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SWAY CONTROL EXPERIMENT OF ROTARY CRANE USING LINEAR TRANSFER TRANSFORMATION MODEL

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Abstract: This paper presents a method to control the sway of a rotary crane by using a linear transfer transformation (LTT) model. The LTT model is built and its parameters are geometrically derived. Optimal control of a rotary crane is presented using an optimization method of Fletcher-Reeves so that the load has no sway at the end point of the transfer. The effectiveness of the proposed control method in the LTT model is demonstrated through simulation and experiments.

Keywords: Crane; Modeling; Optimal control; Vibration control

1. INTRODUCTION

Rotary cranes are indispensable in factories, construction sites, harbors and so on. The fundamental motions of a rotary crane are rotation, boom hoisting and load hoisting. The load sway grows from the centrifugal force that rotary motion induces. As shipping cost is strongly related to a ship's anchorage time, it is desirable to minimize the anchorage time by immediately eliminating sway at the end of a load transfer. Many studies about controlling the sway of a load of a rotary crane have been published. (Sakawa and Nakazuni, 1985) applied the open-loop plus feedback control scheme to let the sway of the load decay at the end point of transfer. (Bahram and Bikdash, 1999) developed a state-space model of the crane from an implicit description without simplifying assumptions, and they designed a fuzzy controller. (Hino and Su, 1998) used a nonlinear control theory to reduce the oscillations of a load. (Yokoyama and Kaneko, 1998) applied an optimum control theory to control the sway of a load. But these papers did not consider the condition of simultaneous rotary motion and boom hoisting motion. Using a rotary crane, (Yamazaki and Hisamura, 1978) presented the idea of a linear transfer that makes the crane tip and the load move on a straight line in the X-Y coordinate by the simultaneous motions of rotation and boom hoisting. It is obvious that linear transfer by a rotary crane uses less space than rotational transfer; in the former, the crane's load is transferred from a starting point to an ending point, although linear transfer is limited to work spaces within 180 degrees per load transfer. However, the problem of bang-bang minimum time control was the only problem considered by means of open-loop inputs. Hence, this paper discusses not only minimum time control but also the general control problem, and the focus is on the motion of a rotary crane in a linear transfer. First, a nonlinear model of a load position and a tip position of the rotary crane represented in three-dimensional space has been built, but we do not present this model here because of space limitations. However, the model allows us to determine the optimum control input for moving a load position along a straight line, i.e., a linear transfer, if the cost function is appropriately given to restrict the load transfer on the straight line. But, when we use the nonlinear model built in three-dimensional space, a lengthy calculation is needed to determine the optimum control input to eliminate residual vibration, due to the model's term of centrifugal force. Secondly, for the linear transfer, we have built a nonlinear model of the swing of a load and the tip position of a boom embedded in two-dimensional space. When a load is moved by two-dimensional transfer, the tip's trajectory of the boom draws a circle in two-dimensional space. We therefore created an imaginary boom angle, an imaginary boom length and an imaginary swing angle of a load, as explained in the next section. In the present paper, this embedded model in two-dimensional space is called a linear transfer transformation (LTT) model. The optimum control problem can then be solved by using the Fletcher-Reeves (FR) method for the LTT model. We demonstrate that actual control inputs of a rotary crane can be obtained by transforming coordination from two-dimensional imaginary coordination in the LTT model to three-dimensional coordination in the actual rotary crane. Finally, we evaluate the validity of the model and the effectiveness of this method, as manifested by both the simulation and the experimental results.

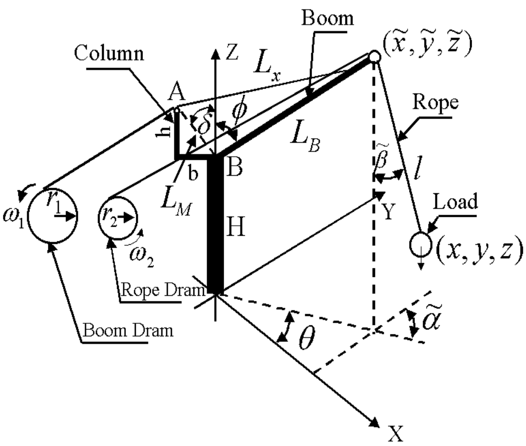

Fig. 1. Schematic diagram of rotary crane

2. DERIVATION OF LTT MODEL

The LTT method makes the crane tip and the load move along a straight line in the X-Y coordinate by the simultaneous motions of rotation and boom hoisting (see Fig. 2). In the analysis of this problem, an embedding method is used in order to reduce the dimensions of the model. Actual boom length L_B is replaced by the imaginary boom

