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# DESIGN OF DISCRETE VARIABLE STRUCTURE CONTROLLER FOR PERTURBED MIMO SYSTEMS

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Abstract: A simple technique of designing a robust model reference discrete-time variable structure output tracking controller for a class of perturbed MIMO linear systems is proposed in this paper. Both an adaptive mechanism and a perturbation estimation process are embedded in the proposed control scheme so that the upper bound of the perturbation estimation error is not required. It is shown that the tracking error will be constrained in a small bounded region, and the stability of the overall controlled system is guaranteed. A numerical example is given to demonstrate the feasibility of the proposed control scheme. *Copyright © 2002 IFAC*

Keywords: model reference, variable structure control, perturbation estimation

## 1. INTRODUCTION

Sliding mode control (SMC) has been studied since early sixties. This control technique is well known to have the invariant property for matched model uncertainties, parameter variations and external disturbances (Utkin, 1977). Lots of researchers have applied SMC to many practical applications, such as servo, robot, and flight control systems (Chern *et al.*, 1996; Fu and Liao, 1990; Singh and Coelho, 1984). Due to the fast development of the personal computers and DSP chips, the usage of computers has appeared in many control applications. Therefore, the design of robust discrete-time variable structure controllers becomes more and more important nowadays.

For solving multi-input discrete-time regulation problems, Spurgeon (1992) proposed a hyperplane design technique for discrete-time variable structure control systems. Fujisaki *et al.* (1994) proposed a controller which consists of a linear state feedback and a switching feedback for systems without perturbation, and select a small switching gain to eliminate the chattering phenomenon. Elmali and Olgac (1992) designed a sliding mode controller with perturbation estimation so that the knowledge of upper bound of perturbation does not required. Cheng *et al.*(2000) designed sliding mode controllers for systems with bounded matching perturbations without the information of upperbound of perturbation.

Some researchers also proposed discrete-time sliding mode adaptive controller for solving stabilization problems. Chen and Fukuda (1999) employed an adaptive sliding mode controller for linear multi-input discrete-time systems with bounded perturbation. Bartolini *et al.*(1995), Chan (1999) transformed a class of continuous-time systems to discrete-time systems, and then designed adaptive

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controller for linear systems with parameter variation.

The objective of this paper is to provide a simple design technique of discrete variable structure controllers for a class of perturbed MIMO linear systems for solving output tracking problems. An adaptive mechanism is employed to overcome the unknown upperbound of the perturbation estimation error, so that the knowledge of the upper bounds of perturbations is not required. In order to increase the tracking accuracy, a perturbation estimation scheme is utilized in the controller. Finally, the stability of the overall controlled system is proved under the proposed control scheme.

## 2. SYSTEM DESCRIPTIONS AND PROBLEM FORMULATIONS

Consider a class of perturbed MIMO linear discrete-time systems, whose dynamic equation can be described by

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) + \Delta\mathbf{P}(\mathbf{x}(k),k), \\ \mathbf{y}(k) &= \mathbf{C}\mathbf{x}(k), \end{aligned} \tag{1}$$

where $\mathbf{x}(k) \in \Re^n$, $\mathbf{u}(k) \in \Re^m$, $\mathbf{y}(k) \in \Re^p$ are the state vector, control input, and output vector respectively. $\mathbf{A} \in \Re^{n\times n}$, $\mathbf{B} \in \Re^{n\times m}$, $\mathbf{C} \in \Re^{p\times n}$ are known constant matrices. The vector $\Delta\mathbf{P}(\mathbf{x}(k),k) \equiv [\Delta P_1(\mathbf{x}(k),k) \quad \Delta P_2(\mathbf{x}(k),k) \quad \cdots \quad \Delta P_n(\mathbf{x}(k),k)]^T \in \Re^n$ represents the lumped modeling uncertainty, parameter variation, nonlinearity and/or disturbance.

For achieving the purpose of model reference control, a reference model is given by

$$\begin{aligned} \mathbf{x}_m(k+1) &= \mathbf{A}_m\mathbf{x}_m(k) + \mathbf{B}_m\mathbf{r}(k), \\ \mathbf{y}_m(k) &= \mathbf{C}_m\mathbf{x}_m(k), \end{aligned} \tag{2}$$

where $\mathbf{x}_m(k) \in \Re^q$ is the reference state variable, $\mathbf{r}(k) \in \Re^l$ is bounded reference input, $\mathbf{y}_m(k) \in \Re^p$ is the desired reference output. $\mathbf{A}_m \in \Re^{q\times q}$, $\mathbf{B}_m \in \Re^{q\times l}$, $\mathbf{C}_m \in \Re^{p\times q}$ are known constant matrices. The following assumptions are assumed to be valid throughout this paper :

