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ON A TWO DEGREE OF FREEDOM GENERALIZED MINIMUM VARIANCE CONTROL

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Abstract: This paper presents a servo strategy of Generalized Minimum Variance Control (GMVC) with a two degree of freedom. If GMVC fasts the transient response by tuning the control signal weight of the cost function, a large overshoot remains. This paper introduces the two degree of freedom GMVC. The two degree of freedom function curbs the overshoot, while the satisfactory transient characteristic holds. This paper presents two type methods for two degree of freedom: One is setting a reference signal filter in the feedforward part of the closed loop systems, the other is setting a rational function into the cost function. This paper recommends two degree of freedom method with the rational function for the simplification.

Keywords: Discrete time Systems, Disturbance Rejection, Minimum Variance Control, Model Reference, Pole Assignment, Predictive Control, Servo Systems, Time delay, Transient response

1. INTRODUCTION

Generalized Minimum Variance Control (GMVC) (Åström *et al.*, 1977; Clarke and Gawthrop, 1979; Wellstead and Zarrop, 1991) is an effective control for plants including a time delay. In particular, GMVC has the control weight in the cost function and it can apply to the nonminimum phase systems while the stability of the closed loop systems holds.

The purpose of the control weight is to curb the input energy to plant. However, if the control signal weight $S(q^{-1})$ of the cost function strengthens, the transient response becomes slowly. In this paper, Jurry stability criterion (Ackermann, 1985) applies to the closed loop systems of GMVC to evaluate the response property. For the satisfactory response, the proposed GMVC curbs the control signal weight and further places the poles of the characteristic equation of the closed loop systems within the desired region. In the continues systems, the desired region of the discrete time systems corresponds to the region within 45 degree line of left side on the s-plane. However, when the control weight is fixed with the Jurry criterion, a large overshoot of the output remains, although the transient response efficiently fasts. The original GMVC can not reject the overshoot as it is.

This paper introduces the two degree of freedom GMVC (Takahashi *et al.*, 1998*b*). The two degree of freedom function curbs the overshoot of the response. This paper presents two type methods for two degree of freedom: One method is that a filter is set in the feedforward part of the closed-loop systems. The transfer function of the filter is derived from the reference model (Yamamoto *et al.*, 1992; Shigemasa *et al.*, 1983). The other method is that the reference signal weight $R(q^{-1})$ of the cost function includes two degree of freedom function instead of setting the filter. Both methods have the same transient characteristic. Therefore, the second method is simpler structure than the first method.

This paper verifies the response property of the proposed GMVC with simulation. The simulation confirms the effects of the control signal weight

$S(q^{-1})$ at the various values and setting the two degree of freedom function. Furthermore, the simulation results are compared with the pole placements of the characteristic equation of the closed loop systems.

2. TWO DEGREE OF FREEDOM GMVC

This chapter verifies the transient characteristic of GMVC. To improve the transient characteristic, this chapter introduces the method for the evaluation of the stability and the two degree of freedom function.

2.1 *A design of Generalized Minimum Variance Control*

Consider a single input single output system, described as the following Controlled Auto Regressive and Integrated Moving Average (CARIMA) model:

$$A(q^{-1})y(k) = q^{-j}B(q^{-1})u(k) + C(q^{-1})\frac{\xi(k)}{\Delta}, \tag{1}$$

$$A(q^{-1}) = 1 + a_1q^{-1} + \cdots + a_nq^{-n}, \tag{2}$$

$$B(q^{-1}) = b_0 + b_1q^{-1} + \cdots + b_mq^{-m}, \tag{3}$$

$$C(q^{-1}) = 1 + c_1q^{-1} + \cdots + c_lq^{-l}, \tag{4}$$

where $y(k)$ is an output signal, $u(k)$ is a control signal, q^{-j} represents a time delay of plants and $\xi(k)$ is a white noise with the zero mean and the variance σ^2. $A(q^{-1})$, $B(q^{-1})$ and $C(q^{-1})$ are polynomials of q^{-1}, such as a backward shift operator. Δ is a difference operator $(1 - q^{-1})$. The generalized output in the cost function $J = E\{h(k+j)^2\}$ for the servo GMVC, based on the internal model principle, is expressed as follows:

$$h(k+j) = P(q^{-1})y(k+j) - R(q^{-1})w(k+j) + S(q^{-1})\Delta u(k), \tag{5}$$

$$P(q^{-1}) = 1 + p_1q^{-1} + \cdots + p_{n_p}q^{-n_p}, \tag{6}$$

$$R(q^{-1}) = r_0 + r_1q^{-1} + \cdots + r_{n_r}q^{-n_r}, \tag{7}$$

$$S(q^{-1}) = s_0 + s_1q^{-1} + \cdots + s_{n_s}q^{-n_s}, \tag{8}$$

where $w(k)$ is a reference signal. The polynomials $P(q^{-1})$, $R(q^{-1})$ and $S(q^{-1})$ are the weights of the cost function. At this point, $y(k+j)$ in the generalized output (5) must be predicted, because it can not be observed at time k. Then, the following Diophantine equation is introduced as

$$P(q^{-1})C(q^{-1}) = E(q^{-1})\Delta A(q^{-1}) + q^{-j}F(q^{-1}), \tag{9}$$

$$E(q^{-1}) = 1 + e_1q^{-1} + \cdots + e_{j-1}q^{-(j-1)}, \tag{10}$$

$$F(q^{-1}) = f_0 + f_1q^{-1} + \cdots + f_hq^{-h}, \tag{11}$$

$$h_1 = \max\{n, n_p + l - j\}. \tag{12}$$

Note that the filter Δ of equations (5) and (9) provides the servo characterisitic against changing the reference signal and loading the disturbance. When both sides of the CARMA model (1) are multiplied by $E(q^{-1})\Delta$ and the Diophantine equation (9) is substituted for the $E(q^{-1})\Delta A(q^{-1})$, the j-steps-ahead optimal prediction is derived from

$$\widehat{y}(k+j \mid k) = \frac{E(q^{-1})\Delta B(q^{-1})u(k) + F(q^{-1})y(k)}{P(q^{-1})C(q^{-1})}. \tag{13}$$

Substituting equation (13) for equation (5) and minimizing the cost function provide the servo GMVC

$$u(k) = \frac{1}{\Delta}\frac{C(q^{-1})R(q^{-1})w(k+j) - F(q^{-1})y(k)}{E(q^{-1})B(q^{-1}) + C(q^{-1})S(q^{-1})}. \tag{14}$$

Figure 1 shows the block diagram of the closed-loop system of GMVC.

2.2 *Problem formulation*

Combine CARMA model (1) and the servo GMVC (14), the transfer characteristics of the closed-loop control system is derived from

$$G_{wy}(q^{-1}) = \frac{B(q^{-1})C(q^{-1})R(q^{-1})}{T(q^{-1})}, \tag{15}$$

$$G_{dy}(q^{-1}) = \Delta q^{-j}B(q^{-1}) \times \frac{E(q^{-1})B(q^{-1}) + C(q^{-1})S(q^{-1})}{T(q^{-1})}, \tag{16}$$

$$T(q^{-1}) = P(q^{-1})B(q^{-1}) + \Delta S(q^{-1})A(q^{-1}) \tag{17}$$

where equation (15) is a transfer function from the reference signal $w(k)$ to the output signal $y(k)$. On the other hand, equation (16) is from the disturbance d to the output signal $y(k)$. $T(q^{-1})$ of equation (17) represents a characteristic equation of the closed loop systems.

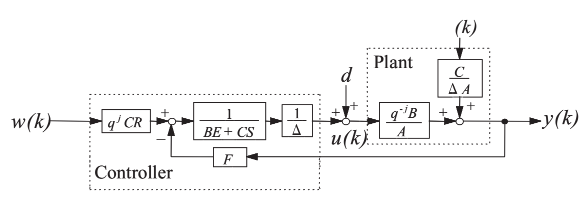

Fig. 1. Block diagram of servo GMVC

However, the reference signal tracking property (15) and the disturbance rejection (16) can not freely change due to the restricted condition $P(1) = R(1)$ (Tuffs and Clarke, 1985; Heath and Wellstead, 1995), because if $R(q^{-1})$ of equation (15) changes to improve the tracking property, $P(q^{-1})$ of the denominator of equation (16) also changes according to the restricted condition $P(1) = R(1)$. In other words, the response properties of both equations (15) and (16) change at the same time.

