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H_∞ DESIGN FOR AN INDUCTION MOTOR [⋆]

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Abstract: A controller for an induction motor is designed using H_∞ control theory and input-output feedback linearization. Because of the special structure of the state-space equations governing the induction motor a Linear Parameter Varying (LPV) feedback controller, scheduled with rotor speed, is used for the inner current loop. An LPV observer is also used to estimate the flux vector. The application is based on the nonlinear model and tracking requirements of a recently published benchmark. The proposed controller delivers high performance over the entire operating range of the induction motor.

Keywords: Induction Motor, Input-Output Linearization; State-Feedback; Linear Matrix Inequalities; H_∞ Optimisation; LPV Gain Scheduling Control.

1. INTRODUCTION

Induction motors are mainly used in industry to transform electrical energy into mechanical energy. However, induction motors are rarely used as actuators because they are significantly more difficult to control than d.c. motors. Nowadays, therefore, there is great interest in developing high performance control laws to make induction motor performance rival that of the d.c. motor in a number of high precision applications (e.g. robotic applications). Induction motors are theoretically challenging for control engineers because as dynamical systems they are highly nonlinear, the flux which is to be controlled (and sometimes the rotor speed) is not available. Also, physical parameter uncertainties, such as the variation of the rotor resistance with temperature, affect significantly the dynamics of the system. Overviews of the important induction motor control techniques are given in (Bose, 1998) and (Taylor, 1994).

In this paper, the design of a controller for an induction motor is based on Linear Parameter Variant H_∞, the so-called LPV theory (Apkarian and Gahinet, 1995), (Gahinet and al, 1995), and input/output feedback linearisation. It is assumed that only the stator currents and the rotor speed are available for measurement. Broadly speaking, the control law consists of a fast LPV inner loop used to track stator current references which are generated by a non-linear input-output linrearization state feedback gain. The flux vector is estimated by an observer which, as we will see, has also an LPV structure. Robust stability and robust performance of the flux observer and the current loop are easy to prove in the LPV context. The non-linear model of the motor and tracking requirements come from the benchmark given in (Ortega *et al.*, 2000). Full non-linear simulations, including parametric variations, saturation limits, time delay and noisy measurements demonstrate that modern H_∞ techniques combined with input/output linearization offer a promising and effective way to design robust controllers for induction motors.

The paper is organized as follows. The model of the induction motor is given in section 2. The controller design is described in section 3. Section

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4 includes the simulations results and conclusions are given in section 5.

2. INDUCTION MOTOR MODEL

We consider a squirrel-cage induction motor whose nominal physical parameters are given in table 1.

Description	Parameter	value	Units
Stator Inductance	L_s	0.47	H
Rotor Inductance	L_r	0.47	H
Mutual Inductance	L_{sr}	0.44	H
leakage factor $\sigma_s = \sigma_r$	σ	0.12	
Stator resistance	R_s	0.8	Ω
Rotor resistance	$\bar{R}_r$	3.6	Ω
Moment of inertia	D_m	0.06	$kg.m^2$
viscous damping constant	R_m	0.04	$N.m.s$
Number of pole pairs	n_p	2	

Table 1. Nominal physical parameters of the induction motor

Under the assumption of linear magnetic circuits, a 5th order non-linear model (stator-fixed frame) of the induction motor is given by

$$G: \begin{cases} \dot{x_1} = a_1(x_2x_5 - x_3x_4) + a_2x_1 + a_3\tau_L \\ \dot{x_2} = a_4x_2 - n_px_1x_3 + a_5x_4 \\ \dot{x_3} = n_px_1x_2 + a_4x_3 + a_5x_5 \\ \dot{x_4} = a_6x_2 + a_7x_1x_3 - \gamma x_4 + a_8u_1 \\ \dot{x_5} = -a_7x_1x_2 + a_6x_3 - \gamma x_5 + a_8u_2 \\ y = [x_1, x_4, x_5]^T \end{cases} \quad (1)$$

