis of the properties of the elastic model shows that high feedback gains on motor position increase load oscillations, a trade off is needed on the loop gain.

In most commercial products, position control (Fig. 2) consists of a position loop with a proportional (P) regulator, implemented for instance in a CNC, cascaded with a velocity loop with proportional-integral (PI) regulator, implemented in the drive electronics. The integral action ensures that the error on motor position for constant setpoint, load torque and torque disturbances (τ_d), vanishes at steady-state. While in the past the velocity was sensed by a tachometer, in current products it is obtained by numerical differentiation of the motor position, sensed by either an encoder or a resolver. To improve the setpoint tracking capabilities, a feedforward derivative action is also inserted from the position setpoint to the velocity one.

It is easy to check that this control scheme is equivalent to a PID controller with real zeros, fed with the motor position error $e_p = \bar{q}_m - q_m$ and with output $\bar{I}$.

Note that in the block diagram of Fig. 2 a block has been added, with transfer function:

$$G_{lm}(s) = \frac{1}{n} \frac{D_{el} s + K_{el}}{J_{lr} s^2 + D_{el} s + K_{el}}, \qquad (6)$$

which allows computation of q_l from q_m.

The velocity loop is designed first. The loop transfer function is:

$$L_v(s) = K_{cv} K_t J_{lr} \frac{(s + 1/T_{iv})(s^2 + 2\xi_z \omega_z s + \omega_z^2)}{s\Delta(s)}$$

and the closed loop transfer function from setpoint to motor velocity is:

$$F_v(s) = K_{cv} K_t J_{lr} \frac{(s + 1/T_{iv})(s^2 + 2\xi_z \omega_z s + \omega_z^2)}{\Delta_v(s)}$$

where $\Delta_v(s)$ is defined as

$$\begin{aligned} \Delta_v(s) = J_{lr} J_m s^4 &+ \begin{bmatrix} (J_m + J_{lr}) D_{el} + \\ J_{lr} D_m + K_{cv} K_t J_{lr} \end{bmatrix} s^3 + \\ &+ \begin{bmatrix} (J_m + J_{lr}) K_{el} + D_m D_{el} + \\ K_{cv} K_t J_{lr} (2\xi_z \omega_z + 1/T_{iv}) \end{bmatrix} s^2 + \\ &+ \begin{bmatrix} D_m K_{el} + \\ K_{cv} K_t J_{lr} (\omega_z^2 + 2\xi_z \omega_z / T_{iv}) \end{bmatrix} s + \\ &+ K_{cv} K_t J_{lr} \omega_z^2 / T_{iv} \end{aligned}$$

For $K_{cv} \to \infty$ the system will be stable, whatever $T_{iv} > 0$, but two poles approach the lightly damped process complex zeros. In this case, also two roots of the characteristic polynomial of the closed position loop, given by:

$$\begin{aligned} \Delta_p(s) = s\Delta_v(s) + K_{cp} K_{cv} K_t J_{lr} (s + 1/T_{iv}) \cdot \\ (s^2 + 2\xi_z \omega_z s + \omega_z^2) \end{aligned}$$

become the process zeros, whatever $K_{cp} > 0$. On the other hand, for smaller values of K_{cv} two of the position loop poles become the process zeros only for $K_{cp} \to \infty$.

If two closed loop poles are equal (or almost equal) to the process zeros, the relevant dynamics is not observable from the motor position, but it reveals with oscillations of the load, because of the poles of $G_{lm}(s)$ (6).

As a conclusion, the controller design requires to trade off between higher gains (in particular K_{cv}), that increase setpoint tracking and torque

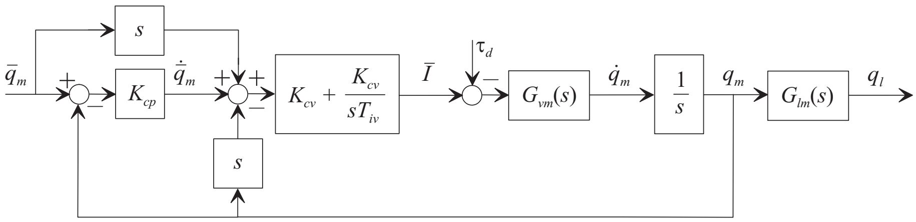

Fig. 2. P/PI control with feedforward action

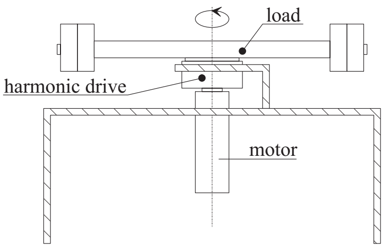

Fig. 3. Prototype positioning system

K_t	J_m	J_l	n	D_m
1.6	1.5×10^{-4}	2.7	100	3.4×10^{-3}

K_{el}	D_{el}
3.05	2.2×10^{-3}

ω_z	ξ_z	ω_p	ξ_p
106.3	3.8×10^{-2}	177.5	$0.105 \div 0.064$

$\lvert s_r \rvert$	T_z
8.13	7.2×10^{-4}

Table 1. Physical parameters

disturbance rejection capabilities, and lower gains, needed to keep the damping of closed loop poles reasonably high.

