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BACKSTEPPING CONTROL OF SATURATED INDUCTION MOTORS

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Abstract: An extensive research activity have been devoted to the problem of induction motor control over the last decade. Most of the proposed controllers have been designed under the assumption that the motor is unsaturated. As this is not the case in realistic situations the expected performances, especially during transient periods, may not be achieved. In this paper, an adequate model that rigorously accounts for the saturation feature in the AC machine with uniform air-gap. Then, a controller is designed using the backstepping technique combined with the usual flux orientation. The obtained controller is shown to meet its objectives, namely motor regional stabilization, reference speed tracking and reference flux regulation.

Keywords: Induction motor modelling, flux saturation, backstepping control

1. INTRODUCTION

Induction motor control has been dealt with following different approaches. These includes simple linear techniques such as field oriented control (Vas, 1986) and more involved nonlinear techniques like input-output linearization (Chiasson, 1995)-(Morino and Tomei, 1995), backsteping (J. Hu and Qu, 1996)-(M. Krstic and P. Kokotovic John Wiley, 1996), (Tan and Chang, 1999), passivity (R. Ortega and Espinosa-Perez, n.d.), sliding mode (J. Hu and Q, 1994). The proposed controllers achieved speed tracking and flux regulation for unsaturated induction motors. Indeed, these controllers are designed under the ideal assumption that the magnetic characteristic is linear. To not violate this assumption, the user have to choose for the flux a low reference value, which limits the achievable motor couple. Furthermore, ignoring the saturation feature may lead to a deterioration of the control performances during transient periods.

This paper focuses on the saturation problem in the induction motor control. The crucial issues are:

i) how the saturation phenomenon is accounted for in the machine model?

ii) how the saturation phenomenon is deal with in the control design?

In (Sullivan and Sanders, 1992) a π-model of the magnetic circuit is used rather than the standard T-model. Furthermore, a current-based control is suggested rather than the usual voltage-based control. Therefore, the pratical implementation of the resulting regulator necessitates a current DC-AC converter with all the related shortcomings (e.g. torque harmonics are generated). These shortcomings are currently avoided using voltage-based control coupled with PWM DC-AC converters. From a theoritical viewpoint, there is no formal analysis of the achieved performances. In (Heinmann and W.Leonhard, 1990), the starting point is the *standard unsaturated model*. This is further simplified by ignoring the current dynamics (which reduces the whole model order) and supposing the stator current i_{sd} and i_{sq} to be equal to their reference values i_{dref} and i_{qref}. Moreover, the analysis presented in (Heinmann and W.Leonhard, 1990) lies upon the crucial assumption that the rotor magnetic flux Φ_{rd} is a only a function of the current i_{sd}. This assumption is not realistic because it ignores the cross-saturation phenomenon; according to this Φ_{rd} is a function of both the stator current (i_{sd}, i_{sq}) and the rotor currents (i_{rd}, i_{rq}).

This paper presents a more deep investigation of the control problem for the saturated induction motor. The controller design includes two major parts. The first one consists in building up a model that accounts appropriately for the flux saturation. In this respect, it is well known that a coupling exists between both axes of an AC-machine, even in the case of a uniform air-gap machine, (Vas, 1986). Such a phenomenon which is known as cross-saturation, is caused by the nonlinear properties of the magntitude materials. As suggested in (M.S. Garrido and Dejaeger, 1988)-(Roberts, 1988), the magnetic characteristic is approximated in this paper, by a *nonlinear function* (polynomial, exponential, arctangent, ...) of the *magnetizing current* i_μ (which includes the contribution of both the stator and rotor currents). Futhermore, contrarily to (Heinmann and W.Leonhard, 1990), the control model is not derived from the standard unsaturated model. It is rather developed from the more rigorous matricial model due to Von Der Embse (**?**). The resulting model turns out to be nonlinear and involves parameters that depend on the system states. The second part in the controller design consists in deriving an appropriate control law based on the saturated machine model. In the present paper, such a control law is obtained using the backstepping design technique combined with the usual field orientation. The state-dependent parameters of the nonlinear control model are computed on-line. It is shown that the resulting closed-loop control system is locally stable with a well characterized attraction region. In addition, speed reference tracking and flux regulation are ensured. To the authors knowledge, it is the first time that such results are achieved for saturated induction motors.