The state vector is $x = [\omega, \phi_a, \phi_b, i_a, i_b]^T = [x_1, x_2, x_3, x_4, x_5]^T$, where ω is the rotor speed, $\phi = [\phi_a, \phi_b]^T$ are the rotor fluxes, $i_s = [i_a, i_b]^T$ are the stator currents and $u_s = [u_1, u_2]^T$ represents the stator voltages. The measured output is $y = [\omega, i_a, i_b]^T$, the control input is $u_s = [u_1, u_2]^T$ and τ_L is the load torque disturbance.
The outputs to be controlled are

- Shaft rotor speed $x_1 = \omega$
- Rotor flux norm $\Phi = \|\phi\|$

The parameters of the induction motor model are defined as follows: $a_1 = n_pL_{sr}/(D_mL_r)$, $a_2 = -R_m/D_m$, $a_3 = -1/D_m$, $a_4 = -1/T_r$, $a_5 = L_{sr}/T_r$, $a_6 = L_{sr}/(T_r\sigma L_sL_r)$,$a_7 = n_pL_{sr}/(\sigma L_sL_r)$ and $a_8 = 1/(\sigma L_s)$ where

$$T_r = \frac{L_r}{R_r}, \quad \gamma = \frac{R_s}{L_s\sigma} + \frac{L_{sr}^2}{L_s\sigma L_rT_r}$$
$$\sigma = 1 - \frac{L_{sr}^2}{L_sL_r} \quad (2)$$

with nominal values given in table 2.
The rotor resistance R_r, which will change in the experiments, and the load torque disturbance τ_L. R_r is assumed to vary in the range $[0.7\bar{R}_r, 1.3\bar{R}_r]$. The change in the rotor resistance affects linearly or affinely the values of parameters a_4, a_5, a_6

Parameter	nominal value
a_1	31.21
a_2	-0.667
a_3	-16.67
$\bar{a}_4$	-7.66
$\bar{a}_5$	3.37
$\bar{a}_6$	127.14
a_7	33.19
a_8	17.73
$\bar{\gamma}$	197.78
$\bar{\tau}_L$	$7(Nm)$

Table 2. Nominal parameter values of the induction motor model

and γ. The load torque disturbance is assumed to vary in the range $[0.25\bar{\tau}_L, \bar{\tau}_L]$. In table 2, the nominal values of parameters subject to variation are denoted by $\bar{a}_4$, $\bar{a}_5$ etc...; similarly, $\bar{R}_r$ in table 1.

3. CONTROLLER STRUCTURE AND CONTROLLER DESIGN

The controller is made up of four elements which will be designed separately. Figure 1 shows the structure considered. K_{lpv} is a current feedback controller which tracks the current setpoint reference i_{sref}. The role of K_{lpv} is to ensure good tracking over the entire operating range (i.e. when ω is varying). It will have an LPV form and be designed using quadratic H_∞ gain scheduling stabilisation (Apkarian and Gahinet, 1995), (Apkarian and Adams, 1998), (Gahinet and al, 1995).

The reference current vector i_{sref} is generated through a static input-output linearization state feedback F_{io}. The flux vector is estimated by G_{obs} which, as we will see has an LPV structure. Finally, K_{lin} is a simple LTI regulator based on the linearized map relating the input ν with the speed and the flux.

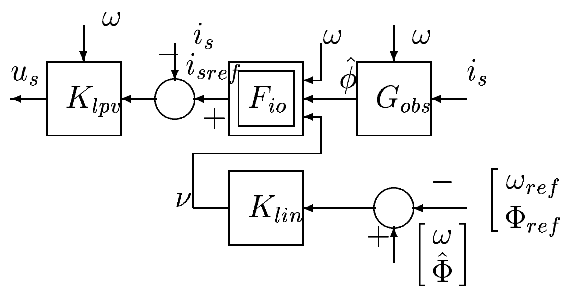

Fig. 1. Controller structure

3.1 *Polytopic H_∞ current feedback*

Let us consider the second, third, fourth and fifth state space equations of the induction motor given in (1). These four equations constitute an affine parameter-dependent plant if the rotor speed $x_1 =$