Table 1. Symbolic notation in Fig. 1

Symbol	Unit	Explanation
θ	rad	rotary angle
$\tilde{\alpha}$	rad	swing angle on the horizontal surface
ϕ	rad	boom angle
$\tilde{\beta}$	rad	swing angle from the vertical direction
l	m	rope length
r_1	m	radius of the boom hoisting drum
L_B	m	length of the boom
r_2	m	radius of the rope hoisting drum
H	m	height of the turn table
ω_1	rad	rotational angle of the rope hoisting drum
m	kg	mass of the load
ω_2	rad	rotational angle of the rope hoisting drum
g	m/s^2	gravity
$(\tilde{x}, \tilde{y}, \tilde{z})$	m	position of the crane tip
F	N	tension of the rope
(x,y,z)	m	position of the load
b	m	distance between axis Z and the column
h	m	height of the column
L_M	m	distance between point A and point B
δ	rad	angle between line L_M and axis Z
L_X	m	distance between point A and point C

length R. Actual rotary input u_θ and hoisting input u_ϕ are also replaced by the imaginary hoisting input u_ψ, while actual swing angles α, β are replaced by the imaginary swing angle ξ (see Fig. 3), so that the differential equations describing the model are reduced to only two variables.
For simplicity, the following assumptions are made:
• The body of the crane is rigid, and the load is a point mass.
• Friction is neglected.
• Rope weight and elongation of ropes due to tensile force are neglected.
• The driving motor used in this paper is assumed to have enough power for the weight of the load. The property of the driving motors is regarded as the following system:

$$\ddot{\theta} = -\frac{1}{T_\theta}\dot{\theta} + \frac{K_\theta}{T_\theta}u_\theta \tag{1}$$

$$\ddot{\phi} = A_\phi \frac{K_\phi}{T_\phi}u_\phi - \frac{1}{T_\phi}\dot{\phi} + B_\phi \dot{\phi}^2 \tag{2}$$

where

$$A_\phi = -\frac{r_1 L_X}{L_M L_B \sin(\phi + \delta)} \quad ,$$

$$B_\phi = \frac{L_M L_B}{L_X^2}\sin(\phi + \delta) - \frac{\cos(\phi + \delta)}{\sin(\phi + \delta)} \quad ,$$

$$L_M = \sqrt{b^2 + h^2} \quad ,$$

$$\delta = \tan^{-1}(\frac{b}{h}) \quad ,$$

$$L_X = \sqrt{L_M^2 + L_B^2 - 2L_M L_B \cos(\phi + \delta)} \quad ,$$

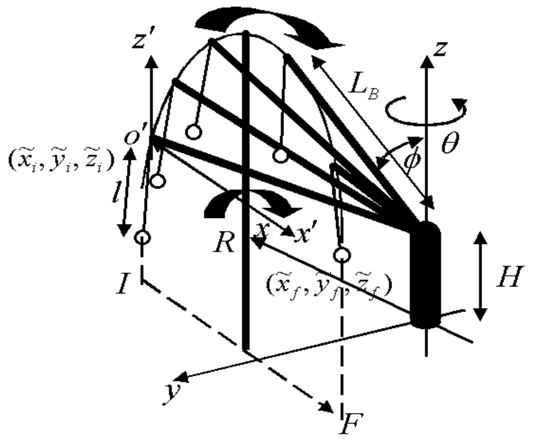

Fig. 2. Linear transfer transformation

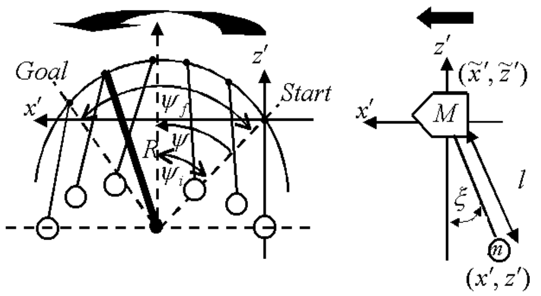

Fig. 3. Linear transfer transformation model of rotary crane

T_θ and T_ϕ are, respectively, the time constant of the rotary motor and that of the hoisting motor. K_θ and K_ϕ represent the gain of each motor, respectively; finally, u_θ is rotary input voltage and u_ϕ is hoisting input voltage. From Fig. 1, the position of the boom tip in the absolute Cartesian coordinate is given by the following equation:

$$\tilde{x} = L_B \sin\phi \cos\theta \quad (3)$$

$$\tilde{y} = L_B \sin\phi \sin\theta \quad (4)$$

$$\tilde{z} = H + L_B \cos\phi \quad (5)$$

2.1 *The Model of LTT*

The concept of LTT is shown in Fig. 2. Supposing the initial place of the boom tip is $(\tilde{x}_i, \tilde{y}_i, \tilde{z}_i)$, and the final place is $(\tilde{x}_f, \tilde{y}_f, \tilde{z}_f)$, then the projection of the orbit of the boom tip is a line I-F on the x-y plane. In order to simply analyze the problem of LTT, a new coordinate, called the linear transfer coordinate, is built according to the following principle:

- The initial place of the boom tip $(\tilde{x}_i, \tilde{y}_i, \tilde{z}_i)$ is the original point of the new coordinate o'.
- Line I-F runs parallel to the z direction and intersects point o'; this line is defined as the x' axis.
- Axis z' is parallel to the axis z.

Figure 3 shows the concepts of imaginary boom length R, the imaginary hoisting angle ψ and the imaginary swing angle ξ. In the linear transfer coordinate, the position of the crane tip can be expressed by the imaginary hoisting angle ψ and the length of the imaginary boom R,

Table 2. Symbolic notation in Fig. 3

symbol	unit	Explanation
l	m	Rope length
ψ_i	rad	Initial imaginary boom angle
ξ	rad	Swing angle
ψ_f	rad	Final imaginary boom angle
R	m	Imaginary boom length
$(\tilde{x}', \tilde{z}')$	m	Position of crane tip
ψ	rad	Imaginary boom angle
x',z')	m	Imaginary boom length
R	m	Position of load
M	kg	mass of cart
m	kg	mass of load

$$\tilde{x}' = R\{\sin(\psi - \psi_i) + \sin\psi_i\} \quad (6)$$

$$\tilde{z}' = R\{\cos(\psi - \psi_i) - \cos\psi_i\} \quad (7)$$

where ψ_i is the initial imaginary hoisting angle. The position of the load is expressed by the swing angle in the direction of linear transfer ξ and the length of the rope l:

$$x' = \tilde{x}' - l\sin\xi \quad (8)$$

$$z' = \tilde{z}' - l\cos\xi \quad (9)$$

The motion equation of Lagrange is given by

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}}\right) - \frac{\partial T}{\partial q} + \frac{\partial U}{\partial q} = 0 \quad (10)$$

Using Eqs.(8) and (9), Eq.(10) is transformed into

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\xi}}\right) - \frac{\partial T}{\partial \xi} + \frac{\partial U}{\partial \xi} = 0 \quad (11)$$

where

$$T = T_x + T_z$$

$$T_x = \frac{1}{2}M\dot{\tilde{x}}'^2 + \frac{1}{2}m\left(\frac{d}{dt}(\tilde{x}' - l\sin\xi)\right)^2$$

$$T_z = \frac{1}{2}M\dot{\tilde{z}}'^2 + \frac{1}{2}m\left(\frac{d}{dt}(\tilde{z}' - l\cos\xi)\right)^2$$

$$U = Mg\tilde{z}' + mg(\tilde{z}' - l\cos\xi)$$

From Eq.(11), $\ddot{\xi}$ can be solved. Suppose that the control input is an imaginary hoisting angle acceleration u_ψ. It follows that:

$$\ddot{\psi} = u_\psi \quad (12)$$

$$\ddot{\xi} = -\frac{g}{l}\sin\xi + \frac{\ddot{\tilde{x}}'}{l}\cos\xi - \frac{\ddot{\tilde{z}}'}{l}\sin\xi \quad (13)$$

$$\ddot{\tilde{x}}' = R\left\{\ddot{\psi}\cos(\psi - \psi_i) - \dot{\psi}^2\sin(\psi - \psi_i)\right\} \quad (14)$$

$$\ddot{\tilde{z}}' = -R\left\{\ddot{\psi}\sin(\psi - \psi_i) + \dot{\psi}^2\cos(\psi - \psi_i)\right\} \quad (15)$$

