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A SLIDING MODE CONTROL PROPOSAL FOR NONLINEAR ELECTRICAL SYSTEMS

Rubén D. Rojas, Oscar E. Camacho,
Ramón O. Cáceres, Alfredo R. Castellano

Postgrado en Automatización e Instrumentación. Facultad de Ingeniería.
Universidad de Los Andes. Mérida – Venezuela. e-mail: rdrojas@ing.ula.ve

Abstract: This article describes the synthesis of a sliding mode controller SMCr based on a second order linear model using an integral-differential surface of the tracking-error. Different from similar strategies, the tuning parameters keep a close relationship with the system dynamics in terms of conventional specifications of transient response. The proposed controller only needs the output feedback of system and could be satisfactorily used in control of single input-single output nonlinear electric systems such as electronic power converters with pulse width modulation. *Copyright ©2002 IFAC*

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1. INTRODUCTION

Numerous physical processes exhibit a behavior whose dynamics can be represented by a single input-single output (SISO), second order model. However, such approach discards process inherent non-linearities that sometimes would cause degradation of control when the conventional PID strategy is used. This becomes more evident when it is working in extreme conditions or in zones where the parametric uncertainty of the model becomes more accented.

In this framework, robustness proper of the sliding mode control strategy (SMC) should provide better performance than conventional control strategies. Properties such as order reduction and invariant dynamics of the system in the sliding mode have stimulated development of multiple procedures for the synthesis of controllers in a wide spectrum of applications. Such are the cases of processes with multi-input / multi-output configuration (DeCarlo et al., 1988), with strong non linearities, with variable dead time or non minimum phase (Camacho, 1996; Camacho and Smith, 1997; Camacho et al., 1999). In addition, excellent results have been reported in control of electric motors and electronic power converters (Utkin, 1993; Cáceres and Barbi, 1999) where discontinuous action in variable structure control (VSC) is compliant with the nature of their elements.

The main drawback in VSC is the chattering generally associated with a high control activity that sometimes could not be tolerated by the system. It could excite high frequency unmodeled dynamics or decrease their efficiency (Utkin, 1993; Sira et al., 1997). This last aspect highlights the convenience of synthesizing the SMCr under an approximately continuous control law.

Based on the SMC robustness, this article shows the synthesis of a controller based on integral-differential surface of error and a continuous approximation of nonlinear part of the controller. The design is pointed to obtain a simple structure that only needs the output feedback making it attractive for control of nonlinear electric systems such as power converters with schemes of pulse width modulation (PWM).

2. SYSTEM MODELING

In a sliding mode control strategy, the system dynamics is forced by the controller to stay confined in a subset of the state space denominated sliding surface, $\sigma(t)$. The system is directed toward and reaches the sliding surface at a finite time due to the control action. Finally, once the system dynamics reach the user-chosen sliding surface, it will behave according to that surface, which is of a lower order than that of the system and independent of the model parametric uncertainties (Utkin, 1977).

Let M be the second order model of system (lower order approximation of the real system Q), with an output variable $\theta_1(t)$ that tracks the reference input $\theta_r(t)$ with a tracking error $e_1(t)$ under control law $U(t)$ of the SMCr . The dynamics of the controlled system M can be described by the following state space equation:

$$\begin{aligned} \dot{\theta}_1(t) &= \theta_2(t) \\ \dot{\theta}_2(t) &= -a_1\theta_1(t) - a_2\theta_2(t) + m(t) + bU(t) \\ y(t) &= \theta_1(t) \end{aligned} \quad (1)$$

where a_1, a_2, b are the parameters of the second order model approximation of the system and $m(t)$ represents the external disturbances. This equation can be rewritten in terms of the tracking error as follows:

$$\begin{aligned} \dot{e}_1(t) &= e_2(t) \\ \dot{e}_2(t) &= -a_1e_1(t) - a_2e_2(t) + f(t) - bU(t) \\ y(t) &= e_1(t) \end{aligned} \quad (2)$$

here, $f(t)$ contains the external disturbances $m(t)$, the reference signal $\theta_r(t)$ and its derivatives. Whenever the external disturbance acts in the same state space of the control U, a control U_f will exist such that:

$$b\,U_f(t) = f(t) \quad (3)$$

and the sliding mode existence will be ensured if a known upper bound, the supreme $F(t)$, for $f(t)$ exists such that for any instant t:

$$f(t) < F(t) \quad (4)$$

3. CONTROLLER SYNTHESIS

The desired performance of the system is usually described in terms of the transient response specifications. This performance must be satisfied in presence of disturbances and set point changes. The transient response specifications are thoroughly used in conjunction with PID control to regulate systems with dominant second order dynamics. This approach is used in this article to evaluate the system response. In general, the most used set of specifications for the transient response can be summarized as follows:

S1. Steady State Error, e_{ss}.
S2. Percent Overshoot, Mp.
S3. Peak Time, Tp.
S4. Settling Time, Ts.