2. MODELIZATION OF THE SATURATION PHENOMENON IN AC MACHINE

The magnetic state of the isotropic saturated sinusoidal machine is completely described by a single magnetic characteristic that relates the flux Φ to the magnetizing current i_μ defined by:

$$i_\mu = i_s + k.i_r \tag{1}$$

where $k = k_r.n_r/k_s.n_s$. Note that such a current includes the contribution to the flux Φ of both the stator and the rotor currents. Let us denote $i_{\mu d}$ and $i_{\mu q}$ the components of i_μ along the dq axes, i.e.

$$i_{\mu d} = i_{sd} + k.i_{rd}\ , \quad i_{\mu q} = i_{sq} + k.i_{rq}\ , \; i_\mu^2 = i_{\mu d}^2 + i_{\mu q}^2 \tag{2}$$

The magnetic characteristic has the general form of figure 1 [1]. It may be approximated using usual mathematical functions such as arctangent, exponential or polynomial. In practice, one can determines points of this characteristic provided that the machine is uncharged and controlled by a varying voltage. Actually, in such operating conditions the magnetizing current i_μ and the flux Φ can be precisely determined by measuring of the active and reactive power

2.1 *Flux equations*

In (M.S. Garrido and Dejaeger, 1988)-(Embse, 1968) it is shown that:

$$\Phi_s = \Phi_{leaking/st} + k_s n_s \Phi \tag{3}$$
$$= l_s i_s + k_s n_s \Phi = l_s.i_s + \lambda_s(i_\mu) \tag{4}$$

The nonlinearity of the magnetic characteristic $f(.)$ gives rise to a coupling between both axes of the AC machine, this is the cross-saturation phenomenon. Consequently, it follows from (4) that (M.S. Garrido and Dejaeger, 1988)-(Roberts, 1988):

$$\Phi_{sd} = l_s.i_{sd} + M_d.i_{\mu d} + M_{dq}.i_{\mu q} + \phi_{d0} \tag{5}$$

where:

. $l_s.i_{sd}$ represents the leakage flux in the stator along the d-axis,

. $M_d.i_{\mu d}$ represents the flux resulting from the stator proper flux along the d-axis together with the mutual flux between the stator d-axis and the rotor d-axis,

. $M_{dq}.i_{\mu q}$ represents the mutual flux between the stator/rotor q-axes, in one hand, and the stator d-axis, in the other hand,

. ϕ_{d0} is an additional term introduced in (M.S. Garrido and Dejaeger, 1988) to define M_d and M_{dq} as being respectively: $\frac{\partial \lambda_{sd}}{\partial i_{\mu d}}$ and $\frac{\partial \lambda_{sd}}{\partial i_{\mu q}}$. Therefore, ϕ_{d0} undergoes the equations:

$$\frac{\partial \phi_{d0}}{\partial i_{\mu d}} = -\frac{\partial M_d}{\partial i_{\mu d}}.i_{\mu d} - \frac{\partial M_{dq}}{\partial i_{\mu d}} i_{\mu q} \tag{6}$$
$$\frac{\partial \phi_{d0}}{\partial i_{\mu q}} = -\frac{\partial M_d}{\partial i_{\mu q}}.i_{\mu d} - \frac{\partial M_{dq}}{\partial i_{\mu q}} i_{\mu q} \tag{7}$$

Similarly, one establishes:

[1] All figures have been omitted. They will be presented in the conference.

$$\Phi_{sq} = l_s.i_{sq} + k_s n_s \Phi_q \quad (8)$$
$$= l_s.i_{sq} + M_q.i_{\mu q} + M_{dq}.i_{\mu d} + \phi_{q0} \quad (9)$$
$$\Phi_{rd} = l_r.i_{rd} + k_r n_r \Phi_d \quad (10)$$
$$= l_r.i_{rd} + k.(M_d.i_{\mu d} + M_{dq}.i_{\mu q} + \phi_{d0}) \quad (11)$$
$$\Phi_{rq} = l_r.i_{rq} + k_r n_r \Phi_q \quad (12)$$
$$= l_r.i_{rq} + k.(M_q.i_{\mu q} + M_{dq}.i_{\mu d} + \phi_{q0}) \quad (13)$$

with ϕ_{q0} satifying the following equations:

$$\frac{\partial \phi_{q0}}{\partial i \mu q} = -\frac{\partial M_q}{\partial i_{\mu q}}.i_{\mu q} - \frac{\partial M_{dq}}{\partial i_{\mu q}} i_{\mu d} \quad (14)$$
$$\frac{\partial \phi_{d0}}{\partial i \mu d} = -\frac{\partial M_q}{\partial i_{\mu d}}.i_{\mu q} - \frac{\partial M_{dq}}{\partial i_{\mu d}} i_{\mu d} \quad (15)$$