ω is taken as the parameter. More precisely, let us define the subsystem G_1 with state vector $\xi = [x_2, x_3, x_4, x_5]^T$ having $i_s = [x_4, x_5]^T$ as output vector and $u_s = [u_1, u_2]^T$ as input vector. From (1) G_1 can be written as follows:

$$G_1 : \begin{cases} \dot{\xi} = (A_0 + \omega A_1)\xi + Bu \\ i_s = C\xi \end{cases} \quad (3)$$

with

$$A_0 = \begin{pmatrix} a_4 & 0 & a_5 & 0 \\ 0 & a_4 & 0 & a_5 \\ a_6 & 0 & -\gamma & 0 \\ 0 & a_6 & 0 & -\gamma \end{pmatrix}, \quad (4)$$

$$A_1 = \begin{pmatrix} 0 & -n_p & 0 & 0 \\ n_p & 0 & 0 & 0 \\ 0 & a_7 & 0 & 0 \\ -a_7 & 0 & 0 & 0 \end{pmatrix} \quad (5)$$

$$B = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ a_8 & 0 \\ 0 & a_8 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (6)$$

and with

$$\omega \in [\omega_{min}, \omega_{max}] = [-110, 110] \quad rad/s \quad (7)$$

Alternatively, $G_1(t)$ admits the following polytopic state-space representation

$$G_1 : \alpha(t)S_1 + (1 - \alpha(t))S_0 \quad (8)$$

with

$$\begin{cases} S_0 = (\tilde{A}_0, B, C, 0_{2\times2}) \\ S_1 = (\tilde{A}_1, B, C, 0_{2\times2}) \\ \tilde{A}_0 = A_0 + \omega_{min} A_1 \\ \tilde{A}_1 = A_0 + \omega_{max} A_1 \\ 0 \leq \alpha(t) = \dfrac{\omega(t) - \omega_{min}}{\omega_{max} - \omega_{min}} \leq 1 \end{cases} \quad (9)$$

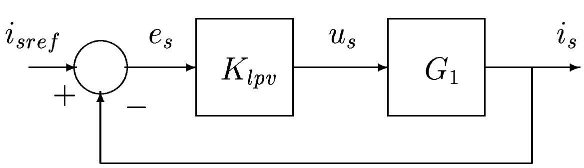

Fig. 2. Current feedback

The robust multivariable LPV controller K_{lpv} has to provide satisfactory performance over the entire operating range of the motor (i.e. when ω varies). The design problem is tackled using the one-degree-of-freedom control structure depicted in figure 2. A mixed-sensitivity T/S loop shaping quadratic-gain H_∞ optimization is proposed for the design of the polytopic regulator K_{lpv}. The optimization problem is to find a stabilizing control law K_{lpv} to minimize, for all ω in (7), in a quadratic H_∞ sense, the cost function

$$\left\| \begin{bmatrix} W_T T \\ W_S S \end{bmatrix} \right\|_\infty \quad (10)$$

W_S and W_T are used to shape the output sensitivity function $S = (I + G_1 K_{lpv})^{-1}$ and the complementarity sensitivity function $T := I - S$.

$$W_S = diag(w_S, w_S), w_S = \frac{s + \omega_{BS} A_S}{1/M_S s + \omega_{BS}} \quad (11)$$

with $A_S = 1/500$, $M_S = 2$, $\omega_{BS} = 550$. The following constant weight was chosen

$$W_T = diag(0.8, 0.8) \quad (12)$$

The polytopic regulator K_{lpv} was computed using the function *hinfgs* in the LMI control toolbox (Gahinet and al, 1995). The H_∞ performance, γ_1, was approximately 1.37. The closed-loop time-responses of the current controlled system for a 1 A step demand in i_a along the rotor speed trajectory

$$\omega(t) = 1000t \quad (13)$$

are given in figure 3. The decoupling is excellent. The settling time is about $60ms$ and the overshoot of 1.1 A is small. The overshoot on the control input is about 27 V which can be considered satisfactory.

