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A PREDICTIVE APPROACH BASED-SLIDING MODE CONTROL

Oscar Camacho

Postgrado en Automatización e Instrumentación. Facultad de Ingeniería.
Universidad de Los Andes. Mérida – Venezuela. e-mail: ocamacho@ing.ula.ve

Abstract: This Paper shows the synthesis of Sliding Mode Controller using model predictive structure of the process. The Smith predictor architecture is combined with the Sliding mode control theory. Two different linear models, with deadtime, are simulated and the performance of the controller is evaluated.

Keywords: Model Predictive Structure, Smith Predictor, Sliding Mode Control, Robustness

1. INTRODUCTION

The predictive structure design of a control system requires the use of a process model, either explicitly or implicitly. Modeling errors are unavoidable and it results in a mismatch between the model and the actual plant. Thus, the result is that the controller designed for a particular model may perform quite differently when it is implemented on the actual process. There exist, a way to address this problem, the design of *robust controllers*, that can deal with model-plant mismatches (Slotine and Li, 1991).

The Sliding Mode Control (SMC) approach, which is also known as variable structure control (VSC), is a nonlinear control technique (Utkin, 1977). Sliding Mode Control design is composed of two steps. At the first step, a custom-made surface is to be designed. While on the sliding surface the plant's dynamics is restricted to the equations of the surface and are robust to match plant uncertainties and external disturbances. At the second step, a feedback control law is to be designed to provide convergence of a system's trajectory to the sliding surface; thus, the sliding surface should be reached in a finite time. The system's motion on the sliding surface is called the sliding mode.

The aim of this paper is to merge for nonlinear systems, the model predictive structure concept on the sliding mode control theory. Therefore, the presented approach provides the predictive characteristic to the SMC structure, improving the transient response for deadtime processes, and SMC gives the robustness to the predictive structure for model mismatches. The Smith Predictor scheme will be analyzed. The SMCr is designed from the delay free model of the process. The sliding surface is designed such a way that it is reached in spite of the presence of modeling errors, which means it is not affected, and therefore the robustness is guaranteed.

The basic assumption of the proposed design is that the robustness of the controller will compensate for modeling errors arising from the linearization of the nonlinear model of the process, and the free delay model chosen for designing the controller.

The paper is organized as follows: Section two shows a brief description of model predictive control structure, and of sliding mode control. The third section shows the procedure used to design the controller. In the fourth section simulations are presented to judge the performance of the proposed controller, in the last part the conclusions are shown.

2. BASIC CONCEPTS

2.1 The model predictive control structure

The predictive control structure scheme is based on a natural way to interpret feedback control. A model of the process to determine the proper adjustment to the manipulated variable is used. Therefore, it is possible to predict the futures values of the controlled variable based on the values of the manipulated variable, the important feedback information is the difference between the predictive model response and the actual process response (Marlin, 1995).

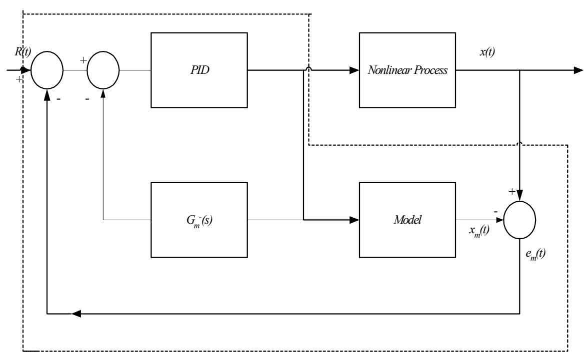

Fig. 1. Predictive Control Scheme

2.2 Sliding Mode Control (SMC)

As has been commented in the introductory section, SMC is a particular technique of VSC (Edwards and Spurgeon, 1998). The control law is composed of two parts: the sliding mode control law and the reaching mode control law
The first of those is responsible for maintaining the controlled system dynamic on a sliding surface, which represents the desired closed loop behavior. The second control law is designed in order to reach the desired surface.

The first step in SMC is choosing the sliding surface, which is usually formulated as a linear function of the system states. *s(t)* can be represented as follows

$$s(t) = f\big(R(t), x_m(t)\big) \qquad (1)$$

Where *R(t)* is the reference and $x_m(t)$ is the model output.