2.2 *Determination of the varying machine parameters*

2.2.1. *Static magnetization parameter* Following (M.S. Garrido and Dejaeger, 1988), let m denote the static magnetization parameter:

$$m = k_s.n_s.\frac{\Phi}{i_\mu} \quad (16)$$

In fact, such a parameter characterizes the operating point on the magnetic characteristic since:

$$k_s.n_s.\Phi = m.i_\mu \quad \Rightarrow \left\{ \begin{array}{l} k_s.n_s.\Phi_d = m.i_{\mu d} \\ k_s.n_s.\Phi_q = m.i_{\mu q} \end{array} \right\} \quad (17)$$

Using (4), one gets from (17) that:

$$m = \frac{k_s.n_s.\Phi_q}{i_{\mu q}} = \frac{\Phi_{sq} - l_s i_{sq}}{i_{\mu q}} \quad (18)$$

This shows that the parameter m becomes measurable, if the following assumptions hold:

H1: The rotor and stator fluxes are measurable.

H2: The coefficients k, l_s, l_r, R_r, R_s are known (as these can be determined performing off-line experiments)

Indeed, it readily follows from H1 and H2 that, except for $i_{\mu q}$, all terms on the right side of (18) are measurable. Furthermore, $(2, 4)$ yield:

$$i_{\mu q} = i_{sq} + k.i_{rq} = i_{sq} + \frac{k}{l_r}.\left(\Phi_{rq} - k.(\Phi_{sq} - l_s.i_{sq})\right) \quad (19)$$

which, together with (18), implies:

$$m = \frac{\Phi_{sq} - l_s i_{sq}}{i_{sq}\left(1 + \frac{k^2 l_s}{l_r}\right) + \frac{k}{l_r}.\left(\Phi_{rq} - k.\Phi_{sq}\right)}$$

where all terms on the right side are measurable.

2.2.2. *Induction coefficients* All the useful induction coefficients can now be expressed in term of the the parameter m:

$$M_d = \frac{\partial \lambda_{sd}}{\partial i_{\mu d}} = m + \frac{\partial m}{\partial i_\mu}.\frac{i_{\mu d}^2}{i_\mu} \quad (20)$$
$$M_{dq} = \frac{\partial \lambda_{sd}}{\partial i_{\mu q}} = m + \frac{\partial m}{\partial i_\mu}.\frac{i_{\mu d}.i_{\mu d}}{i_\mu} \quad (21)$$
$$M_q = \frac{\partial \lambda_{sq}}{\partial i_{\mu q}} = m + \frac{\partial m}{\partial i_\mu}.\frac{i_{\mu q}^2}{i_\mu} \quad (22)$$

with:

$$\frac{\partial m}{\partial i_\mu} = k_s n_s.\frac{i_\mu.\frac{df(i_\mu)}{di_\mu} - f(i_\mu)}{i_\mu^2}$$

3. CONTROL MODEL FOR THE SATURATED AC MACHINE

The machine state vector is chosen to be $[\Phi_{rd}, i_{\mu d}, i_{\mu q}, \omega]$. The common alternative is $[\Phi_{rd}, i_{sd}, i_{sq}, \omega]$. The former choice will prove to be convenient in arriving to a simple model.