In this respect, the common industrial practice of tuning the velocity loop as fast as possible, increasing the gain K_{cv} until an "audible noise" is generated by the motor, proves to be quite dangerous. In fact, this is exactly the way to induce load oscillations, as the high gain velocity feedback places two closed loop poles near the lightly damped process zeros, where they remain also after closing the position loop.

4. AN ILLUSTRATIVE EXAMPLE

The prototype servo positioning system of Fig. 3 is considered as an illustrative example.

It consists of a Control Techniques permanent magnet AC brushless motor, with a nominal torque of 2.3 Nm and a nominal power of 700 W, a Harmonic Drive speed reducer, and a load. Since the load rotates in a horizontal plane the gravity force does not act on the system. The parameters of the testbed are given in Table 4 (in SI units).

Figures 4.a and 4.b show the root loci of the position loop at varying K_{cp}, obtained with $T_{iv} = 1/|s_r|$ (low frequency zero-pole cancellation) and with two different choices of K_{cv}, respectively

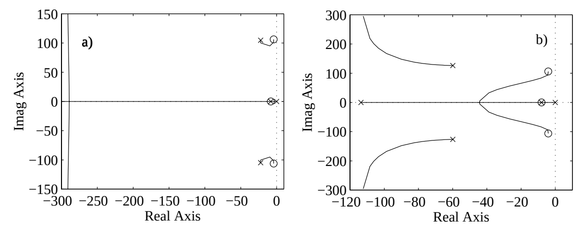

Fig. 4. a) Root locus for $K_{cv} = 5.2 \times 10^{-2}$; b) Root locus for $K_{cv} = 1.84 \times 10^{-2}$

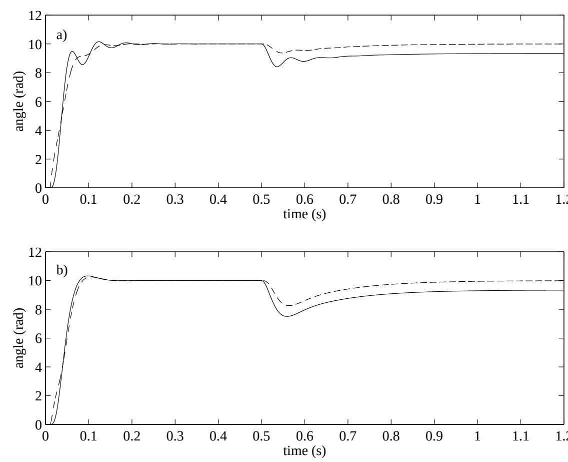

Fig. 5. Time responses of q_m (dashed line) and nq_l (solid line) for a) $K_{cv} = 5.2 \times 10^-2$ and b) $K_{cv} = 1.84 \times 10^-2$

$K_{cv} = 5.2 \times 10^{-2}$, corresponding to a fast but poorly damped velocity loop, and $K_{cv} = 1.84 \times 10^{-2}$, corresponding to a slower but well damped loop. In the first case, two poles of the position closed loop are very close to the process zeros, even for low values of K_{cp}, while in the second case the poles starting from the complex poles of the velocity closed loop go toward infinite. The value of K_{cv} discriminating this behavior, in this example, has been found to be $K_{cv} = 2.4 \times 10^{-2}$, corresponding to $\omega_{cv} = 93.78$, i.e. $\omega_{cv} = 0.52\omega_p = 0.88\omega_z$, which resembles the well known rule of thumb for the choice of ω_{cv}, namely $\omega_{cv} \leq 0.5\omega_p$.

The time responses of q_m and nq_l to steps in setpoint and torque disturbances for both cases, computed with $K_{cp} = 30$ (i.e. approximately $\omega_{cp} = 30$) and without the feedforward derivative action, are given in Figs. 5.a and 5.b respectively. The response to torque disturbances is dominated by the real pole cancelled by the zero of the PI regulator. If the cancellation is avoided, this mode can be made faster. The disturbance rejection is stronger for $K_{cv} = 5.2 \times 10^{-2}$ but the load behavior is more oscillatory.