**A1.** The states of the system (1) are all measurable, and $m \geq p$.

**A2.** The pair $(\mathbf{A},\mathbf{B})$ is completely controllable.

**A3.** There exist matrices $\mathbf{G} \in \Re^{n\times q}$ and $\mathbf{H} \in \Re^{m\times q}$ satisfying the following equation (Shyu and Liu, 1997)

$$\begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \mathbf{G} \\ \mathbf{H} \end{pmatrix} = \begin{pmatrix} \mathbf{G}\mathbf{A}_m \\ \mathbf{C}_m \end{pmatrix} \tag{3}$$

**A4.** There exists an unknown positive constant $g_0$ such that

$$\|\Delta\mathbf{P}(\mathbf{x}(k),k)\| \leq g_0.$$

***Remark 1*:** (Shyu and Liu, 1997) If (3) has no solution for $\mathbf{G}$, another $\mathbf{A}_m$ for the reference model should be chosen.

The objective of control is to design a robust controller so that the output $\mathbf{y}(k)$ can track the reference output $\mathbf{y}_m(k)$ in spite of the existence of perturbation $\Delta\mathbf{P}(\mathbf{x}(k),k)$.

## 3. DESIGN OF CONTROL SCHEME

The design procedures of the proposed control scheme are described as follows.

***Step 1: Design of switching surface***

Let the tracking error be

$$\mathbf{e}(k) \equiv \mathbf{x}(k) - \mathbf{G}\mathbf{x}_m(k). \tag{4}$$

If $\mathbf{e}(k) = \mathbf{0}$, then from (2) and (3) one can obtain $\mathbf{y}(k) = \mathbf{C}\mathbf{x}(k) = \mathbf{C}\mathbf{G}\mathbf{x}_m(k) = \mathbf{y}_m(k)$. In order to track the reference output $\mathbf{y}_m(k)$, the switching surface is designed as

$$\boldsymbol{\sigma}(k) = \mathbf{D}\mathbf{e}(k) - \mathbf{D}(\mathbf{A}+\mathbf{B}\mathbf{K})\mathbf{e}(k-1), \tag{5}$$

where $\mathbf{D} \in \Re^{m\times n}$ is a full rank matrix with constant elements, and $\mathbf{D}\mathbf{B} \in \Re^{m\times m}$ is invertible. The matrix $\mathbf{K} \in \Re^{m\times n}$ is designed such that all the eigenvalues of $(\mathbf{A}+\mathbf{B}\mathbf{K})$ are placed in a unit circle, i.e.,

$$|\lambda(\mathbf{A}+\mathbf{B}\mathbf{K})| < 1. \tag{6}$$

If the matrices $\mathbf{D}$ and $\mathbf{K}$ are designed appropriately, and the proposed controller can drive the trajectory of the switching variable $\boldsymbol{\sigma}(k)$ into a small bounded region (which will be shown in next section), then from (5) one can show that the output $\mathbf{y}(k)$ will closely track the reference output $\mathbf{y}_m(k)$. Then according to (4), it also implies that the stability of the proposed control system is guaranteed.

***Step 2: Design of controller***

The main purpose of this step is to design a control effort so that the trajectory of $\boldsymbol{\sigma}(k)$ will be driven toward zero or into a small bounded region. Before designing the proposed controllers and using the discrete Lyapunov stability theorem for analyzing the stability of the proposed control scheme, a variable $\Delta\boldsymbol{\sigma}(k+1)$ is first defined as

$$\Delta\boldsymbol{\sigma}(k+1) \equiv \boldsymbol{\sigma}(k+1) - \boldsymbol{\sigma}(k), \tag{7}$$

from which the proposed controllers can be designed.

By using (2), (3) and (5), (7) is rewritten as

$$\Delta\boldsymbol{\sigma}(k+1) \equiv \boldsymbol{\phi}(k) + \mathbf{D}\mathbf{B}\mathbf{u}(k) + \mathbf{D}\Delta\mathbf{P}(\mathbf{x}(k),k) \tag{8}$$

where

$$\begin{aligned} &\boldsymbol{\phi}(\mathbf{x}(k),k) \\ &\equiv [\phi_1(\mathbf{x}(k),k) \quad \phi_2(\mathbf{x}(k),k) \quad \cdots \quad \phi_m(\mathbf{x}(k),k)]^T \\ &= \mathbf{D}\mathbf{A}\mathbf{x}(k) - \mathbf{D}\mathbf{G}\mathbf{A}_m\mathbf{x}_m(k) - \mathbf{D}\mathbf{G}\mathbf{B}_m\mathbf{r}(k) - \mathbf{D}\mathbf{e}(k) \end{aligned}$$

$$-\mathbf{D}(\mathbf{A}+\mathbf{BK})\mathbf{e}(k)+\mathbf{D}(\mathbf{A}+\mathbf{BK})\mathbf{e}(k-1)$$

is a vector which is computable.