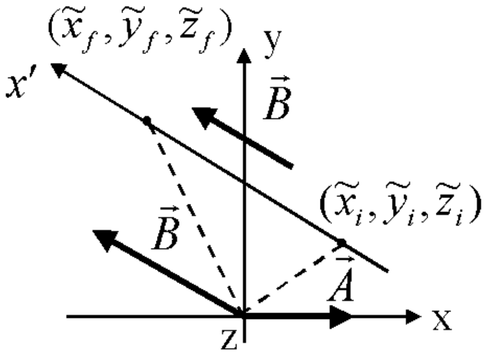

Fig. 4. Rotation angle γ

2.2 *Parameter Derivation of LTT Model*

The parameters of the linear transfer, such as the length of the imaginary boom R, the initial imaginary hoisting angle ψ_i and the final imaginary hoisting angle ψ_f, are easily derived as follows:

$$R = \sqrt{L_B^2 - \frac{(\tilde{x}_i\tilde{y}_f - \tilde{x}_f\tilde{y}_i)^2}{(\tilde{x}_f - \tilde{x}_i)^2 + (\tilde{y}_f - \tilde{y}_i)^2}} \quad (16)$$

The initial imaginary hoisting angle ψ_i becomes

$$\psi_i = \begin{cases} \cos^{-1}\dfrac{\tilde{z}_i - H}{R} & \psi_f \neq \psi_D \\ -\cos^{-1}\dfrac{\tilde{z}_i - H}{R} & \psi_f = \psi_D \end{cases} \quad (17)$$

$$\psi_D = \cos^{-1}\frac{\tilde{z}_f - H}{R} - \cos^{-1}\frac{\tilde{z}_i - H}{R}$$

The final imaginary hoisting angle ψ_f is expressed by

$$\psi_f = \cos^{-1}\left\{1 - \frac{S_{if}}{2L_B^2}\right\}$$

where,

$$S_{if} = (\tilde{x}_f - \tilde{x}_i)^2 + (\tilde{y}_f - \tilde{y}_i)^2 + (\tilde{z}_f - \tilde{z}_i)^2 \quad (18)$$

2.3 *Derivation of Actual Control Inputs*

The variables in the absolute coordinate can de transformed from the variables of the LTT coordinate. As shown in Fig. 4, γ is the angle between linear transfer direction $\overrightarrow{B}$ and x-axis direction $\overrightarrow{A}$; it follows that

$$\gamma = \cos^{-1}\frac{\tilde{x}_f - \tilde{x}_i}{\sqrt{(\tilde{x}_f - \tilde{x}_i)^2 + (\tilde{y}_f - \tilde{y}_i)^2}} \quad (19)$$

Then, the following relation holds:

$$\begin{bmatrix} \tilde{x} & \dot{\tilde{x}} & \ddot{\tilde{x}} \\ \tilde{y} & \dot{\tilde{y}} & \ddot{\tilde{y}} \\ \tilde{z} & \dot{\tilde{z}} & \ddot{\tilde{z}} \end{bmatrix} = \begin{bmatrix} \tilde{x}_i & 0 & 0 \\ \tilde{y}_i & 0 & 0 \\ \tilde{z}_i & 0 & 0 \end{bmatrix} + \begin{bmatrix} \cos\gamma & 0 \\ \sin\gamma & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \tilde{x}' & \dot{\tilde{x}}' & \ddot{\tilde{x}}' \\ \tilde{z}' & \dot{\tilde{z}}' & \ddot{\tilde{z}}' \end{bmatrix} \quad (20)$$

From Eq.(20), the position $(\tilde{x}, \tilde{y}, \tilde{z})$ of the crane tip in the absolute coordinate, velocity $(\dot{\tilde{x}}, \dot{\tilde{y}}, \dot{\tilde{z}})$ and acceleration $(\ddot{\tilde{x}}, \ddot{\tilde{y}}, \ddot{\tilde{z}})$ can be calculated. Dividing Eq.(4) by Eq.(3), it follows that

$$\tan\theta = \frac{\tilde{y}}{\tilde{x}} \quad (21)$$

so that

$$\theta = \tan^{-1}(\frac{\tilde{y}}{\tilde{x}}) \quad (22)$$

Differentiating Eq.(22), it follows that

$$\dot{\theta} = \frac{\dot{\tilde{y}}\tilde{x} - \dot{\tilde{x}}\tilde{y}}{\tilde{x}^2 + \tilde{y}^2} \quad (23)$$

Further, differentiating Eq.(23), it follows that

$$\ddot{\theta} = \frac{\ddot{\tilde{y}}\tilde{x} - \ddot{\tilde{x}}\tilde{y}}{\tilde{x}^2 + \tilde{y}^2} - \frac{2(\dot{\tilde{y}}\tilde{x} - \dot{\tilde{x}}\tilde{y})(\dot{\tilde{x}}\tilde{x} + \dot{\tilde{y}}\tilde{y})}{(\tilde{x}^2 + \tilde{y}^2)^2} \quad (24)$$