2.3 *Decision of the weights in the cost function*

The weights $P(q^{-1})$, $R(q^{-1})$ and $S(q^{-1})$ in the generalized output (5) improve the feedback characteristic. However, GMVC is restricted by the condition $P(1) = R(1)$ (Tuffs and Clarke, 1985; Heath and Wellstead, 1995) that holds the servo mechanism. Therefore, only the control signal weight $S(q^{-1})$ can change. This section presents the decision of the control signal weight $S(q^{-1})$. If the control signal weight $S(q^{-1})$ strengthens, although the input energy to the plant can be curbs, the transient response become slowly. Then, this paper introduces Jurry criterion to evaluate the response property. Figure 2 shows the pole locations for discrete time systems. If all the poles of the characteristic equation is located within the unit circle, the closed loop system is stable. In figure 2, the shaded region represents a desired region (Mori *et al.*, 1995), which means the region within 45 degree line of left side on the s-plane in the continues systems. The purpose is to find the range of $S(q^{-1})$, which includes all the poles of the characteristic equation into the desired region. Furthermore, the minimum value of the derived range of $S(q^{-1})$ is choosen in order to fast the tansient response if possible. For instance, if the derived range of $S(q^{-1})$ is from 2.5 to 11.5, $S(q^{-1}) = 2.5$ is chosen in the proposed GMVC.

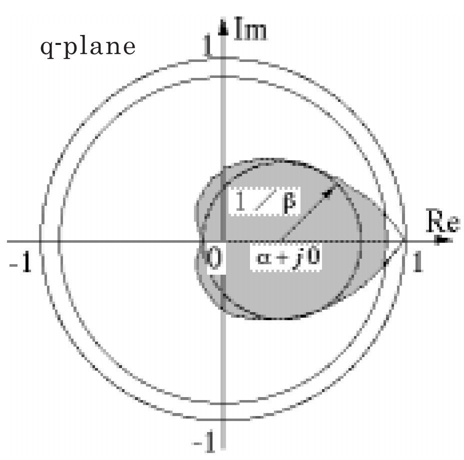

Fig. 2. Desired region of closed loop pole locations

2.4 *Structure of two degree of freedom control systems*

To improve the reference signal tracking property and the disturbance rejection respectively, this section presents a two degree of freedom control GMVC. The two degree of freedom function is derived from the reference signal filter $H(q^{-1})$. Then, the transfer functions of the reference signal tracking property and the disturbance rejection are

$$G_{wy}(q^{-1}) = \frac{H(q^{-1})B(q^{-1})C(q^{-1})R(q^{-1})}{T(q^{-1})}, \quad (18)$$

$$G_{dy}(q^{-1}) = \Delta q^{-j} B(q^{-1}) \times \frac{E(q^{-1}B(q^{-1})) + C(q^{-1})S(q^{-1})}{T(q^{-1})}. \quad (19)$$

The reference signal filter $H(q^{-1})$ matches with a reference model (Yamamoto *et al.*, 1992; Shigemasa *et al.*, 1983)

$$G_m(q^{-1}) = \frac{rB(q^{-1})}{(1 + d_1 q^{-1} + d_2 q^{-2})V(q^{-1})}, \quad (20)$$

$$r = \frac{(1 + d_1 + d_2)V(1)}{B(1)}. \quad (21)$$

The model matching relation $G_{wy}(q^{-1}) = G_m(q^{-1})$ provides

$$H(q^{-1}) = \frac{T(q^{-1})G_m(q^{-1})}{B(q^{-1})C(q^{-1})R(q^{-1})}. \quad (22)$$

Substitute equations (17) and (20) for equation (22), the reference signal filter is expressed as

$$H(q^{-1}) = \frac{rC(q^{-1})\{P(q^{-1})B(q^{-1}) + \Delta S(q^{-1})A(q^{-1})\}}{(1 + d_1 q^{-1} + d_2 q^{-2})C(q^{-1})R(q^{-1})V(q^{-1})}, \quad (23)$$

where d_1, d_2 and $V(q^{-1})$, based on the ref.(Yamamoto *et al.*, 1992), improve the reference signal tracking property. Therefore, if $H(q^{-1})$ changes to improve the reference signal tracking property (18), no $H(q^{-1})$ effects the disturbance rejection (19). Hence, the reference signal tracking property and the disturbance rejection is tuned respectively with the 2 degree of freedom function (Takahashi *et al.*, 1998*a*; Araki, 1985). Figure 3 shows the closed loop system of two degree of freedom GMVC. The reference signal filter $H(q^{-1})$ is set in the feedfoward part of the closed loop system. Then, the two degree of freedom GMVC is derived as