The design problem consists first on choosing the sliding surface so that the system exhibits the desired dynamics defined by the given specifications; and second to guarantee the conditions so that the system reaches that surface at a finite time. There are many options to choose $\sigma(t)$; the Sliding Surface selected in this work is an integral-differential equation acting on the tracking-error (Slotine & Li, 1991) which is represented by the following expression:

$$\sigma(t) = \left(\frac{d}{dt} + \lambda\right)^n \int_0^t e(t)dt \quad (5)$$

The advantage of this surface is that it contains an integral term, which ensures the annulment of the steady state error. Then $\sigma(t)$ is a function of the error between the reference, θ_r, and the output, θ_1, values as was described in Eqn. (2), such that:

$$\begin{gathered} \sigma(t) = \frac{d\,e(t)}{dt} + 2\lambda_1\lambda_0\,e(t) + \lambda_0^2\int_0^t e(t)\,dt = 0 \\ \lambda_0,\, \lambda_1 > 0 \end{gathered} \quad (6)$$

represents the system dynamics in the sliding mode. The satisfaction of expression $\sigma(t)=0$ means that in steady state the tracking-error of the system should decrease to zero with a dynamics that depends on the selection of parameters λ_0 and λ_1.

For all SISO systems (Utkin, 1977), the condition for existence of a sliding mode is satisfied if:

$$\sigma(t)\,\dot{\sigma}(t) < 0 \quad (7)$$

Geometrically, this inequality means that the time derivatives of the state error vector always point toward the sliding surface when system is in reaching mode, and therefore, the system dynamics will approach to the surface dynamics in a finite time. To satisfy the given specifications S1–S4 and Eqn. (6) an augmented equivalent control law (DeCarlo et al., 1988) was used:

$$U(t) = Ueq(t) + U_N(t) \quad (8)$$

$Ueq(t)$ is denominated the Equivalent Control, represents the continuous part of the controller that maintains the output of the system restricted to the sliding surface. $U_N(t)$ is the nonlinear part of the controller that ensure the reach of the sliding mode and therefore, it should satisfy the inequality given in Eqn. (7).

The continuous part $Ueq(t)$ could be determined supposing that in the instant t_0, the state trajectory of the system intercepts the surface and enters the sliding mode. The existence of sliding mode implies that Eqn. (6) is satisfied, so:

$$\dot{\sigma}(t) = \dot{e}_2(t) + 2\lambda_1 \lambda_0 e_2(t) + \lambda_0^2 e_1(t) = 0 \ ; \ \forall \quad t \geq t_0 \quad (9)$$

Then the equivalent control $Ueq(t)$ is obtained from Eqn. (9), after substituting the error derivatives by the state equations (2), when the system is not disturbed ($f(t)$= 0):

$$Ueq(t) = \frac{1}{b}\left[\left(\lambda_0^2 - a_1\right)e_1(t) + \left(2\lambda_0\lambda_1 - a_2\right)e_2(t)\right] \quad (10)$$

As can be appreciated in Eqn. (10), $Ueq(t)$ evidence a proportional-derivative nature, attenuated by the static gain of the model.

Finally, the chattering problem could be solved satisfactorily if the control $U_N(t)$ is designed according to (Zinober, 1994):

$$U_N(t) = Kd \frac{\sigma(t)}{|\sigma(t)| + \delta} \quad ; \quad Kd, \delta > 0 \quad (11)$$

where Kd is a tuning parameter to assure the reach of the sliding surface and δ is set to obtain suppression of chattering. An estimate of the Kd parameter could be obtained from Eqn. (3) and Eqn. (4). The overall control law structure is shown in Fig. 1.

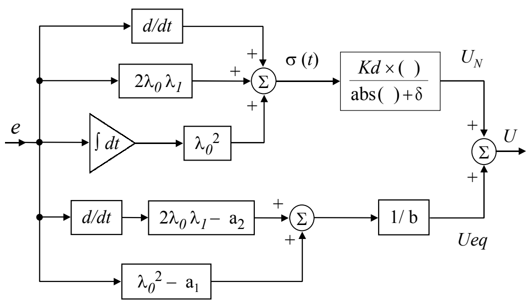

Fig. 1. Structure of the proposed SMCr.