Now, according to (8), a robust discrete variable structure controller for the perturbed system (1) is proposed as

$$\mathbf{u}(k)=\mathbf{u}_f(k)+\mathbf{u}_{est}(k)+\mathbf{u}_s(k)+\mathbf{u}_{adp}(k), \quad (9)$$

where $\mathbf{u}_f(k)$ represents the feedback control part for eliminating the functions $\boldsymbol{\phi}(\mathbf{x}(k),k)$, $\mathbf{u}_{est}(k)$ is the estimation of perturbation, $\mathbf{u}_s(k)$ denotes the switching control part, and $\mathbf{u}_{adp}(k)$ symbolizes the adaptive control mechanism for overcoming the perturbation estimation error. These four control parts $\mathbf{u}_f(k)$, $\mathbf{u}_{est}(k)$, $\mathbf{u}_s(k)$ and $\mathbf{u}_{adp}(k)$ are designed respectively as

$$\mathbf{u}_f(k)=-(\mathbf{DB})^{-1}\boldsymbol{\phi}(\mathbf{x}(k),k), \quad (10)$$

$$\mathbf{u}_{est}(k)=-(\mathbf{DB})^{-1}\mathbf{D}\Delta\mathbf{P}_{est}(\mathbf{x}(k),k), \quad (11)$$

$$\mathbf{u}_s(k)=-(\mathbf{DB})^{-1}\left[q\boldsymbol{\sigma}(k)+\varepsilon\mathbf{sat}(\boldsymbol{\sigma}(k))\right], (12)$$

$$\mathbf{u}_{adp}(k)=-(\mathbf{DB})^{-1}[\mathbf{K}_a(\mathbf{x}(k),k)\mathbf{sat}(\boldsymbol{\sigma}(k))], (13)$$

where $q$ and $\varepsilon$ are designed constants and satisfy $0<q<1$, $\varepsilon>\frac{1-q}{4q}$ respectively, $\Delta\mathbf{P}_{est}=[\Delta P_{1,est}\ \ \Delta P_{2,est}\cdots\ \ \Delta P_{n,est}]^T\in\Re^{n\times 1}$ is the vector of perturbation estimation, which will be introduced in Step 3. The vector $\mathbf{sat}(\boldsymbol{\sigma})$ is defined as $\mathbf{sat}(\boldsymbol{\sigma})\equiv[sat(\sigma_1)\ sat(\sigma_2)\ \cdots\ sat(\sigma_m)]^T\in\Re^{m\times 1}$,

$$sat(\sigma_i)=\begin{cases}1, & if\ \sigma_i>\dfrac{\varepsilon+a_i(k)}{1-q}\\ \dfrac{1-q}{\varepsilon+a_i(k)}\sigma_i, & if\ |\sigma_i|\le\dfrac{\varepsilon+a_i(k)}{1-q}\\ -1, & if\ \sigma_i<-\dfrac{\varepsilon+a_i(k)}{1-q}\end{cases}$$

and the adaptive gain matrix $\mathbf{K}_a(\mathbf{x}(k),k)$ is given by $\mathbf{K}_a(\mathbf{x}(k),k)=diag[a_i(k)]$, where

$$a_i(k)=\hat{r}_i(k)+\frac{\alpha_i}{2}. \quad (14)$$

The adaptive rule of $\hat{r}_i(k)$ with initial condition $\hat{r}_i(0)=0$ is designed as

$$\hat{r}_i(k+1)=\begin{cases}\hat{r}_i(k)+\alpha_i\ , & |\sigma_i(k)|>\dfrac{\varepsilon+a_i(k)}{1-q}\\ \hat{r}_i(k)\ , & else\end{cases} \quad (15)$$

for $i=1,2,\cdots,m$, $\alpha_i$ is positive constant specified by the designer.