$u(k)$

$$= \frac{1}{\Delta} \frac{H(q^{-1})C(q^{-1})R(q^{-1})w(k+j) - F(q^{-1})y(k)}{E(q^{-1})B(q^{-1}) + C(q^{-1})S(q^{-1})}. \tag{24}$$

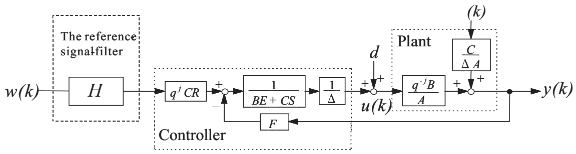

Fig. 3. Structure of GMVC with the reference-signal-filter

3. SIMULATION

This chapter argues the simulation, which GMVC applies to the following model, used in ref.(Palsson *et al.*, 1993; Palsson *et al.*, 1994)

$$\begin{aligned}&(1 - 0.56q^{-1})y(k)\\ &= q^{-2}(0.35 + 0.18q^{-1} + 0.18q^{-2})u(k)\\ &\quad + \xi(k)/\Delta.\end{aligned} \tag{25}$$

The simulations are the time responses of the closed-loop systems. In all the simulations, the reference signal is changing at $k = 20$ from 0 to 1, the disturbance of magnitude 0.1 is loaded at $k = 80$, the variance of noise $\xi(k)$ is 0.01 and the time delay of the plant is 2 steps. Although the weights of the cost function are $P(q^{-1}) = R(q^{-1}) = 1$ due to the restriction (Tuffs and Clarke, 1985; Heath and Wellstead, 1995), the control signal weight $S(q^{-1}) = \lambda$ changes in the respective simulations. In the plant, the range, which includes all the poles of the characteristic equation into the desired region, of the control signal weight is derived from $\lambda(2.41 < \lambda < 11.55)$. Figure 4, figure 5 and figure 6 show the simulation results of $\lambda = 0$, $\lambda = 2.5$ and $\lambda = 11.5$ with the original GMVC. (a) shows the range of λ that includes all the poles into the desired region and the value of λ in respective simulations. (b) locates the poles of the closed loop systems and (c) shows the step responses of the closed loop systems with GMVC. A dotted line is the control signal $u(k)$ and a solid line is the output signal $y(k)$.

Figure 4 is Minimum Variance Control (MVC) because of $\lambda = 0$. Although the output $y(k)$ in figure 4 entirely tracks the reference signal, the control signal $u(k)$ consumes the vast input energy. Therefore, it is not practical.

Figure 6 shows the result of $\lambda = 11.5$. Although all the poles at $\lambda = 11.5$ are included into the desired region, the transient response is slow. On the other hand, figure 5 at $\lambda = 2.5$ shows sufficient transient response. $\lambda = 2.5$ is the minimum value of the range that includes all the poles into the desired region. However, the overshoot remains in the transient response of figure 5.

Figure 7 shows a step response in the case of the two degree of freedom GMVC. In this paper, the Binomial model (Shigemasa *et al.*, 1983), which has no overshoot, applies to the reference model for the two degree of freedom function. The result confirms that the two degree of freedom function curbs the overshoot, while the transient characteristic of $\lambda = 2.5$ holds. Furthermore, two degree of freedom GMVC at $\lambda = 2.5$ also rejects the efficient of disturbance.

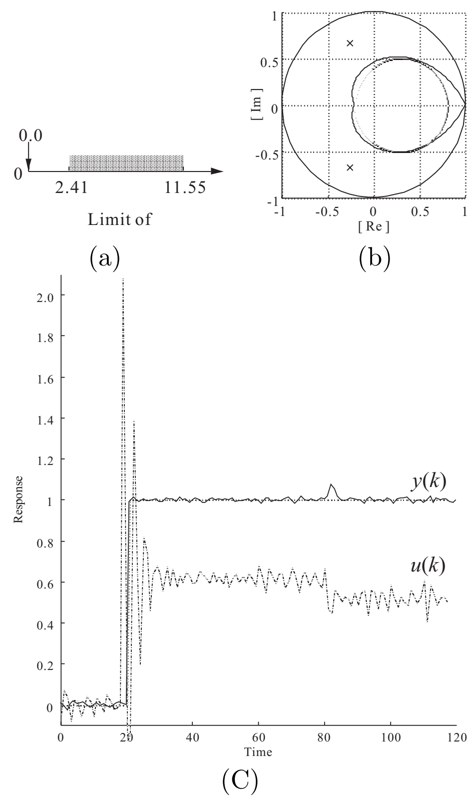

Fig. 4. Step responses of the closed loop system (In the case of $\lambda = 0.0$)

