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MODEL REFERENCE PARAMETRIC ADAPTIVE ITERATIVE LEARNING CONTROL

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Abstract: Most of iterative learning control (ILC) methods requires that the relative degree of the plant is less than 2 for a linear system or the plant is passive for a non-linear system. A new model reference parametric adaptive iterative learning control using the command generator tracker (CGT) theory is proposed in this paper. The method can be applied to control a plant with a higher relative degree and it only requires to iteratively adjust $n_m + 2$ parameters for an SISO plant. Therefore, the ILC control system is very simple. The proposed method is in the spirit of simple adaptive control which has received intensive researches during past two decades. Simulation results show the effectiveness and usefulness of the proposed method.

Keywords: Iterative learning control, Adaptive control, Strictly positive real, Model reference control

1. INTRODUCTION

Iterative learning control (ILC) is a technique to control systems that perform the same task repeatedly. It is learning from the repetitive process like a human being who is learning from his past experiences. Comparing with the feedback control which is usually conducted along the time domain only, iterative learning control is conducted along both the time domain and repetitive trials. It memorizes its previous control information and uses them to compute the next control action so that the system performance is improved. Repetition of trials is essential for this method to work and for this reason ILC is best suited to controlling plants where the same tasks have to be performed repeatedly, e.g. robotic assembly and batch processing in chemical plants.

Arimoto (Arimoto *et al.*, 1984) first noticed this fact and applied the idea to the robot motion control problem. Since then, many research works (Bien and J.-X. Xu, 1998), (Amann *et al.*, 1996), (Owens and Munde, 2000) have been done in the development of various ILC schemes. However, most of ILC approaches require $\mathbf{cb} \neq 0$ which is equivalent to the relative degree of the plant is less than 2. This requirement is equivalent to the condition that the plant is almost strictly positive real (ASPR) (Kaufman *et al.*, 1997). The plant is said to be ASPR if there exists a statict output feedback such that the resulting closed-loop system is strictly positive real (SPR). Although the ASPR charateristics of the plant allow us to control and stabilize the plant robustly with a high gain based ILC output feedback (Owens and Munde, 2000) most practical plants do not satisfy the ASPR condition. Thus such an ASPR condition imposes a strict restriction on the plant with regard to the applicability of the

ILC.

Simple adaptive control (SAC)(Kaufman *et al.*, 1997), which is born from model reference adaptive control, is based on the CGT theory. It gains its name since it only requires to adjust $n_m + 2$ (n_m is the order of the reference model and is much smaller than the order of the plant) parameters for an SISO plant. It has received an intensive research during past two decades (Kaufman *et al.*, 1997), (Deng *et al.*, 2001) and has been widely applied to many industry systems which include large flexible space structures, robot manipulators, ship steering control, DC motors, boilers, aircraft and nonlinear servomechanisms due to its simplicity and robustness. The early version of SAC requires that the plant is ASPR. For a controlled system with a relative degree greater than 2, a robust parallel compensator is introduced in (Deng *et al.*, 2001).

This paper presents an alternative ILC scheme - a parametric ILC scheme for an SISO linear unknown plant. The proposed method is based on the model following structure developed using the CGT theory (Kaufman *et al.*, 1997). The control objective consists in tracking a trajectory specified by a reference model. The approach uses a control structure which is a linear combination of feedforward of the model states and inputs and feedback of the error between plant and model outputs. The convergence of the tracking error is achieved as the number of the trials increases. The proposed method is demonstrated by the simulation results.

2. PROBLEM FORMULATION

Consider a linear SISO system given by the following equations

$$\begin{aligned} \dot{x}(t) &= Ax(t) + bu(t) \\ y(t) &= cx(t) \end{aligned} \tag{1}$$

where $x(t) \in R^n$ denotes the state vector, $u(t)$ and $y(t)$ are the input and output of the system, respectively, A, b, c are matrix and vectors with appropriate dimensions. In this study, the parameters in (1) is assumed unknown and only the knowledge of its relative degree is required for the proposed learning control approach.