4. CONTROLLER TUNING

In sliding mode, system dynamics depend on the sliding surface $\sigma(t)$, and therefore the desired transient response will depend on λ_0 and λ_1 parameters selection. Both parameters could be determined solving Eqn. (6). Depending on λ_1 values range, it is possible to obtain three different solutions for the set of specifications:

4.1 If $0 < \lambda_1 < 1$; $\lambda_0 > 0$

In this case, the dynamics of the surface will present a behavior strongly underdamped and steady-state error will approach to zero after an oscillation around the equilibrium point (see Fig. 2). The response of the system can be evaluated through the following equations:

$$e_{ss} = 0 \quad (12)$$

$$Mp = \frac{100}{\alpha} exp^{-\left(\frac{2\lambda_1\beta}{\alpha}\right)} sin\beta \quad (13)$$

$$Tp = 2\frac{\beta}{\lambda_0 \alpha} \quad (14)$$

$$Ts \approx \frac{4}{\lambda_0 \lambda_1} \quad (15)$$

$$\alpha = \sqrt{1-\lambda_1^2} \ ; \ \beta = tan^{-1}\left(\frac{\alpha}{\lambda_1}\right) \quad (16)$$

Notice that the percent overshoot Mp only depends on λ_1.

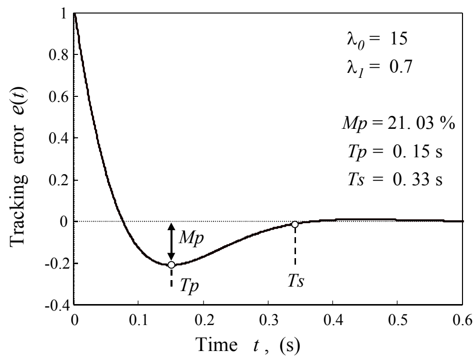

Fig. 2. Dynamics of the sliding surface when $0 < \lambda_1 < 1$.

4.2 If $\lambda_1 = 1$; $\lambda_0 > 0$

In this case, the system response will present a moderate underdamped behavior (see Fig. 3). Because Mp only depends on λ_1, it is constant (13.53%), and the response of the system can be evaluated through the following equations:

$$e_{ss} = 0 \quad (17)$$

$$Mp = 13.53\ \% \quad (18)$$

$$Tp = \frac{2}{\lambda_0} \quad (19)$$

$$Ts = \frac{5.4}{\lambda_0} \quad (20)$$

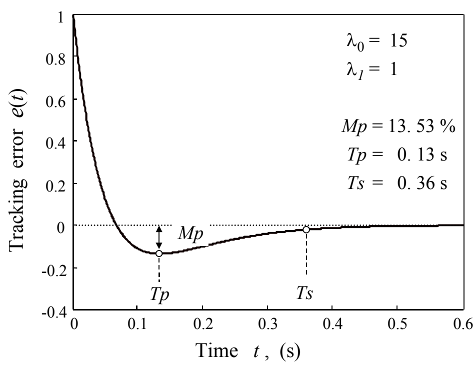

Fig. 3. Dynamics of the sliding surface when $\lambda_1 = 1$.

4.3 If $\lambda_1 > 1$; $\lambda_0 > 0$

In this case, the system response can be evaluated through the following equations:

$$e_{ss} = 0 \quad (21)$$

$$Mp = \frac{50}{\alpha}\left[(\lambda_1 - \alpha)\; exp^{-\left(\frac{\lambda_1}{\alpha} - 1\right) Ln\left(\frac{\lambda_1 + \alpha}{\lambda_1 - \alpha}\right)} - (\lambda_1 + \alpha)\, exp^{-\left(\frac{\lambda_1}{\alpha} + 1\right) Ln\left(\frac{\lambda_1 + \alpha}{\lambda_1 - \alpha}\right)} \right] \quad (22)$$

$$Tp = \frac{1}{\lambda_0\, \alpha} Ln\left(\frac{\lambda_1 + \alpha}{\lambda_1 - \alpha}\right) \quad (23)$$

$$Ts \approx \frac{1}{\lambda_0 (\lambda_1 - \alpha)} Ln\left(\frac{50(\lambda_1 - \alpha)}{\alpha . e_{m\%}}\right) \quad (24)$$

$$\alpha = \sqrt{\lambda_1^{\,2} - 1} \quad (25)$$

where $e_{m\%}$ is the specified error band inside which Ts is specified. The response, Fig. 4, exhibits a reduced overshoot but a larger settling time.