3.1 *Rotor electrical equation in the dq-axes*

$$0 = R_r.i_{rd} + \frac{d\Phi_{rd}}{dt} \quad (23)$$

On the other hand, one gets from (10) and (17) that:

$$i_{rd} = \frac{1}{l_r}.(\Phi_{rd} - k.m.i_{\mu d})$$

which together with (23) yields:

$$\frac{d\Phi_{rd}}{dt} = \frac{-R_r}{l_r}\Phi_{rd} + \frac{R_r}{l_r}k.m.i_{\mu d} \quad (24)$$

3.2 *Stator electrical equation in the dq-axes*

$$V_{sd} = R_s i_{sd} + \frac{d\Phi_{sd}}{dt} - \omega_s \Phi_{sq} \quad (25)$$
$$V_{sq} = R_s i_{sq} + \frac{d\Phi_{sq}}{dt} + \omega_s \Phi_{sd} \quad (26)$$

After several transformations operated on (26), and involving most of equations in section 2, one gets:

$$V_{sq} = (b_1 + a\acute{b}_1\omega_s).i_{\mu q} + (ab_2 + \acute{b}_1\omega_s).i_{\mu d} + b_3.\frac{di_{\mu d}}{dt}$$
$$+ (ab_4 + \acute{b}_4\omega_s).\Phi_{rd} + a.V_{sd} \quad (27)$$

with: $b_1 = R_s\left(1 + \frac{k^2 m}{l_r}\right)$; $\acute{b}_1 = l_s + m + \frac{k^2 l_s m}{l_r}$; $b_2 = -R_s\left(1 + \frac{k^2 m}{l_r}\right) + \frac{k^2 l_s m R_r}{l_r^2}$; $b_3 =$

$M_{dq}\left(1+\frac{k^2 l_s}{l_r}\right)+a.\acute{b}_3$; $a=\frac{l_s+M_q\left(1+\frac{k^2 l_s}{l_r}\right)}{M_{dq}\left(1+\frac{k^2 l_s}{l_r}\right)}$; $\acute{b}_3=-l_s-M_d\left(1+\frac{k^2 l_s}{l_r}\right)$; $b_4=\frac{R_s.k}{l_r}-\frac{k l_s R_r}{l_r^2}$; $\acute{b}_4=-\frac{k l_s}{l_r}$ and:

$$\omega_s=\omega+\frac{R_r}{\Phi_{rd}}\frac{km}{l_r}i_{\mu q} \quad (28)$$

It is worth noticing that equation (28) is introduced to enforce the orientation of the rotor flux along the d-axis; so doing its q-component turns out to be null.

From (27), it readily follows that $i_{\mu d}$ undergoes the following differential equation:

$$\frac{di_{\mu d}}{dt}=-\frac{b_1+a\acute{b}_1\omega_s}{b_3}.i_{\mu q}-\frac{ab_2+\acute{b}_2\omega_s}{b_3}.i_{\mu d} \quad (29)$$

$$-\frac{ab_4+\acute{b}_4\omega_s}{b_3}.\Phi_{rd}-\frac{a}{b_3}.V_{sd}+\frac{1}{b_3}.V_{sq} \quad (30)$$

Furthermore, combining (25), (29) and others from section 2, one gets:

$$\frac{di_{\mu q}}{dt}=(g_1+\acute{g}_1\omega_s).\Phi_{rd}+(g_2+\acute{g}_2\omega_s).i_{\mu d} \quad (31)$$

$$+(g_3+\acute{g}_3\omega_s).i_{\mu q}+g_4.V_{sd}+g_5.V_{sq} \quad (32)$$

with $g_1=g_0.b_4(1-\frac{\acute{b}_3}{b_3})$; $g_0=\frac{1}{M_{dq}}\frac{l_r}{k^2 l_s+l_r}$; $\acute{g}_1=-g_0.\frac{\acute{b}_3\acute{b}_4}{b_3}$; $g_2=g_0.\acute{b}_2(1-\frac{\acute{b}_3}{b_3})$; $\acute{g}_2=-g_0.\frac{\acute{b}_3\acute{b}_2}{b_3}$; $g_3=-g_0.\frac{\acute{b}_3\acute{b}_1}{b_3}$; $\acute{g}_3=g_0.\acute{b}_1(1-\frac{\acute{b}_3}{b_3})$; $g_4=1-\frac{a\acute{b}_3}{b_3}$; $g_5=\frac{\acute{b}_3}{b_3}$.