Given a finite initial state x_0 and a finite time interval $[0, T]$, the control objective is to find an iterative learning control input $u_j(t)$ such that the system output $y_j(t)$, as j tends to infinity, follows the output y_m of the following reference model

$$\begin{aligned} \dot{x}_m(t) &= A_m x_m(t) + b_m u_m(t) \\ y_m(t) &= c_m x_m(t) \end{aligned} \tag{2}$$

where $x_m(t) \in R^{n_m}$ is the model state vector, $u_m(t)$ and $y_m(t)$ are the model input and the model output, respectively, and A_m, b_m, c_m are matrix and vectors with the appropriate dimensions. It is noted that normally $n_m \leq n$.

It is noted that most ILC algorithms (Bien and J.-X. Xu, 1998), (Amann *et al.*, 1996), (Owens and Munde, 2000) requires that $\mathbf{cb} \neq 0$. This condition is equivalent to that the relative degree of the plant is less than 2. However, most practical plants do not satisfy this condition. One of the main contributions of this paper is that we relax this requirement by introducing a robust parallel compensator.

3. COMMANDER GENERATOR TRACKER

The proposed ILC approach is based on the CGT theory (Kaufman *et al.*, 1997). To describe the application of the CGT theory, a new set of system and control trajectories as $x^*(t)$ and $u^*(t)$, respectively, will be used to denote their corresponding values when $y_j(t) = y_m(t)$ for $t \geq 0$ and $j \geq 1$ (i.e. when perfect tracking occurs). From the above description, we have

$$\begin{aligned} \dot{x}^*(t) &= Ax^*(t) + bu^*(t) \\ y^*(t) &= cx^*(t) = y_m(t) = c_m x_m(t) \end{aligned} \tag{3}$$

From the CGT theory, we have

$$\begin{bmatrix} x^*(t) \\ u^*(t) \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} x_m(t) \\ u_m(t) \end{bmatrix} \tag{4}$$

where S_{11}, S_{12}, S_{21}, and S_{22} are the functions of A, b, c, A_m, b_m, and c_m. An adaptation law (Kaufman *et al.*, 1997) can be designed to identify them for the unknown A, b and c. Here we design an iterative learning law to adjust them.

It is noted that although the CGT based analysis requires that $u_m(t)$ is a step command, any command signal which can be described as the solution of a differential equation forced by a step input (or zero) can be used as well (Kaufman *et al.*, 1997).

4. THE ILC CONTROL ALGORITHM FOR AN ASPR CONTROLLED SYSTEM

In this section, we assume that the system (1) is ASPR. That is there exists a constant $\bar{\theta}_e$ such that the transfer function $c(sI - A + b\bar{\theta}_e c)^{-1}b$ is SPR.

It is well known that a necessary and sufficient condition for the transfer function $c(sI - A + b\bar{\theta}_e c)^{-1}b$ to be SPR is that there exist two positive definite symmetric matrices P and Q satisfying the following equations (Kaufman *et al.*, 1997)

$$\begin{aligned} &(A - b\bar{\theta}_e c)^T P + P(A - b\bar{\theta}_e c) = -Q \\ &Pb = c^T \end{aligned} \tag{5}$$

Let $e_{yj}(t) = y_m(t) - y_j(t)$. Our objective is to find a $u_j(t)$ such that $\lim_{j\to\infty} e_{yj}(t) = 0$ for $t \in [0, T]$. Let $e_j(t) = x^*(t) - x_j(t)$. It is easily to show that $\lim_{j\to\infty} e_j(t) = 0$ will lead to $\lim_{j\to\infty} e_{yj}(t) = 0$. Thus our task transfers to design a $u_j(t)$ to achieve $\lim_{j\to\infty} e_j(t) = 0$. Using (1) and (3), we have

$$\begin{aligned}\dot{e}_j(t) &= \dot{x}^*(t) - \dot{x}_j \\ &= A(x^*(t) - x_j(t)) + b(u^*(t) - u_j(t)) \quad (6)\end{aligned}$$