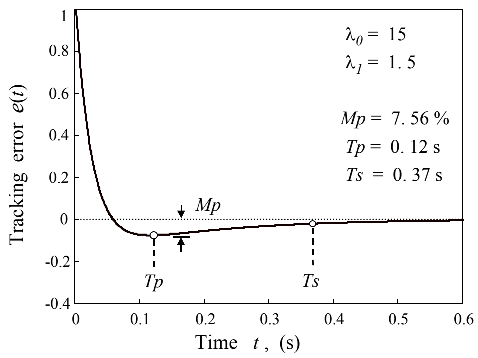

Fig. 4. Dynamics of the sliding surface when $\lambda_1 > 1$.

Due to overdamping, the settling time will be severely influenced by the specified error band. Damping ratio associated with λ_1 parameter is shown in Fig. 5. Note that overshoot only could be reduced if $\lambda_1 >> 1$.

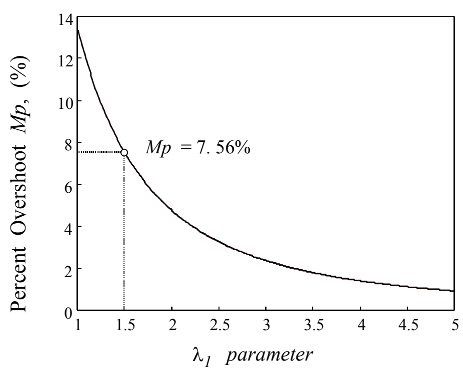

Fig. 5. Overshoot -λ_1 relationship.

As shown in the above results for the three cases, the relationship between the controller parameters and the transient response of the close-loop system is evident. This facilitates the controller tuning considerably. For instance, if a desired overshoot should be lower than 8%, λ_1 >1.5 would be a suitable election (see Fig. 5).

Once defined the percent overshoot Mp, the parameter λ_0 is selected to satisfy the desired time response in terms of peak time or settling time. This tuning procedure is straightforward and it is as simpler as a PID controller tuning.

Another interesting aspect is the strong relationship between the controller parameters and the system natural response. In Eqn. (1), the system approximated by the second order model should contain the dominant poles of Q, so that an alternative representation of M could be obtained in terms of natural frequency ω_n and damping radio ξ from the original system at any operation point, such that:

$$a_1 = \omega_n^{\,2} \quad (26)$$

$$a_2 = 2\,\xi\,\omega_n \quad (27)$$

From Eqn. (10) it is evident that λ_0 is related with ω_n and λ_1 with ξ. As usual, the controlled system must respond faster than the open-loop system and therefore, it is desirable that $\lambda_0 > \omega_n$.

According to Eqn. (13), Eqn. (18) and Eqn. (22), the percent overshoot only depends on λ_1, which provides a straight way to achieve the desired percent overshoot in system response. For instance, choosing $\lambda_1 > \xi$ is convenient if $\xi < 0.707$. Then, next step is to adjust λ_0 to provide the desired response in terms of settling time or peak time, which are totally determined according to the λ_1 range.

A reasonable value for Kd could be determined assuming that in the meantime of an external disturbance or when step change is introduced, the sliding surface value $\sigma(t) >> \delta$. Therefore, the nonlinear part of the controller U_N approximates $Kd.sign(\sigma(t))$ and its magnitude will be Kd. In order to guarantee the reaching condition in any circumstance, from Eqn. (3) and Eqn. (4), it is enough that:

$$Kd > \left| \frac{1}{\mathrm{b}} F(t) \right| \quad (28)$$

For systems with saturated single input, the superior estimate $F(t)$ in Eqn. (4) is given by the control input bounds:

$$\left| f(t) \right| < \left| \mathrm{b}(U_{max} - U_{min}) \right| \quad (29)$$

because is impossible allow, without loss of stability, a disturbance in the system that requires a control magnitude larger than the range of values of $U(t)$ for an indefinite period of time. Finally, the parameter δ (always positive) is adjusted to suppress the chattering.