3.3 *Mechanical equation*

The generated electromagnetic torque undergoes the same equation as in the unsaturated case, i.e.:

$$T_e=\Phi_{rq}i_{rd}-\Phi_{rd}i_{rq}$$

As noticed earlier, condition (28) ensures that $\Phi_{rq}=0$, that is:

$$T_e=-\Phi_{rd}i_{rq}=\Phi_{rd}\frac{km}{l_r}i_{\mu q}$$

The rotor motion equation turns out to be:

$$\frac{d\omega}{dt}=\frac{1}{J}.(\Phi_{rd}\frac{km}{l_r}i_{\mu q}-T_L) \quad (33)$$

The saturated AC machine model thus developped is constituted of equations (24), (29), (32), (33), (28). For convenience, these are rewritten:

$$\frac{d\omega}{dt}=\frac{1}{J}.(\Phi_{rd}\frac{km}{l_r}i_{\mu q}-T_L) \quad (34)$$

$$\frac{di_{\mu q}}{dt}=(g_1+\acute{g}_1\omega_s).\Phi_{rd}+(g_2+\acute{g}_2\omega_s).i_{\mu d} \quad (35)$$

$$+(g_3+\acute{g}_3\omega_s).i_{\mu q}+g_4.V_{sd}+g_5.V_{sq} \quad (36)$$

$$\frac{di_{\mu d}}{dt}=-\frac{b_1+a\acute{b}_1\omega_s}{b_3}.i_{\mu q}-\frac{ab_2+\acute{b}_2\omega_s}{b_3}.i_{\mu d} \quad (37)$$

$$-\frac{ab_4+\acute{b}_4\omega_s}{b_3}.\Phi_{rd}-\frac{a}{b_3}.V_{sd}+\frac{1}{b_3}.V_{sq} \quad (38)$$

$$\frac{d\Phi_{rd}}{dt}=\frac{-R_r}{l_r}\Phi_{rd}+\frac{R_r}{l_r}k.m.i_{\mu d} \quad (39)$$

$$\omega_s=\omega+\frac{R_r}{\Phi_{rd}}\frac{km}{l_r}i_{\mu q} \quad (40)$$

4. BACKSTEPPING CONTROL DESIGN

The controller is designed using the backstepping technique. This is done in two steps: first, the above machine model is reformulated in terms of appropriate tracking and control errors. The performance-oriented model thus obtained suggests a Lyapunov function which is based upon to obtain, in the second step, a stabilizing control law.

4.1 *Step 1.*

Let us introduce the tracking errors on the rotor flux and speed:

$$e_1=\Phi_{rd}-\Phi_{ref} \quad (41)$$

$$z_1=\omega-\omega_{ref} \quad (42)$$

where Φ_{ref} and ω_{ref} denote the corresponding reference signals. We first focus on the flux tracking error. In view of (24), time-derivation of (41) gives:

$$\dot{e}_1=-\frac{R_r}{l_r}\Phi_{rd}-\dot{\Phi}_{ref}+\frac{R_r}{l_r}k.m.i_{\mu d} \quad (43)$$

where the last term will be considered as a virtual control input. This motivates definition of the following control error definition:

$$e_2=\frac{R_r}{l_r}k.m.i_{\mu d}-\alpha_1 \quad (44)$$

where α_1 is a stabilizing function to be defined later. Substituting (44) in (43) yields:

$$\dot{e}_1=-\frac{R_r}{l_r}\Phi_{rd}+e_2+\alpha_1-\dot{\Phi}_{ref} \quad (45)$$

If the virtual control $\frac{R_r}{l_r}k.m.i_{\mu d}$ where effective (in which case $e_2=0$) then the stabilizing function :

$$\alpha_1=-c_1e_1+\frac{R_r}{l_r}\Phi_{rd}+\dot{\Phi}_{ref} \quad (46)$$

would force the flux tracking error to undergo the equation $\mathring{e}_1 = -c_1.e_1$ (with any design real parameter $c_1 > 0$). Unfortunately, $\frac{R_r}{l_r}k.m.i_{\mu d}$ cannot be the effective control because $i_{\mu d}$ is a state variable. Then $e_2 \neq 0$ and, consequently, the stabilizing function (46), together with (45), only gives:

$$\dot{e}_1 = -c_1 e_1 + e_2 \tag{47}$$

Now, let us focus on the speed tracking error $z_1 = \omega - \omega_{ref}$. Time-derivation of z_1 implies, due to (34):

$$\dot{z}_1 = \frac{k.\Phi_{rd}}{j.l_r}.m.i_{\mu q} - \frac{1}{J}C_r - \dot{\omega}_{ref} \tag{48}$$