***Step 3: Estimation of Perturbation*** $\Delta\mathbf{P}$

According to (1), the state equation can be written as

$$\begin{aligned}\Delta\mathbf{P}(\mathbf{x}(k),k)&=\mathbf{x}(k+1)-\mathbf{A}\mathbf{x}(k)-\mathbf{B}\mathbf{u}(k)\\ &\equiv\mathbf{x}(k+1)-\bar{\mathbf{x}}(k+1)\end{aligned} \quad (16)$$

where $\bar{\mathbf{x}}(k)\equiv\mathbf{A}\mathbf{x}(k-1)+\mathbf{B}\mathbf{u}(k-1)$. (16) also indicates

$$\Delta\mathbf{P}(\mathbf{x}(k-1),k-1)=\mathbf{x}(k)-\bar{\mathbf{x}}(k) \quad (17)$$

(17) implies the perturbation of the previous stage can be computed from the present state information. Note that $\bar{\mathbf{x}}(k)$ is calculable since state variable $\mathbf{x}(k-1)$ is measurable. The estimation of the present perturbation $\Delta\mathbf{P}(\mathbf{x}(k),k)$ can be done using the same procedure as proposed in Cheng and Chu (2000), i.e.,

$$\Delta\mathbf{P}_{est}(k)=\Delta\mathbf{P}(k-1)+\sum_{n=1}^{\infty}\frac{1}{n!}\Delta\mathbf{P}^{(n)}(k-1), (18)$$

where

$$\Delta\mathbf{P}^{(n)}(k-1)=\sum_{\lambda=0}^{n}(-1)^{\lambda}C_{\lambda}^{n}\Delta\mathbf{P}(k-\lambda-1),$$

provided that the following assumption is valid :

**A5.** The function $\Delta\mathbf{P}^{(n+1)}(\mathbf{x}(k),k)$ exists for every $\mathbf{x}(k)$ and $k$, and

$$\lim_{n\to\infty}\|\frac{1}{(n+1)!}\Delta\mathbf{P}^{(n+1)}(\mathbf{x}(k),k)\|=0.$$

The above assumption also means that the Taylor series of $\Delta\mathbf{P}$ should converge.

(18) clearly shows that we can use the previous information of perturbation to estimate the present perturbation. In general, the more previous information is utilized, the more precise estimation one can make.

***Remark 2*:** Assumption A5 is a mild assumption. One still can use (18) to estimate the perturbation without assumption A5. If the proposed perturbation estimation scheme is not used in the controller, the stability of the controlled system can still be guaranteed.

## 4. ROBUSTNESS OF SYSTEM'S STABILITY

Before proving that the proposed controller (9)-(13) will drive the trajectory of switching variable $\sigma_i(k)$ into a region $R_{\sigma_i}\equiv\{\sigma_i(k)\in\Re:|\sigma_i(k)|\le\frac{\varepsilon+a_i(k)}{1-q}\}$, a definition of the equivalent control $u_{eq}$ is given first.

***Definition*:** Let the trajectory of $\sigma_i(k)$ in the region $R_{\sigma_i}$ be $\theta_i(k)$, which means $|\theta_i(k)|\le\frac{\varepsilon+a_i(k)}{1-q}$. An equivalent control $u_{eq}(k)$ is defined such that the trajectory of $\sigma_i(k+1)$ driven by $u_{eq}(k)$ is in the region $R_{\sigma_i}$. △

Now the theorem of system's stability and its proof are presented as follows.

**Theorem 1:** Consider the perturbed system (1) with all the aforementioned assumptions. Suppose that there exist unknown positive constants $r_i, i=1,2,\cdots,m$ such that $|\Delta\bar{P}_i(\mathbf{x}(k),k)|\le r_i$ is satisfied for all$(\mathbf{x}(k),k)\in\Re^n\times\Re$, where

$$\begin{aligned}\Delta\bar{P}_i(\mathbf{x}(k),k)\equiv{}&2\sigma_i(k)\Delta\tilde{P}_i+\Delta\tilde{P}_i^2-2q\sigma_i(k)\Delta\tilde{P}_i\\ &-2(\varepsilon+a_i(k))sat(\sigma_i(k))\Delta\tilde{P}_i,\end{aligned} \quad (19)$$

and $\Delta\tilde{\mathbf{P}}=[\Delta\tilde{P}_1\quad\Delta\tilde{P}_2\quad\cdots\quad\Delta\tilde{P}_m]^T\equiv\mathbf{D}(\Delta\mathbf{P}-\Delta\mathbf{P}_{est})$ is the perturbation estimation error. If the proposed controller (9)-(13) and the switching surface function (5) are used, then the trajectory of switching variable $\sigma_i(k)$ will be driven into the bounded region $R_{\sigma_i}$, and the stability of the proposed controlled system is guaranteed.

