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PERFORMANCE MONITORING OF PI CONTROLLERS USING A SYNTHETIC GRADIENT OF A QUADRATIC COST FUNCTION

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Abstract The paper shows how a synthetic gradient of a quadratic cost function can be used to monitor performance. The method requires a model of the closed loop system. In the current paper this is obtained from the recommended tuning rule. The tuning rule requires a simple model of the process. The method is non-invasive, only closed loop data from normal operation is used. By monitoring the gradient, information about the state of the loop is obtained.

Keywords performance monitoring, PID control

1. INTRODUCTION

The need for tools to automatically analyze performance of feedback controllers has been recognized in the process industry where a typical plant might have hundreds or even thousands of controllers. The benefits of having controllers running smoothly is an increase in product quality and reduction in down time because problems are found before they cause problems.

Previous work on performance monitoring has generally focused on identifying the impulse response from a stochastic disturbance added to the output. For a review see (Harris *et al.*, 1999). Using these methods it is possible to compare current performance, measured as variance of output, to the maximally achievable performance while taking into account fundamentally limiting factors of performance such as dead time or non-minimum phase zeros. For a correct usage of these methods knowledge about the limiting factors is required at all operating conditions the control loop is supposed to work in. This information might be considered difficult to obtain.

Any performance monitoring loop is bound to encounter oscillations. Oscillations might occur because loop is marginally unstable or a nonlinearity is present. Oscillating loops might also cause oscillating disturbances in other feedback loops which are “downstream”. In (Horch, 2000) it was shown that the above mentioned methods can fail to indicate poor performance when oscillations occur.

The method presented takes a new approach to the problem. In contrast to comparing the current performance to what is maximally achievable considering the fundamental limitations, the presented method gives information about whether the performance, with the current control parameters and disturbance spectrum, could be improved by a change in the controller parameters. This information is supplied by calculating a synthetic gradient of a quadratic cost function with regard to the controller parameters.

Because the method requires a model of the closed loop, it is suggested to integrate the performance monitoring with a tuning method with the design goal of a certain closed loop transfer function. This is the λ tuning method but it is also shown how the method can be applied when the loop has been manually tuned.

In the current article the focus will be on PI control. Surveys frequently show that more than 90% of controllers in the process industry are controlled with a PI. The synthetic gradient can be used to monitor other types of controllers even though the structure of the PI is taken advantage of. In discrete form the PI is parametrized as

$$C(z) = K\left(1 + \frac{h}{T_i(z-1)}\right) \quad (1)$$

Parameter K is used to increase or decrease the loop gain. It affects both the proportional and integral term. Most control engineers are familiar with what affect it has to change K and it is frequently used for trimming loops. For this reason special emphasis will be on the gradient with respect to K.

The paper is organized in the following manner. In Section 2 it is shown how the synthetic gradient is obtained. In Section 4 it is shown how to use the synthetic gradient. In Section 5 conclusions are drawn.

2. THE SYNTHETIC GRADIENT OF A QUADRATIC COST FUNCTION

2.1 *Iterative Feedback Tuning*

Iterative Feedback Tuning (IFT) has recently emerged as a technology to tune fixed order controllers like the PID by performing experiments on the closed-loop system. The tuning is performed by calculating the gradient of a quadratic cost function with respect to the controller parameters and modifying the parameters in the descent direction of this cost function. Because of lack of space only the aspects of IFT that are used directly in the proposed method will be introduced. Every thing else about IFT can most probably be found in (Hjalmarsson *et al.*, 1998).