Similarly, we introduce the control error:

$$z_2 = \frac{k.\Phi_{rd}}{J.l_r}.m.i_{\mu q} - \gamma_1 \tag{49}$$

where γ_1 is a stabilizing function to be defined later. Substituting (49) in (48) gives:

$$\dot{z}_1 = z_2 + \gamma_1 - \frac{1}{J}C_r - \dot{\omega}_{ref} \tag{50}$$

As previously, if $\frac{k.\Phi_{rd}}{J.l_r}.m.i_{\mu q}$ where an effective control (in which case $z_2 = 0$) then the stabilizing function

$$\gamma_1 = -d_1 z_1 + \frac{1}{J}C_r + \dot{\omega}_{ref} \tag{51}$$

would ensure for z_1 the trajectory $\mathring{z}_1 = -d_1.z_1$ (where $d_1 > 0$ is any design real parameter). As, $\frac{k.\Phi_{rd}}{J.l_r}.m.i_{\mu q}$ cannot be an effective control (which means that $z_2 \neq 0$), the stabilizing control (51) together with (50), only yields:

$$\dot{z}_1 = -d_1 z_1 + z_2 \tag{52}$$

4.2 *Step2*

Deriving the error e_2, with respect to time readily yields, due to equation (44):

$$\dot{e}_2 = k.\frac{R_r}{l_r}.\left(\dot{m}.i_{\mu d} + m.\frac{di_{\mu d}}{dt}\right) - \dot{\alpha}_1 \tag{53}$$

Now, from (17,2) and (46) one gets:

$$\dot{m} = \frac{dm}{di_\mu}.\frac{1}{i_\mu}.\left(i_{\mu d}.\frac{di_{\mu d}}{dt} + i_{\mu q}.\frac{di_{\mu q}}{dt}\right) \tag{54}$$

$$\dot{\alpha}_1 = -c_1 e_2 + c_1 e_1^2 + \frac{R_r}{l_r}.\dot{\Phi}_{rd} + \ddot{\Phi}_{ref} \tag{55}$$

Substituting (54),(55),(36),(38) and (39) in (53) gives:

$$\dot{e}_2 = \beta_1 + \lambda_1.v_{sd} + \lambda_2.v_{sq} \tag{56}$$

where β_1 includes all measurable quantities, i.e.:

$\beta_1 = k.\frac{R_r}{l_r}M_d[-\frac{b_1+\acute{b}_1\omega_s}{b_3}.i_{\mu q} - \frac{b_2+\acute{b}_2\omega_s}{b_3}.i_{\mu d} - \frac{b_4+\acute{b}_4\omega_s}{b_3}.\Phi_{rd}]$ +

$k.\frac{R_r}{l_r}.(M_{dq}-m).[(g_1+\acute{g}_1\omega_s).\Phi_{rd} + (g_2+\acute{g}_2\omega_s).i_{\mu d} + (g_3+\acute{g}_3\omega_s).i_{\mu q}]$

$+c_1 e_2 - c_1 e_1^2 - \left(\frac{R_r}{l_r}\right)^2.(-\Phi_{rd} + k.m.i_{\mu d}) + \ddot{\Phi}_{ref}$

and:

$$\lambda_1 = -k.\frac{R_r}{l_r}\left[M_d\frac{a}{b_3} - (M_{dq}-m).g_4\right] \tag{57}$$

$$\lambda_2 = k.\frac{R_r}{l_r}\left[M_d.\frac{1}{b_3} + (M_{dq}-m).g_5\right] \tag{58}$$

Similarly, due to (49), time-derivation of the speed control error z_2 gives:

$$\mathring{z}_2 = \frac{k}{j.l_r}.\left(\dot{\Phi}_{rd}.m.i_{\mu q} + \Phi_{rd}.\dot{m}.i_{\mu q} + \Phi_{rd}.m.\frac{di_{\mu q}}{dt}\right) - \dot{\gamma}_1 \tag{59}$$

Then, using (39), (36), (38), (54) and (51), equation (59) becomes

$$\dot{z}_2 = \beta_2 + \lambda_3 v_{sd} + \lambda_4 v_{sq} \tag{60}$$

where β_2 includes all measurable terms:

$\beta_2 = \frac{k}{J.l_r}.m.i_{\mu q}.\frac{R_r}{l_r}.(-\Phi_{rd} + k.m.i_{\mu d}) + d_1 z_2 - d_1^2 z_1 - \frac{1}{J}\mathring{T}_L - \ddot{\omega}_{ref}$

$\frac{k}{J.l_r}.\Phi_{rd}.M_q.[(g_1+\acute{g}_1\omega_s).\Phi_{rd} + (g_2+\acute{g}_2\omega_s).i_{\mu d} + (g_3+\acute{g}_3\omega_s).i_{\mu q}]$ +

$+\frac{k}{J.l_r}.\Phi_{rd}.(M_{dq}-m)[-\frac{b_1+\acute{b}_1\omega_s}{b_3}.i_{\mu q} - \frac{b_2+\acute{b}_2\omega_s}{b_3}.i_{\mu d} - \frac{b_4+\acute{b}_4\omega_s}{b_3}.\Phi_{rd}]$

and

$$\lambda_3 = -\frac{k}{J.l_r}.\Phi_{rd}.\left((M_{dq}-m).\frac{a}{b_3} + M_q g_4\right) \tag{61}$$

$$\lambda_4 = \frac{k}{j.l_r}.\Phi_{rd}\left((M_{dq}-m).\frac{1}{b_3} + M_q g_5\right) \tag{62}$$

To analyse the error system (47), (52) ,(56) and (60), let us consider the Lyapunov function:

$$V(X) = \frac{1}{2}e_1^2 + \frac{1}{2}e_2^2 + \frac{1}{2}z_1^2 + \frac{1}{2}z_2^2 \tag{63}$$

with $X = [e_1, e_2, z_1, z_2]^T$. Deriving (63), with respect to time, and using (47), (52), (56) and (60), yields:

$$\begin{aligned}\dot{V} = & -c_1 e_1^2 - c_2 e_2^2 - d_1 z_1^2 - d_2 z_2^2 \\ & + e_2(c_2 e_2 + e_1 + \beta_1 + \lambda_1.v_{sd} + \lambda_2.v_{sq}) \\ & + z_2(d_2 z_2 + z_1 + \beta_2 + \lambda_3 v_{sd} + \lambda_4 v_{sq})\end{aligned}$$

This shows that if v_{sd} and v_{sq} are such that:

$$\begin{bmatrix} \lambda_1 & \lambda_2 \\ \lambda_3 & \lambda_4 \end{bmatrix} . \begin{bmatrix} v_{sd} \\ v_{sq} \end{bmatrix} = \begin{bmatrix} -c_2 e_2 - e_1 - \beta_1 \\ -d_2 z_2 - z_1 - \beta_2 \end{bmatrix} \quad (64)$$

then one gets:

$$\dot{V} = -c_1 e_1^2 - c_2 e_2^2 - d_1 z_1^2 - d_2 z_2^2 \quad (65)$$

which clearly establishes asymptotic stability of the origin $(e_1, e_2, z_1, z_2) = (0, 0, 0, 0)$. The attraction region is the state domain where the algebraic equation (64) deos have a solution. It is easily seen that such a domain includes all states where:

$$W(X) \neq 0 \quad (66)$$

with $W(X) = \lambda_1.\lambda_4 - \lambda_2.\lambda_3$. In such a domain the control law turns out to be the following:

$$\begin{bmatrix} v_{sd} \\ v_{sq} \end{bmatrix} = \begin{bmatrix} \lambda_1 & \lambda_2 \\ \lambda_3 & \lambda_4 \end{bmatrix}^{-1} . \begin{bmatrix} -c_2 e_2 - e_1 - \beta_1 \\ -d_2 z_2 - z_1 - \beta_2 \end{bmatrix} \quad (67)$$

5. CLOSED-LOOP STABILITY ANALYSIS

Theorem 1. Consider the control system consisting of the above model in closed-loop with the backstepping control law (67). There exist nonzero bounds c and δ such that if: $0 < \delta < \Phi_{ref} < c$, $\left|\dot{\Phi}_{ref}\right| < c$, $|\omega_{ref}| < c$, $|\dot{\omega}_{ref}| < c$ and $\left|\frac{T_L}{J}\right| < c$; then:

i) the closed-loop system is stable,

ii) the flux and the speed errors, z_1 and e_1, are asymptotically vanishing.

Proof. See the full version of the paper.

6. REFERENCES

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