**Proof:** Substituting the proposed control (9)-(13) into (8) yields

$$\Delta\boldsymbol{\sigma}(k+1) = -q\boldsymbol{\sigma}(k) - \varepsilon\mathbf{sat}(\boldsymbol{\sigma}(k)) \\ -\mathbf{K}_a(\mathbf{x}(k),k)\mathbf{sat}(\boldsymbol{\sigma}(k)) + \Delta\tilde{\mathbf{P}} \quad (20)$$

Let $\tilde{r}_i(k) \equiv \hat{r}_i(k) - r_i$ be the errors of adaptive gains, and a Lyapunov function candidate $\mathbf{V}(k) \equiv [V_1(k) \quad V_2(k) \quad \cdots \quad V_m(k)]^T$ be

$$V_i(k) = \sigma_i^2(k) + \frac{\alpha_i^{-1}}{2}\tilde{r}_i^2(k).$$

Then

$$V_i(k+1) - V_i(k) = \sigma_i^2(k+1) - \sigma_i^2(k) \\ + \frac{\alpha_i^{-1}}{2}\left[\tilde{r}_i^2(k+1) - \tilde{r}_i^2(k)\right]. \quad (21)$$

By using (20), the first two terms of (21) is computed as

$$\sigma_i^2(k+1) - \sigma_i^2(k) \\ = -2q\sigma_i^2(k) - 2(\varepsilon + a_i(k))\sigma_i(k)sat(\sigma_i(k)) \\ +q^2\sigma_i^2(k) + (\varepsilon + a_i(k))^2 sat^2(\sigma_i(k)) \\ +2q(\varepsilon + a_i(k))\sigma_i(k)sat(\sigma_i(k)) \\ +\Delta\bar{P}_i(\mathbf{x}(k),k) \quad (22)$$

If $|\sigma_i(k)| > \frac{\varepsilon + a_i(k)}{1-q}$, then from (15), the third term of (21) can be simplified as

$$\tilde{r}_i^2(k+1) - \tilde{r}_i^2(k) = 2\alpha_i\left[\hat{r}_i(k) - r_i\right] + \alpha_i^2, \quad (23)$$

Now using (22), (23) and (19), if $|\sigma_i(k)| > \frac{\varepsilon + a_i(k)}{1-q}$, from (21) it is seen that

$$V_i(k+1) - V_i(k) \\ = -2q\sigma_i^2(k) - 2(\varepsilon + a_i(k))|\sigma_i(k)| + q^2\sigma_i^2(k) \\ +(\varepsilon + a_i(k))^2 + 2q(\varepsilon + a_i(k))|\sigma_i(k)| \\ +\Delta\bar{P}_i(\mathbf{x}(k),k) + [\hat{r}_i(k) - r_i] + \frac{\alpha_i}{2} \\ \leq -2q\sigma_i^2(k) - 2(\varepsilon + a_i(k))|\sigma_i(k)| + q^2\sigma_i^2(k) \\ +(\varepsilon + a_i(k))^2 + 2q(\varepsilon + a_i(k))|\sigma_(k)| + r_i \\ + [\hat{r}_i(k) - r_i] + \frac{\alpha_i}{2} \\ = -2q\sigma_i^2(k)(1-q) - 2(\varepsilon + a_i(k))|\sigma_i(k)|(1-q) \\ -q^2\sigma_i^2(k) + (\varepsilon + a_i(k))^2 + a_i(k) \\ < -2q\left[\frac{\varepsilon + a_i(k)}{1-q}\right]^2 (1-q) + (\varepsilon + a_i(k))^2 \\ -2(\varepsilon + a_i(k))\left[\frac{\varepsilon + a_i(k)}{1-q}\right](1-q) + a_i(k) \\ = \frac{-2q}{1-q}(\varepsilon + a_i(k))^2 - 2(\varepsilon + a_i(k))^2 \\ +(\varepsilon + a_i(k))^2 + a_i(k) \\ < \frac{-2q}{1-q}(\varepsilon^2 + a_i^2(k)) + \frac{-2q}{1-q}2\frac{1-q}{4q}a_i(k) + a_i(k) \\ = \frac{-2q}{1-q}(\varepsilon^2 + a_i^2(k)) < 0$$