4. RATIONAL FUNCTION OF THE REFERENCE SIGNAL WEIGHT $R(Q^{-1})$ FOR TWO DEGREE OF FREEDOM

If the reference signal weight of the cost function can form a rational function, the reference signal weight substitute for the filter, such as two degree of freedom function, in sec. 2.4. The relation between $H(q^{-1})$ and $R(q^{-1})$ is expressed as

$$R'(q^{-1}) = H(q^{-1})R(q^{-1}). \tag{26}$$

Figure 8 shows the relation in (26). The reference signal weight $R'(q^{-1})$ in figure 8 has the two degree of freedom function instead of the

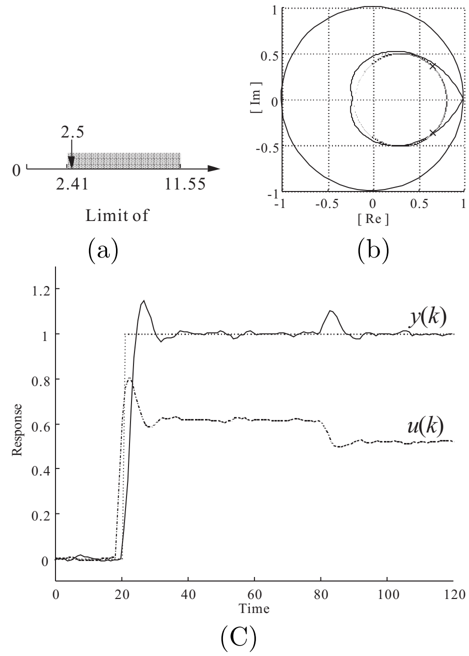

Fig. 5. Step responses of the closed loop system (In the case of $\lambda = 2.5$)

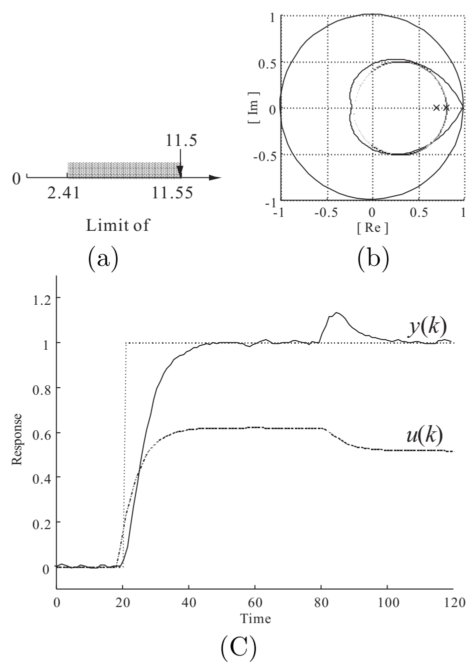

Fig. 6. Step responses of the closed loop system (In the case of $\lambda = 11.5$)

independent filter $H(q^{-1})$. In the case of the rational function $R'(q^{-1})$, this section verifies that whether $R'(q^{-1})$ satisfies the restricted condition $R'(1) = P(1)$. Substitute the filter (22) for the relation (26) of the rational function

$$R'(q^{-1}) = \frac{T(q^{-1})G_m(q^{-1})}{B(q^{-1})C(q^{-1})}. \tag{27}$$

Further, substitute the characteristic equation (15) $T(q^{-1})$ and the reference model (20) $G_m(q^{-1})$ for equation (27)

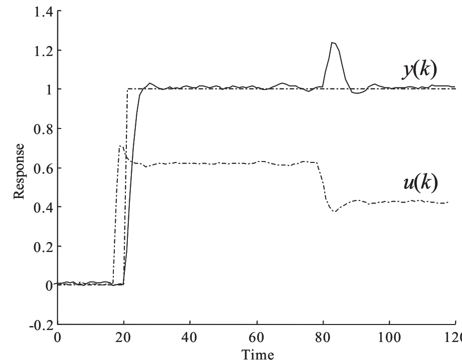

Fig. 7. Step responses of the two degree of freedom GMVC(In the case of $\lambda = 2.5$)