From (4), it is noted that $u^*(t) = S_{21}x_m(t) + S_{22}u_m(t)$. The proposed parametric ILC control algorithm is

$$\begin{aligned}u_j(t) &= \theta_j^T(t)z_j(t) \\ &= \left[\, \theta_{ej} \ \ \theta_{xj}^T \ \ \theta_{uj} \,\right] \left[\, e_{yj} \ \ x_m^T \ \ u_m \,\right]^T \quad (7)\end{aligned}$$

and the parameter error vector

$$\begin{aligned}\tilde{\theta}_j(t) &= [\, \tilde{\theta}_{ej}(t) \ \ \tilde{\theta}_{xj}(t)^T \ \ \tilde{\theta}_{uj}(t) \,]^T \\ &= [\, \bar{\theta}_e - \theta_{ej}(t) \ \ S_{21} - \theta_{xj}(t)^T \ \ S_{22} - \theta_{uj}(t) \,]^T\end{aligned}$$

Putting the control law (7) into (6), we have the error equation

$$\begin{aligned}\dot{e}_j(t) &= (A - b\bar{\theta}_e c)e_j(t) + b(\tilde{\theta}_{ej}(t)e_{yj}(t) \\ &\quad + \tilde{\theta}_{xj}(t)x_m(t) + \tilde{\theta}_{uj}(t)u_m(t)) \\ &= (A - b\bar{\theta}_e c)e_j(t) + b\tilde{\theta}_j(t)z_j(t) \quad (8)\end{aligned}$$

The proposed parametric iterative learning law is

$$\theta_j(t) = \theta_{j-1}(t) - \ \ z_j(t)e_{yj}(t) \quad (9)$$

where   is a positive constant. Define a cost function of the parameter errors as

$$J_j = \frac{1}{\ } \int_0^t \tilde{\theta}_j^T(\tau)\tilde{\theta}_j(\tau)d\tau \quad (10)$$

From (9) and (10), we have

$$\begin{aligned}\Delta J_j &= J_j - J_{j-1} \\ &= \frac{1}{\ } \int_0^t [\tilde{\theta}_j^T(\tau)\tilde{\theta}_j(\tau) - (\tilde{\theta}_j(\tau) \\ &\quad + \ \ z_j(\tau)e_{yj}(\tau))^T(\tilde{\theta}_j(\tau) + \ \ z_j(\tau)e_{yj}(\tau))]d\tau \\ &= -\int_0^t [\ \ z_j^T(\tau)z_j(\tau)e_{yj}^2(\tau)]d\tau \\ &\quad - 2\tilde{\theta}_j^T(\tau)z_j(\tau)e_{yj}(\tau))d\tau \quad (11)\end{aligned}$$

Define a positive Lyapunov-like function

$$V_j(t) = e_j^T(t)Pe_j(t) \quad (12)$$

Taking the derivative of (12) with respective to time and using (8) and (5), we have

$$\begin{aligned}\dot{V}_j(t) &= e_j^T(t)[(A - b\bar{\theta}_e c)^T P + P(A - b\bar{\theta}_e c)]e_j(t) \\ &\quad + 2e_j^T(t)Pb\tilde{\theta}_j^T(t)z_j(t) \\ &= -e_j^T(t)Qe_j(t) + 2\tilde{\theta}_j^T(t)z_j(t)e_{yj}(t) \quad (13)\end{aligned}$$

Integrating (13) from 0 to t and putting it into (11) and considering $V_j(0) = 0$, we have

$$\begin{aligned}\Delta J_j = &-\int_0^t (\ \ z_j^T(\tau)z_j(\tau)e_{yj}^2(\tau) - \int_0^t e_j^T(\tau)Qe_j(\tau)d\tau \\ &- V_j(t) + V_j(0) \le -V_j(t) \le 0 \quad (14)\end{aligned}$$