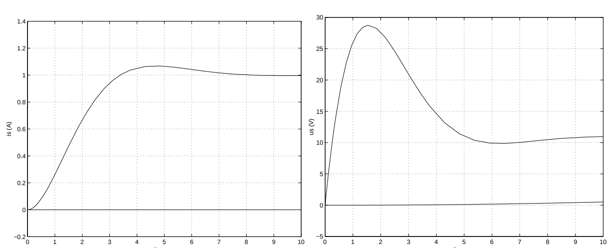

Fig. 3. Unit step closed-loop responses of the current gain scheduled feedback system along the rotor speed trajectory given in (13). Stator currents (left) and corresponding stator tensions (right)

3.2 *Input-ouput linearization*

Let $Y = [\omega, \Phi]$ with $\Phi = \sqrt{\phi_a^2 + \phi_b^2}$. For the induction motor, the relative degree is well defined and the zero dynamics are stable; thus the input-output linearization can be performed. Let us consider the three first equations of (1). Assuming that the rotor speed and the flux vector ϕ are available for measurement, our aim is to construct a non-linear relation to produce an ideal reference stator current vector i_{sref} to make the closed-loop

input-output map $T_{\nu,Y}$ linear.
Let ν_2 be the first derivative of the flux. From (1) and using the definition of Φ, we get

$$\dot{\Phi} = \frac{\phi_a(\bar{a}_4\phi_a + \bar{a}_5 i_{aref}) + \phi_b(\bar{a}_4\phi_b + \bar{a}_5 i_{bref})}{\Phi} := \nu_2 \tag{14}$$

Let ν_1 be the first derivative of the rotor speed $\omega = x_1$. ν_1 is given by

$$\dot{\omega} = a_1(\phi_a i_{bref} - \phi_b i_{aref}) + a_2\omega + a_3\bar{\tau}_L := \nu_1 \tag{15}$$

(14) and (15) can be rewritten as

$$\begin{bmatrix} \phi_a\bar{a}_5 & \phi_b\bar{a}_5 \\ -a_1\phi_b & a_1\phi_a \end{bmatrix} \begin{bmatrix} i_{aref} \\ i_{bref} \end{bmatrix} = \begin{bmatrix} \Phi(\nu_2 - \bar{a}_4\Phi) \\ \nu_1 - a_3\bar{\tau}_L - a_2\omega \end{bmatrix} \tag{16}$$

Thus,

$$F_{io}(\omega,\phi,\nu) := \begin{bmatrix} i_{aref} \\ i_{bref} \end{bmatrix} = \frac{1}{a_1\bar{a}_5\Phi^2} \begin{bmatrix} a_1\phi_a & -\phi_b\bar{a}_5 \\ a_1\phi_b & \phi_a\bar{a}_5 \end{bmatrix} \begin{bmatrix} \Phi(\nu_2 - \bar{a}_4\Phi) \\ \nu_1 - a_3\bar{\tau}_L - a_2\omega \end{bmatrix} \tag{17}$$

The nonlinear state feedback control law $i_{sref} = [i_{aref}, i_{bref}]^T$ given in (17) is well defined whenever $\Phi \neq 0$ and results in

$$G_2 : \begin{cases} \dot{\omega} = \nu_1 \\ \dot{\Phi} = \nu_2 \\ Y = [\omega, \Phi] \end{cases} \tag{18}$$

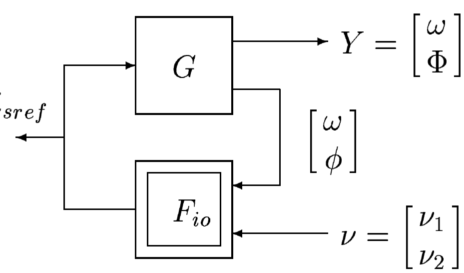

Fig. 4. Input-Ouput linearization

3.3 *LPV flux observer design*

An LPV flux estimator can be designed using both tension and current measurements. The design is based on the plant G_3 which is equal to G_1 but with $C = I_4$, where a_4 is the uncertain parameter which is linearly dependent on the rotor resistance. The aim is to design an observer able to maintain a good flux estimation at any speed despite uncertainty in the rotor resistance. We know that G_1 admits a polytopic representation (S_0, S_1) where a_4 enters linearly in S_0 and S_1. Therefore, we can extract a_4 from the polytopic representation using the concept of Linear Fractional Transformations (see e.g (Zhou *et al.*, 1995)). Our aim is to take explicitly into account the uncertainty due to the rotor resistance variation in the design of the flux observer. Let us consider the block diagram of figure 5. The matrix δI_2 represents the rotor resistance uncertainty. It is normalized in such a way that $\|\delta\|_\infty \leq 1$.