The method deals with SISO linear systems on the form

$$y_t = G_0 u_t + v_t \quad (2)$$

v_t is the process disturbance. In this paper the controller is restricted to be of one degree of freedom. The control signal is given by

$$u_t = C(\rho)(r_t - y_t) \quad (3)$$

ρ is a vector containing the controller parameters. Given that the controller is given by Eq.(1) the parameter vector would be

$$\rho = [K\ T_i]$$

The closed loop system is then given by

$$\begin{aligned} y_t &= \frac{C(\rho)G_0}{1+C(\rho)G_0} r_t + \frac{1}{1+C(\rho)G_0} v_t \\ &= T(\rho) r_t + S(\rho) v_t \end{aligned} \quad (4)$$

$T(\rho)$ is referred to as the complimentary sensitivity function while $S(\rho)$ is the sensitivity function. It is easy to check that they satisfy $T(\rho) + S(\rho) = 1$. The time argument of the signals will now be omitted but they will be written as a function of controller parameters ρ.

Putting $\tilde{y}(\rho) = r - y(\rho)$ the cost function that is monitored is of quadratic type (LQG with tracking),

$$J(\rho) = \frac{1}{2N} E\left[\sum_{t=1}^{N} \tilde{y}(\rho)^2 + \delta \sum_{t=1}^{N} u^2(\rho)\right] \quad (5)$$

The expectation is with respect to the weakly stationary disturbance v. The goal within the IFT framework is to minimize $J(\rho)$ by finding the solution ρ so that

$$\begin{aligned} 0 &= \frac{\partial J}{\partial \rho} \\ &= \frac{1}{N} E\left[\sum_{t=1}^{N} \tilde{y}(\rho)\frac{\partial \tilde{y}(\rho)}{\partial \rho} + \delta \sum_{t=1}^{N} u(\rho)\frac{\partial u(\rho)}{\partial \rho}\right] \end{aligned}$$

This is done by obtaining a unbiased estimate of $\partial J/\partial \rho$ and modifying ρ in the negative gradient direction.

$$\rho_{i+1} = \rho_i - \gamma_i R_i^{-1} \frac{\partial J}{\partial \rho_i} \quad (6)$$

where R_i is some appropriate positive definite matrix, typically an estimate of the Hessian of J and γ is the step size.

In what follows, δ is set to zero making it in effect a minimum variance cost function that is monitored. With $\delta > 0$ a cost function is obtained that would result in a more conservative controller. The effect of δ is of interest but will not be pursued further in this paper.

Within the IFT framework the gradient is found by performing two experiments (when controller is one-degree of freedom) under which input and output data is collected. The resulting data series are of length N and are referred to as u^i, y^i for $i = 1, 2$.

The first experiment consists of collecting data under normal operation. This results in data series u^1, y^1. In the second experiment y^1 is subtracted from the reference value and the resulting signal, $r - y^1$, is fed to the closed loop system. This generates data series u^2, y^2.

When a synthetic gradient is calculated the last experiment is omitted. In stead $r - y^1$ is fed through a model of the closed loop system. This model will be referred to as $\widehat{T}(\rho)$ to distinguish it from the actual $T(\rho)$.

The equation for the synthetic gradient is now given for $\delta = 0$.

$$\frac{\widehat{\partial J}}{\partial \rho} = \frac{1}{N}\sum_{t=1}^{N}[r_t - y_t^1]\frac{1}{C}\frac{\partial C}{\partial \rho}\widehat{T}(\rho)[r_t - y_t^1] \quad (7)$$

Remark Since it is the minimum of the cost function that is of interest, in what follows when referring to the gradient, the direction to that minimum is what is meant, i.e. the negative of the traditional gradient.

Remark Notice that for parameter K in the PI algorithm this equation becomes particularly simple, namely

$$-\widehat{\frac{\partial J}{\partial K}} = -\frac{1}{N}\sum_{t=1}^{N}[r_t - y_t^1]\frac{1}{K}\widehat{T}(\rho)[r_t - y_t^1] \quad (8)$$

A positive value of this gradient indicates loop gain could be increased to reduce $J(\rho)$.