5. CONTROL FOR A BOOST CONVERTER

In general, if energy conversion is carried out in continuous conduction mode, the boost converter can be described in the average state space according to:

$$\begin{aligned} \frac{d\,i_L(t)}{dt} &= \frac{\mathrm{Vs}}{\mathrm{L}} - \frac{1}{\mathrm{L}} v_0(t)\,\big(1-u(t)\big) \\ \frac{d\,v_0(t)}{dt} &= -\frac{1}{\mathrm{RC}} v_0(t) + \frac{1}{\mathrm{C}} i_L(t)\,\big(1-u(t)\big) \end{aligned} \qquad (30)$$

where $i_L(t)$ is the inductor average current, $v_0(t)$ is the output voltage, R is the load in ohms, Vs is the external dc source voltage and $u(t)$ is the control input. This control input is assumed a saturated continuous function in the open interval (0, 1).

To derive an approximate model of the converter, the electronic switch is replaced by the *PWM* switch (Timersky et.al, 1988), resulting in the circuit shown in Fig. 6.

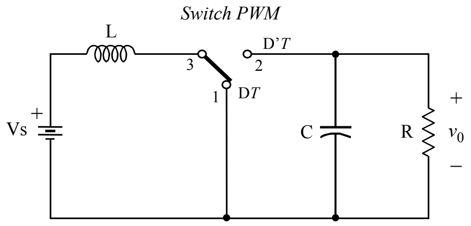

Fig. 6. Equivalent circuit of the boost converter

Substituting the *PWM* switch by its fundamental frequency model, around a stationary point of operation D, the dynamics of system in Eqn. (30) is described approximately as follows:

$$\frac{v_0(s)}{d(s)} = -\frac{\mathrm{Vs}}{\mathrm{RC}(1-\mathrm{D})^2} \frac{\left(s - \frac{\mathrm{R}}{\mathrm{L}}(1-\mathrm{D})^2\right)}{\left(s^2 + \frac{1}{\mathrm{RC}} s + \frac{(1-\mathrm{D})^2}{\mathrm{LC}}\right)} \qquad (31)$$

where $d(s)$ represents the incremental duty cycle. Equation (31) characterizes a non minimum phase system with respect to the output voltage, and therefore, it's difficult to control.

The behavior of the open-loop system corresponding to reference step changes and load disturbances is shown in Fig. 7.

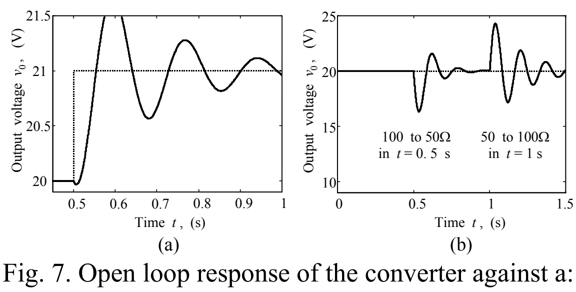

Fig. 7. Open loop response of the converter against a:
(a) Reference step change response (20 a 21V)
(b) Step change in the load resistor (±50%R)

To obtain the tuning parameters of the controller, the system in Eqn. (31) is described in terms of the natural response of the approximate model *M*:

$$\omega_n = \frac{(1-\mathrm{D})}{\sqrt{\mathrm{LC}}} \quad ; \quad \xi = \frac{\sqrt{\mathrm{LC}}}{2\mathrm{RC}(1-\mathrm{D})} \qquad (32)$$

where D is the steady state duty cycle. A parameter summary is shown in Table 1, according to the following circuit elements: inductor, 170mH; capacitor, 1000μF; resistor, 100 Ω; and power supply voltage, 10Vdc. The duty cycle D is computed for an output voltage of 20 V.

Table 1. Model parameters *M*(*s*) (D = 0.5)

Parameter	Value
b	5.882×10^4
a_1	1.471×10^3
a_2	10
ω_n, natural frequency	38.35
ξ, damping ratio	0.130

Lower limits for λ_0 and λ_1 are immediately obtained from the natural response:

$$\lambda_0 > \omega_n = 38.35 \quad \text{y} \quad \lambda_1 > \xi = 0.130$$

Considering the closed-loop gain restriction that imposes an unstable zero dynamics of the system, is advisable to choose moderate gains, for instance, $\lambda_0 = 1.05\omega_n$ or $1.1\omega_n$ and then increase λ_1 from a value equal to ξ until the desired response is achieved. The gain *Kd* is obtained from the control input bounds ($0 < u < 1$). Hence, to reach the sliding surface is enough to choose *Kd* = 1.