The previous derivation clearly shows that, as $k$ increases, $V_i(k)$ is a decreasing function if the trajectory of $\sigma_i(k)$ is outside the region $R_{\sigma_i}$. If $\sigma_i(0)$ is outside the region $R_{\sigma_i}$, then there exists a finite time $k_1$ such that the trajectory of $\sigma_i(k)$ will enter the region $R_{\sigma_i}$ at time $k_1$ (even if the region $R_{\sigma_i}$, or $a_i(k)$, is increasing during $k \in [0, k_1]$). Note that $V_i(k)$ is a bounded function since $V_i(k) < V_i(0)$, $k \in [0, k_1]$. Once the trajectory of $\sigma_i(k)$ enters the region $R_{\sigma_i}$, from (15) one can see that $\hat{r}_i(k)$ is bounded, then according to (14), $a_i(k)$ is also bounded. If $\sigma_i(0)$ is inside the region $R_{\sigma_i}$, then obviously $a_i(k)$ is bounded in accordance with (14) and (15). All these analyse show that $\sigma_i(k)$ is bounded and will be driven into the region $R_{\sigma_i}$ eventually.

On the other hand, one can show that the error function $\mathbf{e}(k)$ is also bounded under the proposed control strategy. According to (1), (2), (3) and (4), the error function $\mathbf{e}(k+1)$ can be written as

$$\mathbf{e}(k+1) = \mathbf{A}\mathbf{e}(k) + \mathbf{B}\mathbf{u}(k) + \Delta\mathbf{P}(\mathbf{x}(k),k) \\ -\mathbf{B}\mathbf{H}\mathbf{x}_m(k) - \mathbf{G}\mathbf{B}_m\mathbf{r}(k) \quad (24)$$

Substituting (24) into (5) yields

$$\boldsymbol{\sigma}(k+1) = \mathbf{D}\mathbf{B}\mathbf{u}(k) + \mathbf{D}\Delta\mathbf{P}(\mathbf{x}(k),k) \\ -\mathbf{D}\mathbf{B}\mathbf{H}\mathbf{x}_m(k) - \mathbf{D}\mathbf{G}\mathbf{B}_m\mathbf{r}(k) - \mathbf{D}\mathbf{B}\mathbf{K}\mathbf{e}(k) \quad (25)$$

According to the definition of $\mathbf{u}_{eq}$, the $\mathbf{u}_{eq}(k)$ can be solved from (25) as

$$\|\boldsymbol{\sigma}(k+1)\| = \|\mathbf{D}\mathbf{B}\mathbf{u}(k) + \mathbf{D}\Delta\mathbf{P}(\mathbf{x}(k),k) \\ -\mathbf{D}\mathbf{B}\mathbf{H}\mathbf{x}_m(k) - \mathbf{D}\mathbf{G}\mathbf{B}_m\mathbf{r}(k) - \mathbf{D}\mathbf{B}\mathbf{K}\mathbf{e}(k)\| \\ \leq \|\frac{\varepsilon + \mathbf{K}_a(\mathbf{x}(k),k)}{1-q}\|,$$

which indicates

$$\mathbf{u}_{eq} = \mathbf{H}\mathbf{x}_m + \mathbf{K}\mathbf{e} - (\mathbf{D}\mathbf{B})^{-1}\mathbf{D}\Delta\mathbf{P} \\ +(\mathbf{D}\mathbf{B})^{-1}\mathbf{D}\mathbf{G}\mathbf{B}_m\mathbf{r} + (\mathbf{D}\mathbf{B})^{-1}\boldsymbol{\theta} \quad (26)$$

Hence the closed-loop error dynamic equation after system entering the region $R_{\sigma_i}$ can be obtained by substituting (26) into (24), and the resultant equation is

$$\mathbf{e}(k+1) = (\mathbf{A} + \mathbf{B}\mathbf{K})\mathbf{e}(k) + \boldsymbol{\rho}(k) \quad (27)$$

where

$$\boldsymbol{\rho}(k) \equiv -\mathbf{B}(\mathbf{D}\mathbf{B})^{-1}\mathbf{D}\Delta\mathbf{P} + \mathbf{B}(\mathbf{D}\mathbf{B})^{-1}\mathbf{D}\mathbf{G}\mathbf{B}_m\mathbf{r} \\ +\Delta\mathbf{P} - \mathbf{G}\mathbf{B}_m\mathbf{r} + \mathbf{B}(\mathbf{D}\mathbf{B})^{-1}\boldsymbol{\theta}$$

is a bounded function. (27) clearly shows that the tracking error $\mathbf{e}(k)$ is a bounded function if (6) is satisfied, and hence (4) implies $\mathbf{x}(k)$ is also bounded. Therefore, one can conclude that the robust stability is guaranteed under the proposed control scheme. □

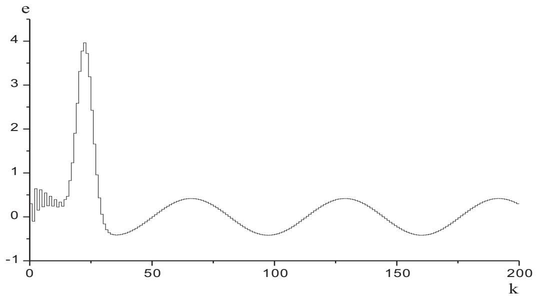


Fig. 1. Tracking error $e$ (no estimation of perturbation).