2.2 *Tuning rule*

The tuning rule used is λ-tuning and applies specially for PI controllers. Among the first references to λ-tuning is (Dahlin, 1968). The process is modeled as

$$G(s) = \frac{K_p}{Ts+1}e^{-Ls}$$

In λ-tuning the closed loop transfer function is specified as

$$T(s) = \frac{1}{T_{cl}s+1}e^{-sL} \quad (9)$$

where $T_{cl} = \lambda T$. The main tuning parameter is λ. By approximating e^{-sL_n} with $1 - sL$ and using pole placement, then given λ, the PI parameters are given as

$$K = \frac{1}{K_p}\frac{T}{L+T_{cl}} \qquad T_i = T \quad (10)$$

Within the pulp and paper industry the recommended values for λ is $1 - 3$. To calculate the gradient with Eq. (7) the transfer function given by Eq. (9) is used as $\widehat{T}(\rho)$

Manual tuned loop Assume the loop has been tuned manually. If dead time, L, is known and it is possible to characterize the tuning with λ then the relations $T_{cl} = \lambda T$ and $T_i = T$ can be used to find $\widehat{T}(\rho)$.

3. USING THE GRADIENT FOR PERFORMANCE MONITORING

In the current section it is shown how the gradient can be used for performance monitoring. This is best illustrated by an example. Observations will be made regarding the behavior of the gradient under normal conditions and abnormal conditions. From these observations, the performance monitoring algorithm will be suggested in later sections.

Within the IFT methodology, the gradient is calculated with regard to what disturbances are actually affecting the loop. A frequency domain approximation of the cost function $J(\rho)$, with $r = 0$ and $\delta = 0$, can be found by Parseval's relation

$$J(\rho) \approx \frac{1}{4\pi}\int_{-\pi}^{\pi}|S(\rho)|^2\Phi_{vv} \quad (11)$$

Φ_{vv} is the power spectrum of the disturbances. The controller parameters affect the sensitivity function by lifting it up or dragging it down on specific frequency intervals. If the power spectrum of the disturbances is concentrated in a frequency region where an increase in a parameter increases the gain of the sensitivity function, the gradient with regard to this parameter will be negative. If the sensitivity function gain is reduced with a positive change in the parameter the gradient will be positive.

In what follows, the gradient will be calculated for a number of different disturbances. It is not implied that a typical loop in a plant is affected only by disturbances of this kind even though an attempt has been made to cover many cases. In reality the disturbances affecting a loop can vary greatly in frequency content and intensity. Sometimes they can best be described in a stochastic fashion while other times more deterministic disturbances seem to be affecting the loop. The deterministic disturbances might be related to the operating points of the plant or other factors.

Plant Assume the process is a typical monotonic process,

$$G_0(s) = \frac{e^{-5s}}{(10s+1)(s+1)} \quad (12)$$

A PI controller is found with Eq. (10) after identifying the FOPDT parameters from a step response.

3.1 *Disturbance is sinusoidal*

A sinusoidal was applied to the system through v_t, $v_t = sin(\omega_0 t)$. As there is no stochastic component in this disturbance, variance is zero. The result is shown in Fig. 1.

The gradient is negative when the frequency is above 0.08. This can be explained from Fig. (2) and by considering Eq. (11) while keeping in mind that the power spectrum of a sinusoidal is like a delta function around the disturbance frequency. There the sensitivity function is shown when gain is increased by a factor 2 or reduced by a factor 2. For those frequencies that the sensitivity function increases with increase in gain, the gradient will be negative. This is for frequencies higher than 0.08. For lower frequencies, increasing the gain, drags the sensitivity function down. This gives a negative gradient.

A negative gradient indicates that the disturbances have much power in the frequency range where the sensitivity function can be reduced by reducing the gain. This is typically high frequency. A positive gain means the disturbances are of low frequency character and by increasing gain performance can be improved.

Notice that the smallest frequency is well below the system bandwidth and can be compared with a slow load disturbance.