Therefore, we have that $J_j \le J_{j-1}$. Adding (14) from 0 to j, we have

$$\Sigma_{j=1}^k V_j(t) \le -J_k(t) + J_0(t) \le J_0(t) < \infty \quad (15)$$

This leads to $\lim_{j\to\infty} V_j(t) = 0$. From the definition of $V_j(t)$, we have that $\lim_{j\to\infty} e_j(t) = 0$. Thus

$$\begin{aligned}\lim_{j\to\infty} e_{yj}(t) &= \lim_{j\to\infty} (y_m(t) - y_j(t)) \\ &= \lim_{j\to\infty} ce_j(t) = 0 \quad (16)\end{aligned}$$

Next, we will show that all the signals of the closed-loop system are bounded. From (10) and (14), we have $0 \le J_j \le J_{j-1} \le J_0$. This leads to the conclusion that the $\theta_j(t)$ is bounded. Since $x_m(t), u_m(t), e_{yj}(t)$ and $\theta_j(t)$ are bounded, from (7) we conclude that u_j is bounded. Since $x^*(t)$, $u^*(t)$ and $e_j(t)$ are bounded, we have shown that $x_j(t)$ is bounded.

At this point, we would like to make the following remark to the proposed control approach.

The proposed learning control algorithm is different with the other approaches. It iteratively learns the parameters instead of the control input and it only adjusts three parameters for an SISO plant. It uses the output of the plant, the input and states of the reference model only, and it does not use identifiers or observers in the control loop.

5. ROBUST PARALLEL FEEDFORWARD COMPENSATOR

The ILC algorithm proposed in section 4 requires that the plant is ASPR. In this section, we introduce a robust parallel compensator proposed in (Deng *et al.*, 2001) to relax this requirement. The transfer function of Eq. (1) is

$$G_p(s) = \frac{kN(s)}{D(s)} \quad (17)$$

where N(s) and D(s) are

$$D(s) = s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0$$

$$N(s) = b_m s^m + b_{m-1}s_{m-1} + \ldots + b_1 s + b_0$$

It is assumed that the range of the unknown plant parameters are known as follows

$$a_i^- \leq a_i \leq a_i^+, i = 0, 1, \quad \ldots, n-1.$$

$$b_i^- \leq b_i \leq b_i^+, \quad i = 0, 1, \ldots, m.$$

If the relative degree $n - m > 1$, the method proposed in section 4 can not be applied since the plant is not ASPR. For a non-ASPR plant, we design a PFC F(s) which makes the resulting augumented controlled system, $G_a(s) = G_p(s) + F(s) = \frac{N_a(s)}{D_a(s)}$, ASPR. Consider the following parallel compensator

$$F(s) = \sum_{i=1}^{\gamma-1} \delta^i F_i(s) = \sum_{i-1}^{\gamma-1} \frac{\delta^i \beta_i n_i(s)}{d_i(s)} \tag{18}$$

where $= n - m$ and $\delta > 0$, $d_i(s)$ is a monic Hurwitz polynomial of order n_{di}, $n_i(s)$ is a monic polynomial of order $m_{ni} = n_{di} - (\; - i) > 0$, and β_i is chosen to satisfy that the polynomials $\beta_{\gamma-1}s^{\gamma-1} + \cdots + \beta_1 s + b_m^-$ and $\beta_{\gamma-1}s^{\gamma-1} + \cdots + \beta_1 s + b_m^+$ are Hurwitz polynomials. It has been proved in (Deng *et al.*, 2001) that we can choose a reasonable small δ to make $G_a(s)$ be ASPR.

6. ILLUSTRATIVE EXAMPLE

In order to demonstrate the proposed control algorithm, we consider the following example. The transfer function of the plant is

$$G_p(s) = \frac{d_1 s + d_0}{s^2 + a_1 s + a_0}$$

where d_1, d_0, a_1, and a_0 are unknown parameters. The reference model is chosen as

$$G_m(s) = \frac{1}{s+1}$$

Its state representation is

$$\begin{aligned} \dot{x}_m(t) &= x_m(t) + u_m(t) \\ y_m(t) &= x_m(t) \end{aligned} \tag{19}$$

We will carry out the simulation study in two cases. In case 1, we assume that $d_1 > 0$ and $d_0 > 0$. Thus $G_p(s)$ is ASPR. In case 2, we assume that $d_1 = 0$ and $d_0 > 0$ that is $G_p(s)$ is no longer ASPR.