5. ON THE USE OF THE NOTCH FILTER

Commercial drives frequently enhance the P/PI control scheme using a notch filter, namely a filter with transfer function:

$$G_{nf}(s) = \mu \frac{s^2 + 2\xi_n \omega_n s + \omega_n^2}{s^2 + 2\xi_d \omega_d s + \omega_d^2}$$

Sometimes the filter is inserted within the velocity loop, in series to the PI, sometimes outside. In principle, the filter can be used to avoid the excitation of oscillatory modes through the control variable. However, in industry there are controversial opinions about the effectiveness of the notch filter and on the best way to exploit it. Frequently, people rely on the notch filter to counteract oscillations which cannot get rid of otherwise. It is also suggested to use the notch filter to cancel the open loop complex poles and replace them with more damped poles (Vukosavic and Stojic, 1998). Precise cancellation however is difficult because of uncertainties on the servo system parameters, especially the damping coefficient. Moreover, poles are cancelled in the transfer functions involving the setpoint, but they remain unchanged in the closed loop and may be excited by other inputs, especially by torque disturbances.

Following the above root locus analysis, it is suggested in this contribution to place the filter outside the velocity loop and to select its zeros so as to cancel the poles of the velocity loop. The position loop root locus, for the case $K_{cv} = 5.2 \times 10^{-2}$, becomes as in Fig. 6.

The closed loop dominant poles, again with $K_{cp} = 30$, are now well damped, even if the poorly damped poles of the velocity loop are still observable in the response to torque disturbances. As a result, the step responses of q_m and nq_l are as in Fig. 7. There is a clear improvement in the response to the setpoint (the overshoot is due to the step input, which must be avoided) and a smaller one in that to the torque disturbance, whose dynamics is dominated by the cancelled poles.

As a conclusion, designing jointly the velocity regulator and the notch filter, a good trade off can be obtained between velocity loop bandwidth and damping of the closed loop dominant poles. The velocity loop gain K_{cv} should be first chosen to get a desired damping of the velocity loop poles. Then, these poles are cancelled (i.e., blocked) by the notch filter. Finally, the position loop gain K_{cp}

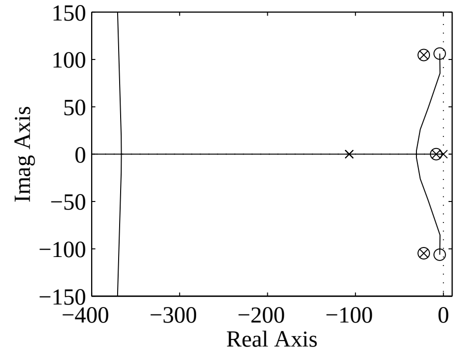

Fig. 6. Root locus of the position loop with P/PI plus notch filter

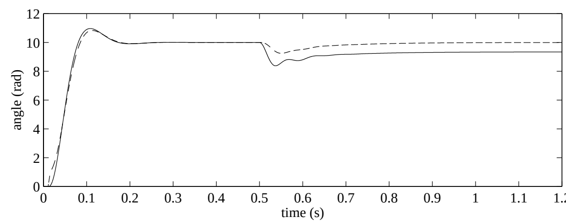

Fig. 7. Time responses of q_m (dashed line) and nq_l (solid line) for $K_{cv} = 5.2 \times 10^{-}2$ with the notch filter

is selected to achieve the desired damping of the dominant poles of the servo system.

The filter design proved also to be robust to errors in model parameters, since the notch filter zeros cancel out closed loop poles (the velocity loop ones), which are less sensitive to process parameter variations, than the open loop ones.

6. CONCLUSIONS AND FUTURE DIRECTIONS

Modelling and control design of an elastic servo positioning drive have been discussed in this paper. By feeding back only the motor position it is relatively easy to obtain satisfactory control of the motor position and velocity. Nevertheless this does not guarantee a satisfactory control of the load position and velocity for demanding motion control applications. This happens especially during slow motion, when the frequencies of the pulsating disturbances generated by the motor and by the gear reducer match, and thus the servo structural resonance is excited.

It has been shown how to design the widely used P/PI control to achieve a balanced trade off between a fast motor response and a damped behavior of the load position. This helps matching the strong requirement of a smooth, non oscillating, slow velocity motion, typical of robotics, machine tools and other applications, but also points out the limits of P/PI control. P/PI control happens to be rather robust with respect to system parameter variations but its performance may not be fully satisfactory in those applications where the load velocity ripple is a major problem. To this purpose a new way to exploit a notch filter has been proposed, that not only prevents harmonic signals (at the notch frequency) from entering the velocity loop but also improves the damping of the closed loop (dominant) poles.

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