Filippov's construction of the equivalent dynamics is the method normally used to generate the equivalent sliding mode control law (Utkin, 1977). It consists of satisfying the following sliding condition

$$\frac{ds(t)}{dt} = 0 \qquad (2)$$

and substituting it into the system dynamic equations, the control law is thereby obtained. To design the reaching mode control law, the signum function of $s(t)$ affected by a constant gain can be used. However, this produces the undesirable effect of chattering, normally not tolerated by the actuators. A more appropriate solution is to use the sigmoid-like function, instead of the signum one, to smooth the discontinuity and to obtain a continuous approximation to the surface behavior (Camacho et al, 1999) and avoid chattering in the control signal when the surface is (pseudo) reached. This is known in the literature as reaching a pseudo-sliding mode. The expression for the reaching mode control law can then be expressed as:

$$U_{reach}(t) = K_D \frac{s(t)}{s(t)+\delta} \qquad (3)$$

where K_D is the tuning parameter responsible for the speed with which the sliding surface is reached, and δ is used to reduce the chattering problem

3. SYNTHESIS OF THE PREDICTIVE SLIDING MODE CONTROLLER (PSMCr)

Most processes in industry can be modeled by an FOPDT model described by

$$G(s) = \frac{K}{\tau s+1} e^{-t_o s} \qquad (4)$$

where K is the process modeled static gain, τ is the time constant or process modeled lag, and t_o is the modeled deadtime or delay.

This model can be represented in the following way

$$G_m(s) = G_m^+ G_m^- \qquad (5)$$

where $G_m^+(s)$ corresponds to the noninvertible term of the model, and $G_m^-(s)$ is the free delay part. They can be represented as:

$$G_m^+ = e^{-tos} \qquad (6)$$

$$G_m^- = \frac{K_m}{\tau s+1} \qquad (7)$$

Using the free delay part, $G_m^-(s)$, can be easily demonstrated that the characteristic equation is free of deadtime, so the SMCr can be designed from it. Thus, the performance can be improved over the conventional SMCr without delay compensation.

Let us propose the following sliding surface

$$s(t) = e_1(t) \tag{8}$$

Where, $e_1(t)$ is the error between the reference, $R(t)$, and the free delay part of model output, $x_m^-(t)$, considering a perfect model, i.e., $G(s) = G_m$. Thus, the sliding surface is given as a function of the reference and the free delay model output. This representation is very important because the controller design does not consider the process deadtime. In spite of the previous considerations, it is assumed that the robustness of the controller will compensate for these modeling errors.

From the sliding condition, $\frac{ds(t)}{dt} = 0$,

$$\frac{ds(t)}{dt} = \frac{dR(t)}{dt} - \frac{dx_m^-(t)}{dt} = 0 \tag{9}$$

From equation 4, and putting it into differential equation form, which represents the process model

$$\tau \frac{dx_m^-(t)}{dt} + x_m^-(t) = Ku(t) \tag{10}$$

adding equations (9) and (10), results

$$\frac{dR(t)}{dt} + \frac{x_m^-(t)}{\tau} = \frac{K}{\tau}u(t) \tag{11}$$

and the equivalent control law, is given by

$$u_e(t) = \frac{\tau}{K}\left[\frac{dR(t)}{dt} + \frac{x_m^-(t)}{\tau}\right] \tag{12}$$

In (Camacho and Smith, 2000) is shown that the derivatives of the reference value can be discarded, without any effect on the control performance, resulting in a simpler controller. The Smith predictor scheme based-SMCr is given by the following equation

$$u(t) = \left[\frac{x_m^-(t)}{K}\right] + K_D \frac{s}{|s| + \delta} \tag{13}$$

The following equations are used to tune the controller:

$$K_D = \frac{0.72}{|K|}\left(\frac{\tau}{t_o}\right)^{0.76} \tag{14}$$

$$\delta = 0.68 + 0.12|K|K_D \frac{t_o + \tau}{t_o \tau} \tag{15}$$

Proof

The reaching condition is given by

$$s\frac{ds(t)}{dt} < 0 \tag{16}$$

$$\frac{ds(t)}{dt} = \frac{dR(t)}{dt} - \frac{dx_m^-(t)}{dt} \tag{17}$$

$$\frac{ds(t)}{dt} = \frac{dR(t)}{dt} - \left[\frac{K}{\tau}u(t) - \frac{x_m^-(t)}{\tau}\right] \tag{18}$$

substituting in the previous equation, it is obtained

$$\frac{ds(t)}{dt} = -K^* \frac{s}{|s| + \delta} \tag{19}$$

where

$$K^* = \frac{KK_D}{\tau} > 0 \tag{20}$$

therefore

$$s\frac{ds(t)}{dt} < 0 \ \text{ for all } t>0$$

which shows that the motion is enforced to reach the sliding surface and keep on it in a stable behavior.