For the hoisting angle, from Eq.(5), it follows that

$$\phi = \cos^{-1}\frac{\tilde{z} - H}{L_B} \quad (25)$$

Differentiating Eq.(25), it follows that

$$\dot{\phi} = -\frac{\dot{\tilde{z}}}{L_B^2 - (\tilde{z} - H)^2} \quad (26)$$

Further, differentiating Eq.(26), it follows that

$$\ddot{\phi} = -\frac{\ddot{\tilde{z}}}{\{L_B^2 - (\tilde{z} - H)^2\}^{1/2}} - \frac{\dot{\tilde{z}}^2(\tilde{z} - H)}{\{L_B^2 - (\tilde{z} - H)^2\}^{3/2}} \quad (27)$$

From Eqs.(1) and (2), the actual rotary input voltage u_θ and that of hoisting input voltage $u_\phi$becomes

$$u_\theta = \frac{T_\theta}{K_\theta}\ddot{\theta} + \frac{\dot{\theta}}{K_\theta} \quad (28)$$

$$u_\phi = \frac{T_\phi}{A_\phi K_\phi}(\ddot{\phi} + \frac{\dot{\phi}}{T_\phi} - B_\phi\dot{\phi}^2) \quad (29)$$

2.4 *Optimal Control*

Consider nonlinear dynamical system described by

$$\dot{\mathbf{x}} = \mathbf{f}(t, \mathbf{x}(t), \mathbf{u}(t)) \quad (30)$$

where $\dot{\mathbf{x}} = (x_1, \cdots, x_n)^T$ is a state vector, $\mathbf{f} = (f_1, \cdots, f_n)$ is a nonlinear n-vector function, and $\mathbf{u}(t)$ is a control input vector.

Performance index (or cost function) is given by

$$J = \mathbf{g}(t_f, \mathbf{x}(t_f)) + \int_{t_0}^{t_f} \mathbf{f}_0(t, \mathbf{x}(t), \mathbf{u}(t))dt \quad (31)$$

where t_0 is an initial time, and t_f is a final time. If a Hamiltonian is given by

$$\mathbf{H} = \mathbf{f}_0(t, \mathbf{x}(t), \mathbf{u}(t)) + \mathbf{p}^T(t)\mathbf{f}(t, \mathbf{x}(t), \mathbf{u}(t)) \quad (32)$$

then an adjoint equation is derived as

$$\left.\begin{aligned} \dot{\mathbf{p}}(t) = -(\frac{\partial \mathbf{H}}{\partial \mathbf{x}})^T &= -\mathbf{f}_x^T(t, \mathbf{x}(t), \mathbf{u}(t)) \\ &-\mathbf{f}_{0_x}^T(t, \mathbf{x}(t), \mathbf{u}(t)) \\ \mathbf{p}(t_f) = \frac{\partial g}{\partial \mathbf{x}(t_f)} &\equiv \mathbf{p}_f \end{aligned}\right\} \quad (33)$$

where $\mathbf{f}_x \equiv (\partial \mathbf{f})^T/(\partial \mathbf{x})$.

Gradient of a performance index J is defined by

$$J_u = \frac{\partial \mathbf{H}}{\partial \mathbf{u}} = \mathbf{f}_{0_u}(t, \mathbf{x}(t), \mathbf{u}(t))$$

$$+\mathbf{p}^T(t)\mathbf{f}_u(t, \mathbf{x}(t), \mathbf{u}(t)) \quad (34)$$

Therefore, an optimal control problem is reduced to a two-point boundary value problem constructed of Eqs.(30) and (33). Optimal solution can be numerically obtained on a fixed time interval by using Fletcher-Reeves (FR) method (Yoshino, 1995). When the load arrives at the desired position, it is hoped that the sway of the load stops at once and there is no residual sway. So, in this paper, the performance index is given by

$$J = \mathbf{x}(t_f)^T W \mathbf{x}(t_f), \quad (35)$$

where $\mathbf{x}(t_f) = \left[\psi - \psi_{t_f}, \dot{\psi}, \xi, \dot{\xi}\right]$ and $W = diag\left[10^5, 10^5, 10^5, 10^5\right]$. Thus, first, an optimum control is sovled by FR method for Eqs.(12) and (13) of LTT model. Nextly, via Eqs.(19)-(27), the actual optimal control input voltage can be obtained by using Eqs.(28) and (29).