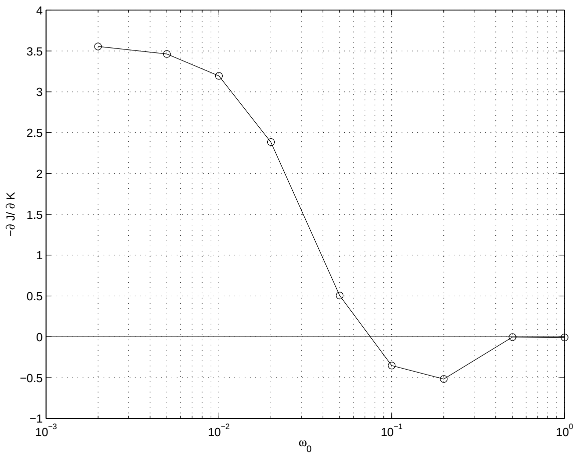

Fig. 1 $-\partial J/\partial K$ as a function of frequency

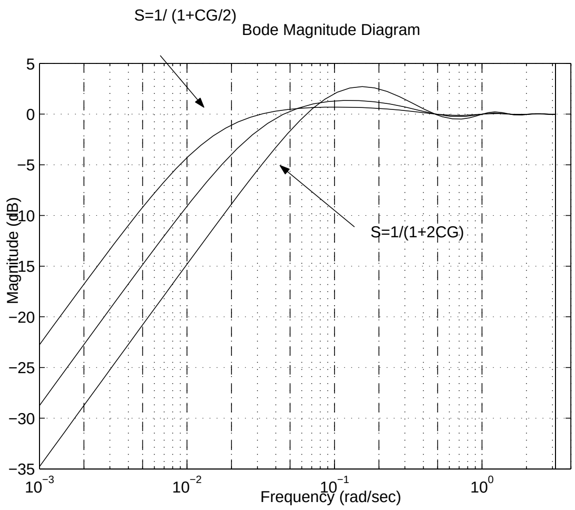

Fig. 2 Sensitivity function S with the frequencies as in Fig. 1 marked with vertical, dashed lines.

3.2 *Stochastic disturbance*

The disturbance model was

$$v_t = G_d(z)e_t = G_0(z)\frac{1}{z-1}e_t \tag{13}$$

The disturbance is driven by noise e with variance 1. In Fig.4 the level curves of the cost function $J(\rho)$ are shown. The cost function has a minimum for $K = 2.1$ and $T_i = 15$ approximately. The thick dashed line farthest to the right is the stability boundary. Robustness of the controllers within this parameter space changes with parameters as expected. As a measure of robustness the maximum sensitivity is used.

$$M_s = \max_{\omega} |S(i\omega)| \tag{14}$$

Shown in Fig. 4 are three lines for equal maximum sensitivity, namely $M_s = [1.4, 2, 5]$. The tuning for $\lambda = 1$ and $\lambda = 3$ is shown as $\bigcirc$ and $\times$ in the lower left corner.

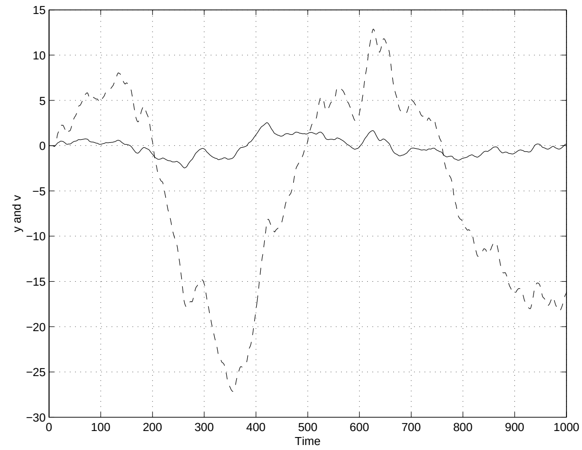

Fig. 3 Typical disturbance v ($--$) and output signal y ($-$) for Example 1 with $\lambda = 3$.