Case 1. In this study we assume that the parameters of the plant are $d_1 = 1$, $d_0 = 1$, $a_1 = 1$, $a_0 = -2$. This knowledge is used for the simulation study only and it is not used in the ILC design. It is clearly to see that the plant is unstable.

For this case, the ILC control law is shown in (7), the parametric learning law is shown in (9), the initial values of the learning parameters are chosen as zero, and the learning gain $= 10$.

Figure 1 shows the tracking performance for the first trial for case 1 and Fig. 2 shows the input profile at this trial. Figures 3 and 4 show the tracking performance and control input at fifth trial in case 1.

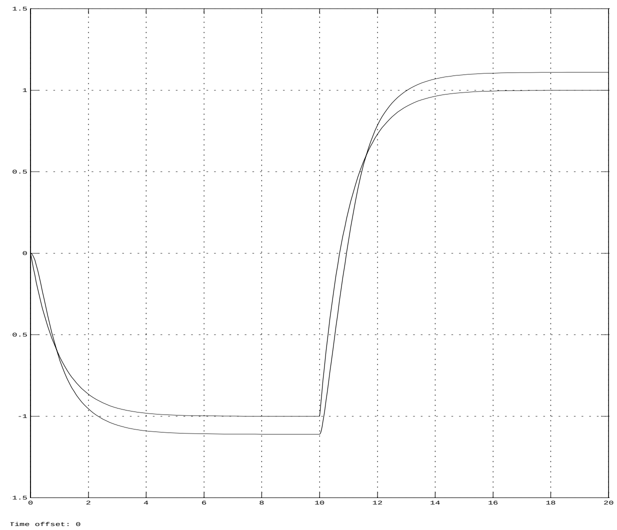

Fig. 1. Tracking performance at the first trial of case 1

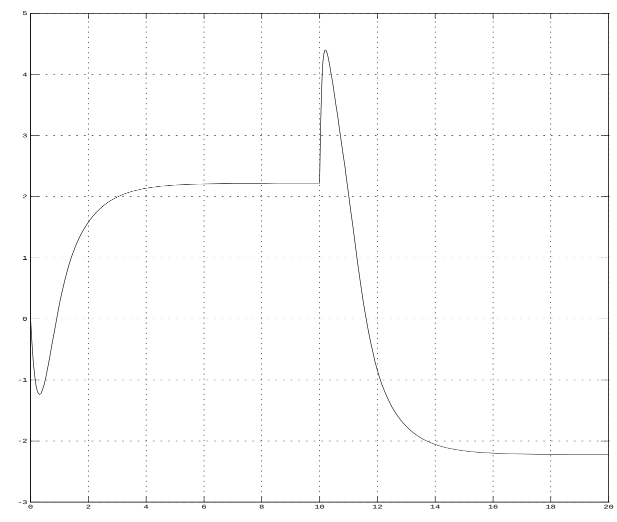

Fig. 2. Control input at the first trial of case 1

Case 2. In this study we assume that $d_1 = 0$ and the rest parameters of the plant are the same as in Case 1. It is clearly to see that the system is not ASPR since its relative degree is 2. We design a PFC using the approach in section 5 as follows,

$$F(s) = \frac{s}{s+\alpha_0} \cdot \frac{\delta\beta_0}{s+\alpha_1} \tag{20}$$

In the simulation $\beta_0 = 1$, $\alpha_0 = 5$, $\alpha_1 = 1$ and $\delta = 0.5$. For this case, the ILC control law and the