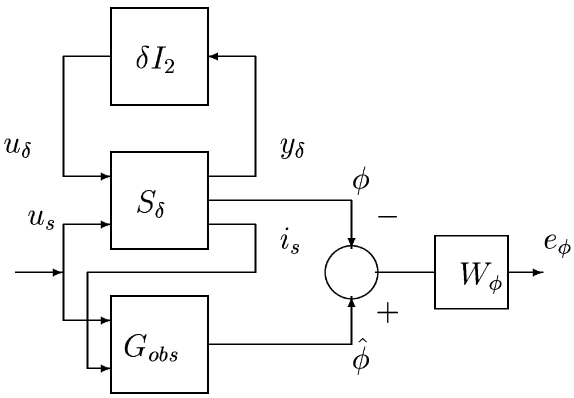

Fig. 5. Flux observer interconnection structure

More precisely, the uncertain parameter a_4 can be rewritten as $a_4 = \bar{a}_4(1+\delta_a)$, $|\delta_a| < \delta_{max}$, $\delta_{max} = \frac{a_{4max} - a_{4min}}{2\bar{a}_4}$. Let us consider, the following LTI system which corresponds to S_0 (the vertex for $\omega = \omega_{min}$).

$$S_{0\delta} : \begin{cases} \dot{x} = (\bar{a}_4 A_2 + \omega_{min} A_1)x + P_{12}u_\delta/\delta_{max} + Bu \\ y_\delta = P_{21}x \\ y = x \end{cases} \tag{19}$$

with

$$A_2 = \begin{pmatrix} 1 & 0 & -L_{sr} & 0 \\ 0 & 1 & 0 & -L_{sr} \\ -\frac{L_{sr}}{\sigma L_s L_r} & 0 & \frac{L_{sr}^2}{\sigma L_s L_r} & 0 \\ 0 & -\frac{L_{sr}}{\sigma L_s L_r} & 0 & \frac{L_{sr}^2}{\sigma L_s L_r} \end{pmatrix} \tag{20}$$

$$P_{12} = \begin{pmatrix} 1 & 0 & -\frac{L_{sr}}{\sigma L_s L_r} & 0 \\ 0 & 1 & 0 & -\frac{L_{sr}}{\sigma L_s L_r} \end{pmatrix}^T \tag{21}$$

$$P_{21} = \begin{pmatrix} 1 & 0 & -L_{sr} & 0 \\ 0 & 1 & 0 & -L_{sr} \end{pmatrix} \tag{22}$$

and where A_1 and B are given in (5) and (6) respectively.

It is easy to show that

$$S_0 = F_l(\delta I_2, S_{0\delta}) \tag{23}$$

and

$$S_1 = F_l(\delta I_2, S_{1\delta}) \tag{24}$$

where $S_{1\delta}$ is the same as $S_{0\delta}$ if one replaces ω_{min} by ω_{max} in (19) and where $\delta := \delta_a/\delta_{max}$.

The observer design is based on the interconnection of figure 5. The regulated output is the weighted flux estimation error e_ϕ. The measured outputs are the current vector and the supply voltage. Set

$$w = \begin{bmatrix} u_\delta \\ u_s \end{bmatrix} \quad z = \begin{bmatrix} y_\delta \\ e_\phi \end{bmatrix} \quad u = \hat{\phi} \quad y = \begin{bmatrix} u_s \\ i_s \end{bmatrix} \qquad (25)$$

The observer design consists of finding $u = G_{obs} y$ to minimize, for all admissible speed trajectories, the closed-loop quadratic H_∞ performance from w to z (γ_3). According to the small gain theorem, robust stability and robust performance will be achieved whenever $\gamma_3 < 1$. As is customary in μ-synthesis e.g. (Zhou *et al.*, 1995), (Balas and al, 1993),(Boyd *et al.*, 1994) we can enforce a better minimization of T_{e_ϕ,u_s}, by scaling u_δ by $d_s = diag(d_1, d_2)$ with $d_1 > 0$, $d_2 > 0$ and y_δ by d_s^{-1} without affecting performance and stability.

W_ϕ was chosen as the identity matrix and after some trial and error d_s was selected as

$$d_s = diag(3.7, 3.7) \qquad (26)$$

The H_∞ performance, γ_3, was approximately 1.17. It is easy to check that $\|T_{y_\delta,u_\delta}\|_\infty < 1$. Therefore the observer is robustly, quadratically stable (i.e. stable for any variation of the rotor resistance and for any speed trajectory). The frequency responses of T_{e_ϕ,u_s} for various values ω and a_4 as given in figure 6.