4. SIMULATIONS

This section simulates the control performance of the SMCr designed and given in Eqs. 8 and 13. The following process models with deadtime will be used in the simulations of this article:

$$G_1(s) = -0.78\frac{e^{-5s}}{(2s+1)} \tag{21}$$

$$G_2(s) = \frac{0.4e^{-24s}}{(13.8s+1)(6.1s+1)(3.9s+1)} \tag{22}$$

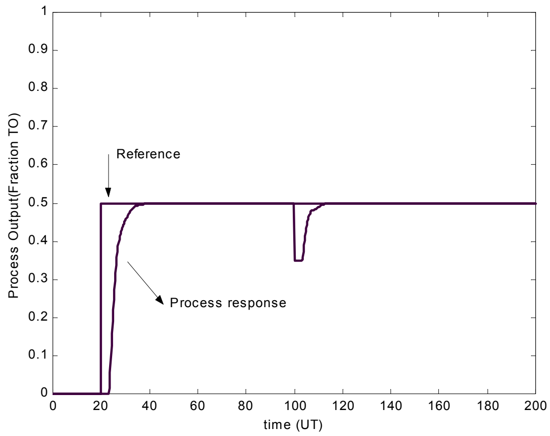

Fig. 2. Process response, for set point and disturbances changes applied to $G_1(s)$ without modeling errors.

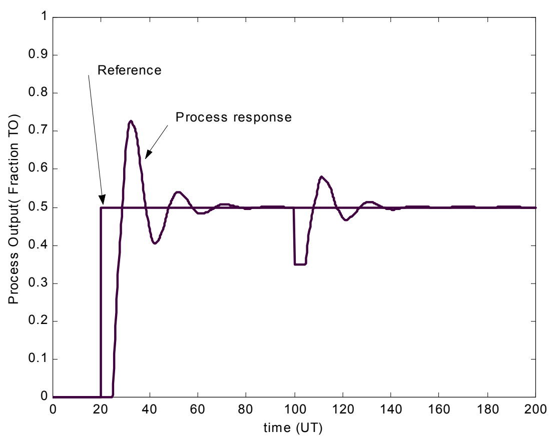

Fig. 3. Process response, for set point and disturbances changes applied to G1(s) with 40% in modeling errors.

Figure 2 depicts the process response when a set point and disturbance changes are applied to the model $G_1(s)$. The process output is close to critically damped. When, for some reason, a model error of + 40% occurs in the model parameters (K, τ, and t_o), in spite of the errors, the system oscillates but becomes stable, Figure 3. The controllability relationship t_o/τ is 2.5 and the system all the times is stable despite modeling errors.

A higher order system with dead time is also simulated. In this case, also, the relationship t_o/τ is close to 2.5. Figures 4 and 5. As can be observed in both figures, similar results are obtained.

6. CONCLUSIONS

This paper has shown the synthesis of a sliding mode controller based on the predictive control structure. The controller obtained is of fixed structure. A set of equations obtains the first estimates for the tuning parameters. The examples presented indicate that the PSMCr performance is stable and quite satisfactory in spite of the modeling errors.

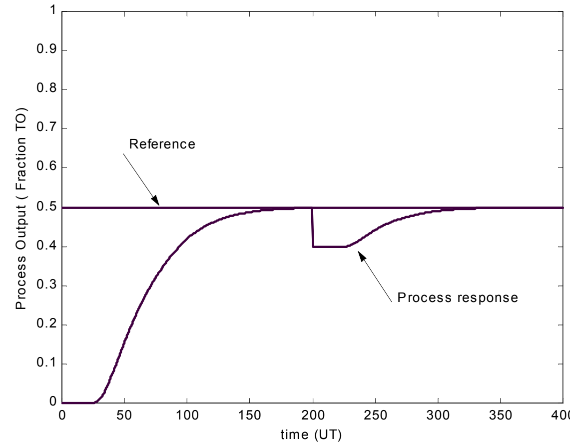

Fig. 4. Process response, for set point and disturbances changes applied to $G_2(s)$ without modeling errors.

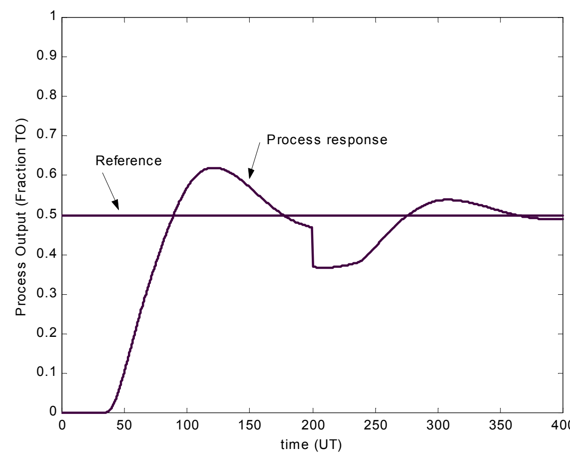

Fig. 5. Process response, for set point and disturbances changes applied to G2(s) with 40% in modeling errors

The controller law, Eqs. 8 and 13 should be rather easy to implement in any computer system (DCS) (Camacho and Rojas, 2000)

In order to compare, the performance of the proposed predictive scheme, in future works a comparison between this controller approach against the version given in (Perez de la Parte et al, 2002), should be done.

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