According to (14) and (15), one can easily see that larger value of $q$ and smaller values of $\alpha_i$ and $\varepsilon$ will in general reduce the upperbound of $\sigma_i(k)$, which also means that the tracking accuracy will be increased. However, the adaptive speed will be reduced when smaller $\alpha_i$ is used.

## 5. EXAMPLE AND SIMULATION

Consider the following unstable linear discrete-time system

$$\mathbf{x}(k+1) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 2 & 4 \\ 0 & 3 & 1 \end{pmatrix} \mathbf{x}(k) + \begin{pmatrix} 0 & 0 \\ 1 & 1 \\ 1 & 0 \end{pmatrix} \mathbf{u}(k) + \Delta\mathbf{P}(k)$$
$$y(k) = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \mathbf{x}(k),$$

where $\mathbf{x} = \begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix}^T$, $\mathbf{u} = \begin{pmatrix} u_1 & u_2 \end{pmatrix}^T$, $\Delta\mathbf{P}(k) = \begin{pmatrix} d_1(k) & d_2(k) & d_3(k) \end{pmatrix}^T$, and the unknown perturbations are assumed to be $d_1(k) = 0.02 \sin(0.1k)$, $d_2(k) = 5e^{-(0.1k-2)^2/0.2} + 0.5\cos(0.1k)$, and $d_3(k) = 0.5x_2 \cos(0.1k) + \sin x_1$. Note that there is an unexpected large disturbance in $d_2(k)$ at $k = 20$.

The desired reference model is given by

$$\mathbf{x}_m(k+1) = \begin{pmatrix} 0 & 1 \\ -0.25 & 1 \end{pmatrix} \mathbf{x}_m(k) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} r(k)$$
$$y_m(k) = \begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x}_m(k), \; r(k) = 0.2\sin(0.1k)$$

The objective is to employ the proposed control technique so that the output $\mathbf{y}(k)$ can track the reference signal $\mathbf{y}_m(k)$.

The switching surface $\boldsymbol{\sigma}$ is given by (5), where $\mathbf{D} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$, $\mathbf{K} = \begin{pmatrix} 0 & -3 & -1.5 \\ 0.3 & 0.5 & -2.5 \end{pmatrix}$. The matrix $\mathbf{G}$ in (3) is $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \end{pmatrix}^T$. The controller is given by (9), where $q = 0.99$, $\varepsilon = 0.003$ and $\alpha_i = 0.0005$. If only one previous stage information is used to estimate the perturbation $\Delta\mathbf{P}(k)$, then according to (18), the perturbation estimation $\Delta\mathbf{P}_{est}$ is

$$\Delta\mathbf{P}_{est}(k) = \Delta\mathbf{P}(k-1) = \mathbf{x}(k) - \bar{\mathbf{x}}(k).$$

If five previous stages information are used to estimate the perturbation $\Delta\mathbf{P}(k)$, the perturbation estimation $\Delta\mathbf{P}_{est}$ is designed as

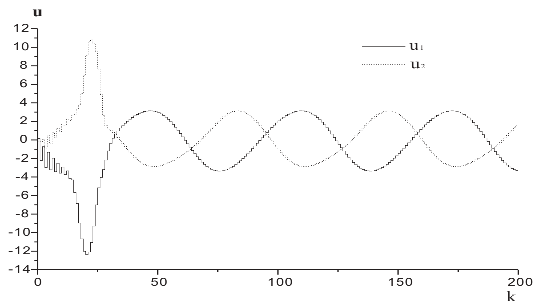


Fig. 2. Control effort $\mathbf{u}$ (no estimation of perturbation).