Table 1 Mean and standard deviations of gradient and other important numbers

	$-\frac{\partial J}{\partial K}$	$-\frac{\partial J}{\partial T_i}$	$\Delta J(\rho)$	$\left\|\frac{\partial J}{\partial \rho}\right\|$
Mean	1.81	-0.04	0.75	1.81
Std	0.41	0.014	0.067	0.41

The gradient, $-\partial J/\partial \rho$ was calculated using the approximate model of the closed loop given by Eq. (9) for 100 different disturbance realizations that were filtered through the actual sensitivity function. Sampling time was 1, and 1000 points were collected. The signal was normalized to have variance estimate equal to 1. A typical realization can be seen in Fig. 3. The mean and standard deviation for the components of the gradient are shown in Table.1.Also shown is the mean and standard deviation the length of the gradient. The statistical properties of the gradient estimate can be obtained by considering Eq.(8) as an estimate of the cross correlation between two signals for time shift equal to zero. As expected from Fig. 4 the gradient recommends increase in loop gain. The steepest decent is in the positive K direction.

Some observation can be made at this point.

- When a stochastic disturbance affects the system, the gradient estimate is a stochastic variable as well. Drop in variance should mean that a more deterministic disturbance is affecting the loop.
- The tuning is quite conservative for the disturbances affecting the system. This is not limited to the tuning method presented. Most tuning method used for PID controllers would result in a conservative controller.
- Direction is an indicator of where the current tuning is close to instability or not. If $-\partial J/\partial K$ is negative, meaning reducing loop gain would

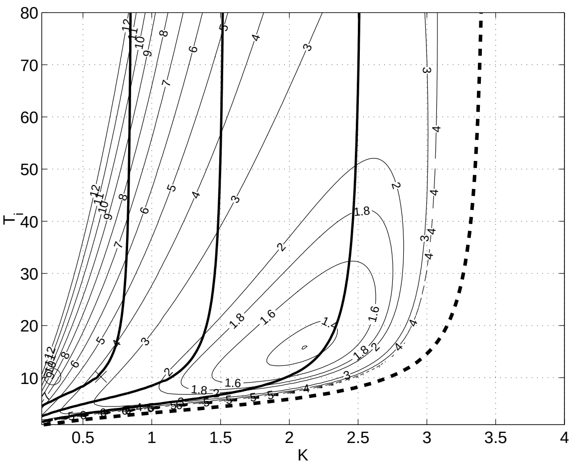

Fig. 4 Contour curves with $\delta = 0$ with example 1.

Table 2 Mean and standard deviations of loop gain gradient as a function of ΔK.

ΔK	Mean($-\partial J/\partial K$)	Std($-\partial J/\partial K$)
2	1.1	0.32
4	0.41	0.20
6	0.17	0.16
8	-0.23	0.09

increase performance one is closer to the region of instability than if it is positive

- Length of gradient is an indicator how far from the minimum one is. The IFT method aims at finding the parameter ρ so that $\partial J/\partial\rho = 0$. As the gradient becomes longer, one can assume one is moving away from better performance to worse.

3.3 *Increase in gain*

Assume that the static gain of the process is increased. This is described by the equation

$$y_t = G_0 \Delta K u_t + v_t \tag{15}$$

where ΔK corresponds to the increase in gain. In Table. 2 the loop gain gradient is shown. 10 disturbance realizations were simulated for each ΔK again through the real sensitivity function while the gradient was calculated with Eq. (8). In Fig. 5 typical realizations are shown for each case.

As expected, the gradient becomes smaller until finally it becomes negative. The variance is also reduced greatly. The increase in gain means the disturbances with frequency where the amplitude of S is larger than 1 are amplified further as an increase in ΔK pushes the peak of S up. With $\Delta K = 8$ the system has very small amplitude marginal.