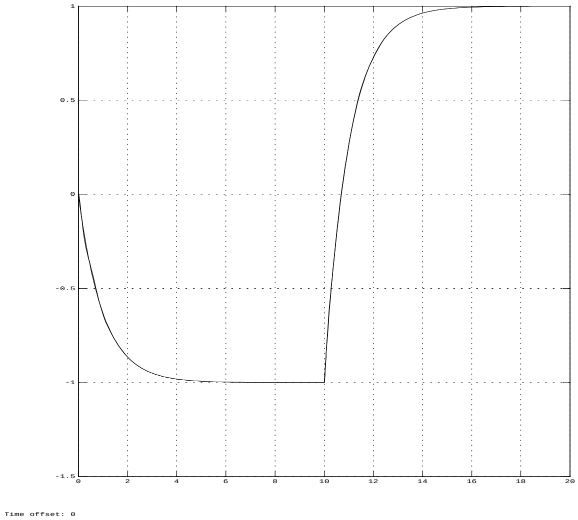

Fig. 3. Tracking performance at the fifth trial of case 1

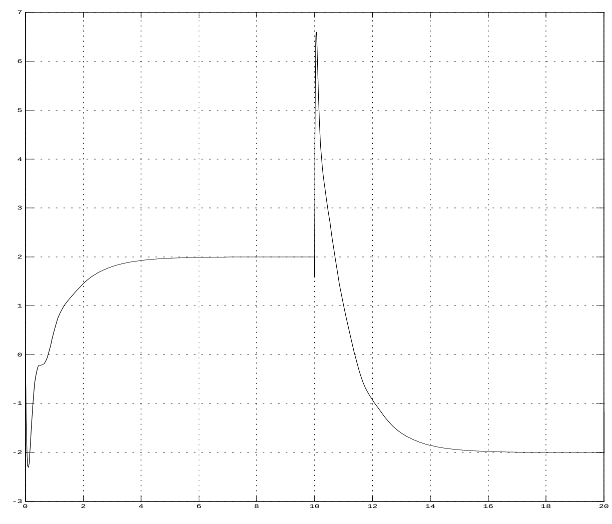

Fig. 4. Control input at the fifth trial of case 1

parametric learning law are same as in case 1.

Figures 5 and 6 show the tracking performance and the control input profile at the first trial for case 2. Figures 7 and 8 show the tracking performance and control input at the thirtieth trial in case 2.

From the simulation results, the system learns very fast for an ASPR plant. For a non-ASPR plant we can see that it takes longer to learn. It may get worse first before it achieve a good tracking performance.

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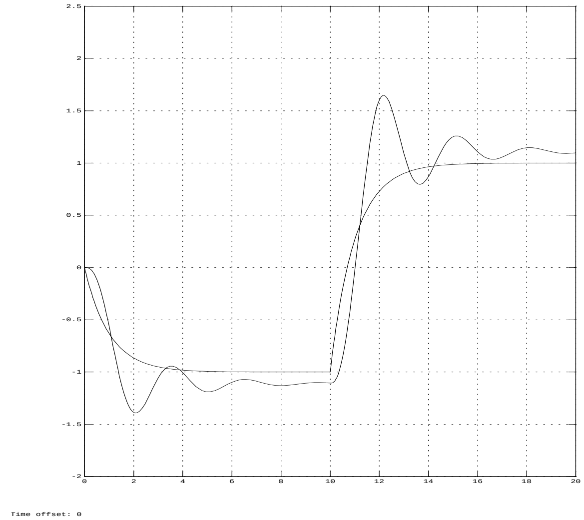