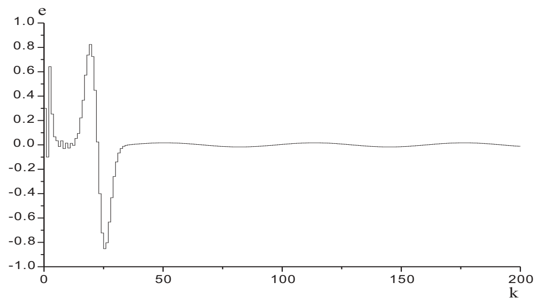


Fig. 3. Tracking error $e$ (controller has perturbation estimation).

$$\Delta\mathbf{P}_{est}(k) = \frac{65}{24}\Delta\mathbf{P}(k-1) - \frac{8}{3}\Delta\mathbf{P}(k-2)$$
$$+ \frac{5}{4}\Delta\mathbf{P}(k-3) - \frac{1}{3}\Delta\mathbf{P}(k-4) + \frac{1}{24}\Delta\mathbf{P}(k-5)$$

Noted that 2-norm is used to compute $\|\mathbf{x}\|$. The simulation results are given from Fig. 1 to Fig. 6 with initial condition $\mathbf{x}(0) = (0.3 \quad -0.1 \quad 0.1)^T$.

Fig. 1 to Fig. 2 show the results of the proposed control scheme without the estimation of perturbation. Fig. 3 to Fig. 4 show the results of the proposed control scheme with five stages of previous information being utilized in the estimation of perturbation. Obviously, the controller with the estimation of perturbation can achieve better tracking accuracy even when an unexpected large disturbance $d_2$ happens around $k = 20$ in this case. The control input are shown in Fig. 2 and Fig. 4. It is clearly shown that there is a peak at $k = 20$ for the case when the perturbation estimation algorithm is not employed, whereas this peak is effectively reduced if the estimation process is utilized. Fig. 5 shows the adaptive gains, both two gains approach to a constant, respectively. Fig. 6 shows the comparison of tracking errors between one stage and five stages of previous information being utilized in the estimation of perturbation. It is clearly shown that more stages of previous information are used in the perturbation estimation process, the perturbation estimation's error is in general smaller, and the tracking accuracy can also be increased in this case.

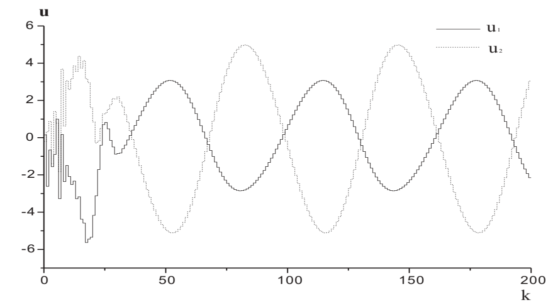


Fig. 4. Control effort **u** (controller has perturbation estimation).

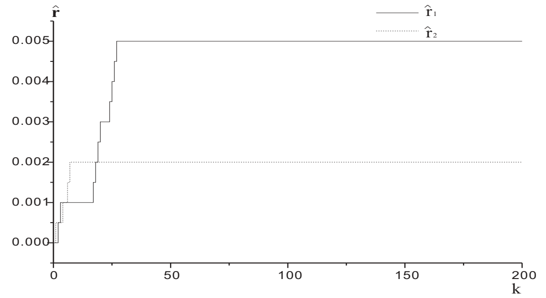


Fig. 5. Adaptive gain $\hat{\mathbf{r}}$ (controller has perturbation estimation).

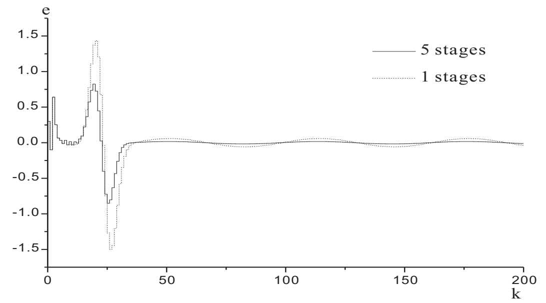


Fig. 6. Tracking error $e$ with different previous stage information.

## 6. CONCLUSIONS

A discrete variable structure control method is successfully proposed to solve the robust tracking problem for a class of perturbed MIMO linear discrete time systems. Due to the utilization of the adaptive mechanism in the proposed controller, the knowledge of the upper bound of perturbation is not needed beforehand, the adaptive gain of the proposed control scheme needs only to overcome the upper bound of perturbation estimation error instead of that of the perturbations. The advantages of using the perturbation estimation is that in general the control energy can be reduced as well as the tracking accuracy can also be increased. The disadvantages of the proposed perturbation estimation scheme is that it is not quite effective for estimating faster varying perturbations, and it has to store the previous information of state variables. For increasing the tracking accuracy, more memory devices are needed.

## 7. ACKNOWLEDGMENTS

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