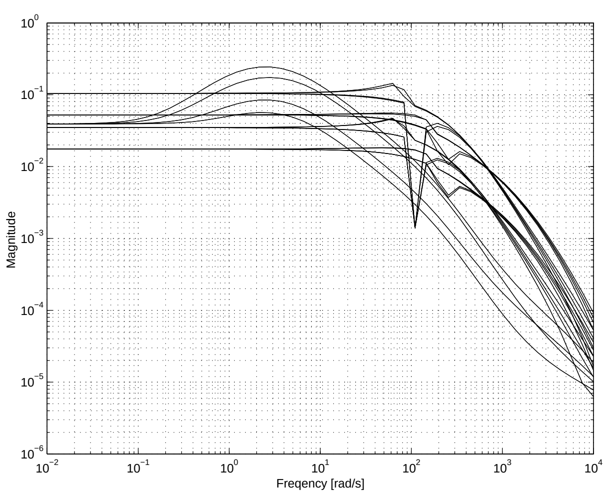

Fig. 6. Singular value plot of T_{e_ϕ,u_s} for 5 and 4 equally spaced values of ω and a_4

3.4 *Control of the speed and the flux*

To control the speed and the flux, one has to consider the plant G_2 given in (18). It is a chain of 2 integrators, representing the nominal closed-loop input-output map $T_{Y,\nu}$ when the condition $i_s = i_{sref}$ holds. The regulator can be designed using any classical linear method. In this example, a simple diagonal gain was found to provide good tracking and sufficient robustness. Trial and error led to $K_{lin} = -diag(343, 286)$, for which the closed-loop bandwidth is approximately $300 rad/s$.

4. SIMULATION RESULTS

Full non-linear simulations were carried out for the speed and flux step demand profiles and for parameter variations shown in figure 7. Figure 7 represents the worst case rotor resistance changes which may correspond to breaks in the rotor bars. Saturation limits on the stator tensions and on the the reference currents were included to prevent peaks especially at initialization. Zero-order holds with a sampling frequency of $F_s = 4kHz$ and a unit time delay $T_s = 1/F_s$ were added in the feedback loop to simulate a digital implementation. The saturation limits on the reference current components were taken as

$$|i_{srefi}| < 7A, \quad i = 1, 2 \qquad (27)$$

and for the control signal

$$|u_i| < 210V, \quad i = 1, 2 \qquad (28)$$

The flux and speed demands were filtered through the following second order filters

$$F_{ref} := \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \qquad (29)$$

with $\omega_n = 8$, $\xi = 0.8$.

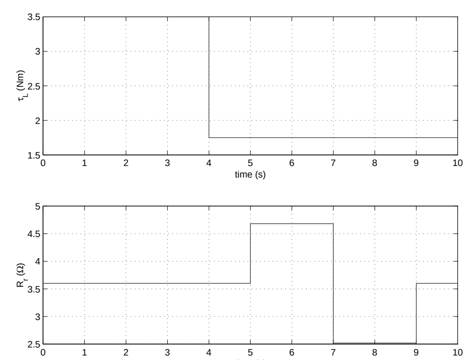

Fig. 7. Load torque and rotor resistance variations

4.1 *Controller using the LVP observer*

The estimation of the flux is provided by the LPV observer. The response of the rotor speed, given in figure 8. Figure 9 shows that the modulus of the control signal (u_s) stays within the limits