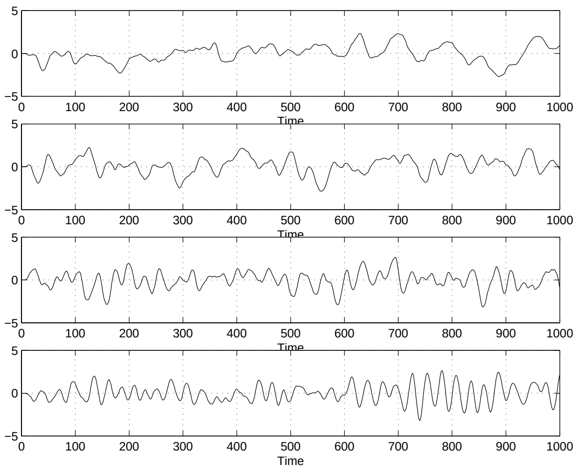

Fig. 5 Normalized responses for gain case, gain goes from 2 to 8 from increasing from top to bottom..

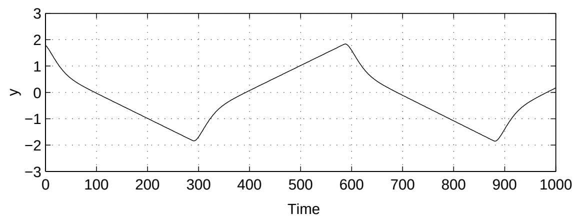

Fig. 6 Limit cycle caused by friction.

3.4 *Limit cycle because of friction*

A simple model of a valve with Coulomb friction was added in front of the plant. The model that was used was similar to the one presented in (Horch, 2000). The stiction force was 0.1 while the Coulomb force was half that.

In Fig.6 a simulation can be seen of the resulting limit cycle. No noise was added to the system so that the dominating frequencies would be the one from the friction limit cycle. The gradient was calculated when starting at 10 different time points of the period of the limit cycle. The resulting mean and standard deviation of $-\partial J/\partial K$ were

$$\begin{aligned} \text{Mean}(-\partial J/\partial K) &= 2.97 \\ \text{Std}(-\partial J/\partial K) &= 0.04 \end{aligned}$$

The limit cycle causes a drop in variance. The gradient has a high positive value similar to when sinusoidal frequencies of low frequencies was added to the system.

4. ALGORITHM

It is suggested that the loop gain gradient is calculated periodically with certain frequency, for example using 1000 data points or 20 to 30 times T_{cl}. The variance of

y, σ_y^2, is estimated for the same period. Alarms should not be sounded if σ_y^2 is very low.

The **mean** of the gradient should be monitored and an alarm should be sounded if it becomes negative. The stochastic character of the gradient should be taken into account when determining if the gradient is negative. A negative gradient means the system might be close to instability. A softer alarm should be sounded if it becomes to long since this indicates performance is very far from optimal.

The **variance** should be monitored and an alarm could be sounded when there is a drop in it. A drop in variance indicate disturbances are more deterministic such as load disturbances or disturbances with one frequency dominant (such as limit cycles).

Methods within statistical process control might be appropriate to monitor these things (such as $\bar{x}$ and R control charts). Operators in control rooms should be able to plot the gradient over a long period of time, over all operating regions. Notice that this plot can give valuable information regarding possibilities to trim the loop further. If the gradient is always positive and far from zero, this indicates the loop gain could be increased which is a good indication the loop could be made faster.

5. CONCLUSIONS

A method to monitor performance of PID controller has been suggested in much reduced form because of lack of space. The monitoring is done by calculating the synthetic gradient of a quadratic cost function by using a model of the closed loop obtained from the tuning method.

With this method it is possible to get valuable information about whether the current controller is suitable for the type of disturbances affecting the loop. Abnormal operating conditions such as if system is close to instability, or if loop is in a limit cycle, can be detected. The data collected give valuable information for trimming the loop. The method is non-invasive and requires little prior information to be used. The parameter values of a well trimmed PI are sufficient.

Focus is on PI even though the method is not limited to this structure. The fact that control engineers are familiar with this structure should make the method more useful.

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