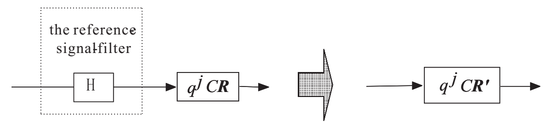

Fig. 8. The relation of the reference signal filter $H(q^{-1})$ and the reference signal weight $R'(q^{-1})$

$$\begin{aligned} &R'(q^{-1}) \\ &= \frac{rC(q^{-1})\{P(q^{-1})B(q^{-1}) + \Delta S(q^{-1})A(q^{-1})\}}{(1 + d_1 q^{-1} + d_2 q^{-2})C(q^{-1})V(q^{-1})}. \end{aligned} \tag{28}$$

If the final value theorem applies to equation (28), the steady state characteristic is derived from

$$R'(1) = P(1), \tag{29}$$

which satisfies the restricted condition $R'(1) = P(1)$. Suppose that the rational function of the reference signal weight $R'(q^{-1})$ is expressed as

$$R'(q^{-1}) = \frac{R_n(q^{-1})}{R_d(q^{-1})}. \tag{30}$$

Then, the generalized output (5) is rewritten as

$$\begin{aligned} &h(k+j) \\ &= P(q^{-1})y(k+j) - \frac{R_n(q^{-1})}{R_d(q^{-1})}w(k+j) \\ &\quad + S(q^{-1})\Delta u(k). \end{aligned} \tag{31}$$

Multiply both sides of equation (31) with $R_d(q^{-1})$, it provides

$$\begin{aligned} &h'(k+j) \\ &= R_d(q^{-1})P(q^{-1})y(k+j) - R_n(q^{-1})w(k+j) \\ &\quad + R_d(q^{-1})S(q^{-1})\Delta u(k), \end{aligned} \tag{32}$$

where $y(k+j)$ is predicted with the Diophantine equation (9). Minimizing the generalized output (32) provides

$$u(k) = \{R_n(q^{-1})C(q^{-1})w(k+j)$$

$$
\begin{aligned}
&- R_d(q^{-1})F(q^{-1})y(k)\} \\
&\times [\Delta R_d(q^{-1})\{B(q^{-1})E(q^{-1}) \\
&+ C(q^{-1})S(q^{-1})\}]^{-1}.
\end{aligned} \tag{33}
$$

Arrange equation (33) with $R_d(q^{-1})$

$$
\begin{aligned}
&u(k) \\
&= \frac{1}{\Delta}\frac{R'(q^{-1})C(q^{-1})w(k+j) - F(q^{-1})y(k)}{B(q^{-1})E(q^{-1}) + C(q^{-1})S(q^{-1})},
\end{aligned} \tag{34}
$$

where the reference signal weight $R'(q^{-1})$ represents the rational function. This argument confirms that the proposed GMVC (34) has the same effect as two degree of freedom GMVC (14) with the filter. In the GMVC with the rational function $R'(q^{-1})$, the offsets due to the reference signal changing and the disturbance are

$$
\begin{aligned}
E[e(\infty)] &= \lim_{q \to 1}\{w(k) - y(k)\} \\
&= \lim_{q \to 1}[[B(q^{-1})\{P(q^{-1}) - R(q^{-1})\} \\
&\quad + \Delta S(q^{-1})A(q^{-1})]T'^{-1}(q^{-1})] \\
&= 0, \qquad (35) \\
y_d(\infty) &= \lim_{q \to 1}[\Delta q^{-j}B(q^{-1})\{B(q^{-1})E(q^{-1}) \\
&\quad + C(q^{-1})S(q^{-1})\}\{C(q^{-1})T(q^{-1})\}^{-1}] \\
&= 0, \qquad (36)
\end{aligned}
$$

which no offset remains even if changing the reference signal and loading the disturbance. In other words, the two degree of freedom servo GMVC can be derived from the rational reference signal weight.

5. CONCLUSION

Jurry criterion applied to the characteristic equation of the closed-loop systems to improve the transient characteristic. This paper pointed out that the overshoot remains cause of the restricted structure of the original GMVC. We introduced the two degree of freedom GMVC with the reference signal filter to remove the overshoot. Furthermore, this paper transformed the filter into the reference signal weight that is the rational function. Both the two degree of freedom GMVC strategies have the same response property. The latter proposed GMVC with the rational function is superior to the GMVC with the filter because of the simple structure. The simulation confirmed that the two degree of freedom GMVC satisfies both the transient characteristic and the overshoot reduction.

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