In systems with high natural frequency, λ_0 will be larger than λ_1 and therefore, the instantaneous value of the surface $\sigma(t)$ could reach an unusual magnitude. Because δ y $\sigma(t)$ have the same order of magnitude, for practical implementation purposes, the sliding surface $\sigma(t)$ can be normalized with respect to the natural frequency without degrades the U_N behavior.

6. RESULTS

Using the average description according to Eqn. (30), the system was simulated using MATLAB® with step time equal to 1×10^{-4} s. The controller synthesis was carried out with the following parameters: $\lambda_0 = 42.18$ ($1.1\omega_n$); $\lambda_1 = 0.417$ (3.2 ξ); *Kd* = 1; δ = 1.5; the error surface was normalized with respect to a factor of $10\omega_n$.

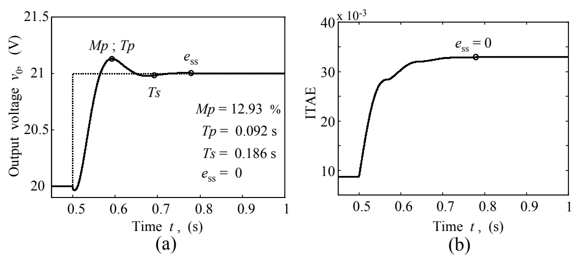

Fig. 8. System step response.
(a) From 20 up to 21 V, in t = 0.5 s
(b) ITAE performance index

Figure 8, shows the system step response when a reference step change is introduce at t = 0.5 s. It shows a small overshoot and a short settling time even better than the predicted by the tuning equations. The ITAE performance index, Fig. 8 (b), becomes constant confirming the annulment of steady state error.

The good system response behavior to load disturbances is shown in Fig. 9 (a), when the output voltage is set to 20V. The load disturbances are simulated by step changes of ±50% of the nominal load. Figure 9 (b) shows that the steady state error is annulled in similar times.

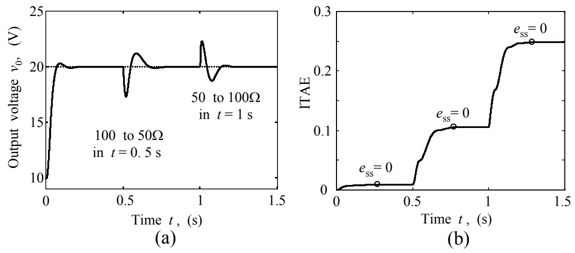

Fig. 9. Attenuation of load disturbances.
(a) Step changes ± 50% nominal load
(b) ITAE performance index.

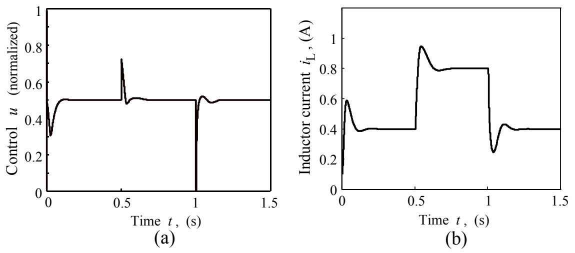

Fig. 10. Load step change response.
(a) Control input magnitude (normalized)
(b) Average inductor current

Figure 10 shows the control signal and the average inductor current when the load change was simulated. The results are very similar to those obtained using and SMC adaptive strategy (Escobar et al., 1997) evaluated on experimental prototype of the boost converter.

7. CONCLUSIONS

The designed SMC controller, based on an integral-differential surface of the tracking-error, performed very well when reference and load step changes were introduced, with zero steady state error and without chattering.

Its main attribute is the close relationship between the controller tuning parameters and the desired closed-loop transient response. The possibility to obtain the desired percent overshoot unilaterally (adjusting λ_1) allows achieving demanding agreements between percent overshoot and settling time.

Deduced from a second order model of the system, the control law has a simple structure and it requires only an output feedback loop. This could be suitable in design of control schemes of minor complexity, especially for power converters with schemes of pulse width modulation (PWM), where the current feedback is usual in the control strategy.

Just one parameter (δ) in the controller structure is subject to the designer selection. Once a satisfactory *Kd*/δ relationship has been determined, the sliding surface $\sigma(t)$ can be normalized with respect to the natural frequency, without degrades the U_N behavior because it only depends on the magnitude and sign of the sliding surface $\sigma(t)$ with respect to the numeric value of δ. This aspect is very important for practical implementation of the SMCr in analog schemes.

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