Fig. 5. Tracking performance at the first trial of case 2

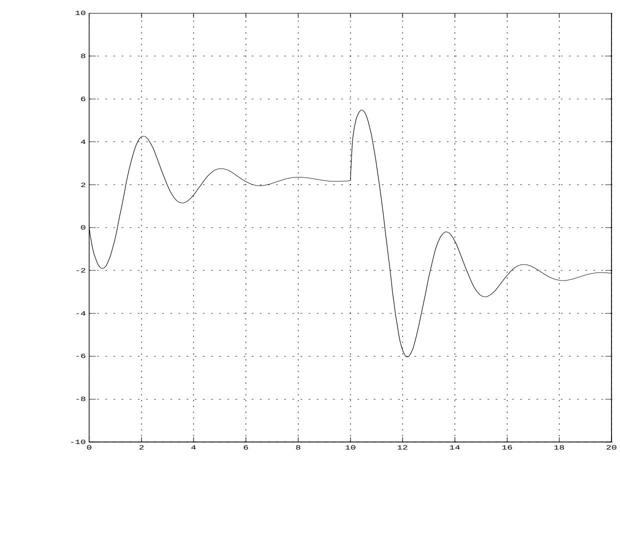

Fig. 6. Control input at the first trial of case 2

REFERENCES

Amann, N., D.H. Owens and E. Rogers (1996). Iterative learning control using optimal feed-back and feedforward actions. *Int. J. Control* **65**, 277–293.

Arimoto, S., S. Kawamura and F. Miyazaki (1984). Bettering operation of robots by learning. *J. Robot. Syst.* **1**, 123–140.

Bien, Z. and Eds. J.-X. Xu (1998). *Iterative Learning Control: Analysis, Design, Integration and Applications*. Kluwer Academic Publishers. Boston.

Deng, M., H. Yu and T. Yang (2001). Robust parallel compensator for mimo plants with structured uncertainty. *Proc. of 6th European Control Conference* pp. 3404–3409.

Kaufman, H., I. Bar-Kana and K. Sobel (1997). *Direct Adaptive Control Algorithms - Theory and Applications*. Springer-Verlag. New York.

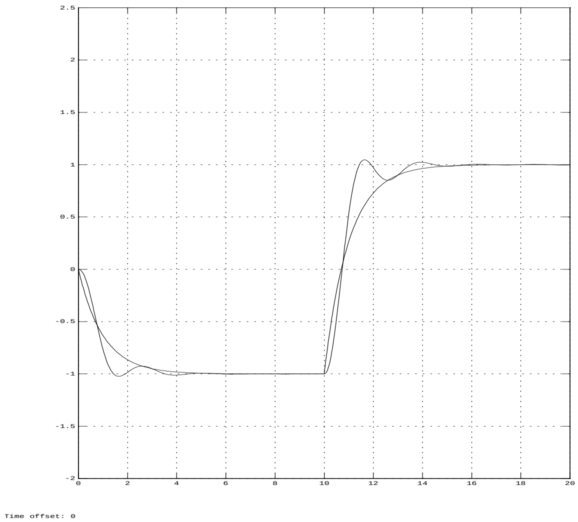

Fig. 7. Tracking performance at the thirtieth trial of case 2

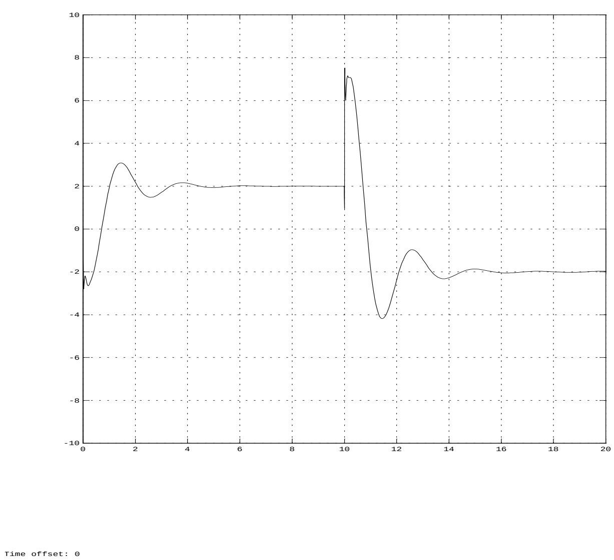

Fig. 8. Control input at the thirtieth trial of case 2

Owens, D.H. and G. Munde (2000). Error convergence in an adaptive iterative learning controller. *Int J. Control* **73**, 851–857.