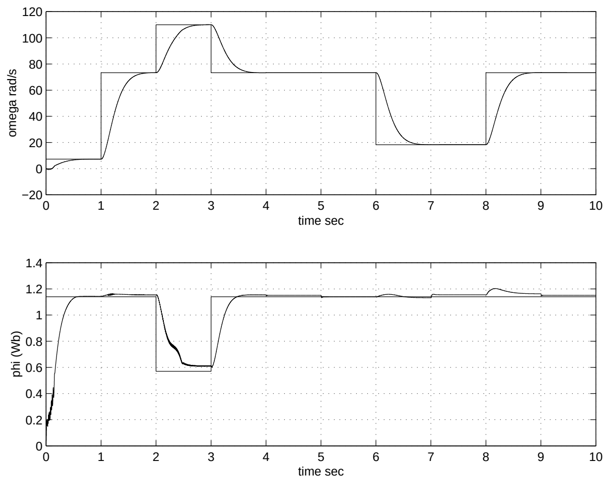

Fig. 8. Speed and flux tracking with the LPV observer G_{obs}

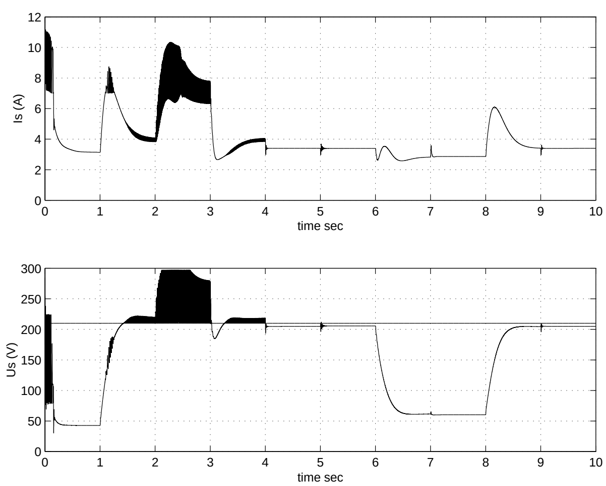

Fig. 9. Stator voltage modulus $\|u_s\|$ and stator current modulus $\|i_s\|$ with limits (LPV observer)

except between $t = 2s$ and $t = 3s$ when the speed demand is $110rad/s$. The stator current modulus remains below the limit of $12A$.

5. CONCLUSIONS

This paper has presented a non-linear controller design for an induction motor. It was assumed that only the stator currents and the rotor speed were available for measurement. The controller has a structure made up of four different subsystems: an LPV feedback current loop, a non-linear linearizing state feedback, an LPV observer and an LTI regulator. An LMI-based approach, has been proposed to design a quadratically stable flux observer and an output feedback regulator to track the stator currents. In both cases we have obtained a scheduled time varying system (LPV) which ensures a finite L_2 attenuation for a given closed-loop transfer function which represents the design requirements (tracking robustness for the LPV current regulator and flux estimation error for the LPV observer). The main advantage of using LPV methods is that they provide a systematic way of designing an H_∞ flux observer for the induction motor assuming that the rotor speed is available. Stability of the flux estimator was demonstrated using small-gain based analysis. The results for the benchmark were found to be satisfactory. Due to the very low order of the its components (K_{lpv} is a 2 by 2 speed dependent regulator with only 6 states, the speed dependent flux observer has only 4 states, 4 inputs and 2 outputs) the proposed control law is easy to implement and can work with relatively slow DSP cards. A drawback of the proposed controller is that it requires a measure of the rotor speed to update the LPV parts of the control law.

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6. REFERENCES

Apkarian, P. and P. Gahinet (1995). A convex characterization of gain-scheduled H_∞ controllers. *IEEE Trans. Aut. Contr.* **40**, 853–864.

Apkarian, P. and R. J Adams (1998). Advanced gain-scheduling techniques for uncertain systems. *IEEE Trans Contr Sys Technology.* **6**(1), 21–32.

Balas, G. and al (1993). μ-analysis and synthesis toolbox. *The Math Works.*

Bose, B. K. (1998). High performance control of induction motor drives. *IEEE Industrial Electronics Society Newsletter* **45**(3), 7–11.

Boyd, S., L. El Ghaoui, E. Feron and V. Balakrishnan (1994). *Linear Matrix Inequalities in System and Control Theory.* SIAM. Studies in Applied Mathematics.

Gahinet, P. and al (1995). LMI control toolbox. *The Math Works.*

Ortega, R., G. Chang and E. Mendes (2000). A benchmark for induction motor control. *International Journal of Adaptative Control and Signal Processing.*

Taylor, D. G. (1994). Nonlinear control of electric machines: An overview. *IEEE Control Systems* pp. 41–51.

Zhou, K., J. Doyle and K. Glover (1995). Robust and optimal control. *Prentice Hall.*