3. EXPERIMENTAL APPARATUS AND RESULTS

3.1 *Experimental Apparatus*

As shown in Fig. 5, When the load is transferred, the swing angle α' and β' can be measured by the two pairs of forks and encoders. It must be noted that in the simulation program the swing angles of the load are α and β; according to Fig. 5, the relationship between (α, β) and the load position (x, y, z), as well as the position of the crane tip $(\tilde{x}, \tilde{y}, \tilde{z})$, can be obtained as follows.

The parameters of the crane are shown in Table 4.

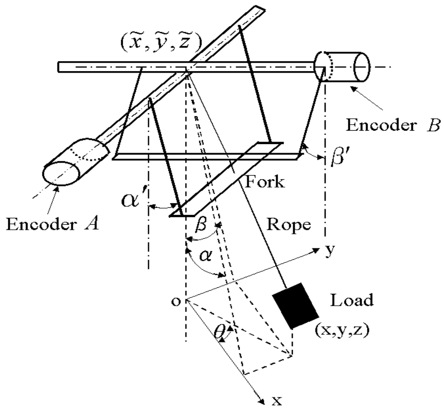

Fig. 5. Measurement of swing angle

Table 3. Symbolic notation in Fig. 5

Symbol	Unit	Explanation
α	rad	load swing angle in x direction
β	rad	load swing angle in y direction
α'	rad	load swing angle measured by encoder A
β'	rad	load swing angle measured by encoder B

Table 4. Parameters of the crane

physical value	numerical value	unit
L_B	1.143	m
H	0.627	m
l	0.9	m
r_1	0.045	m
b	0.338	m
h	0.506	m
T_θ	0.015	s
K_θ	0.3117	rad/sV
T_ϕ	0.008	s
K_ϕ	0.6266	rad/sV

$$\alpha = \sin^{-1}(\frac{x - \tilde{x}}{\sqrt{l^2 - (y - \tilde{y})^2}}) \quad (36)$$

$$\beta = \sin^{-1}(\frac{y - \tilde{y}}{\sqrt{l^2 - (x - \tilde{x})^2}}) \quad (37)$$

But the measured swing angles from the sensors are α' and β'. From Fig. 6, the relationship between α , β and α', β' can be obtained

$$\begin{bmatrix} \alpha' \\ \beta' \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \quad (38)$$

3.2 *Experimental Results*

The initial position of the crane tip is (1,0,1.18)[m] (the initial rotary angle $\theta_i = 0°$, the initial hoisting angle $\phi_i = 61°$), the final position is (0,1,1.18)[m] (the final rotary angle $\theta_f = 90°$, the final hoisting angle $\phi_f = 61°$), the control time is 4[s]. According to Eqs.(16)∼(18), the imaginary boom length R=0.898[m], the initial imaginary hoisting angle $\psi_i = 52°$(0.9065[rad]), the final imaginary hoisting angle $\psi_f = 104°$(1.8132[rad]), and $\gamma = 135°$(2.3562[rad]).

Fig. 6 and Fig. 7 show the experimental and simulation results which the load's velocity is trapezoid

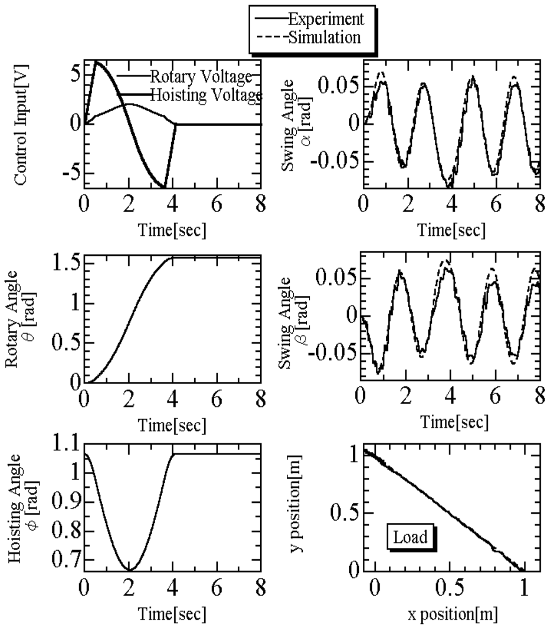

Fig. 6. Experimental results of linear transfer with trapezoid velocity

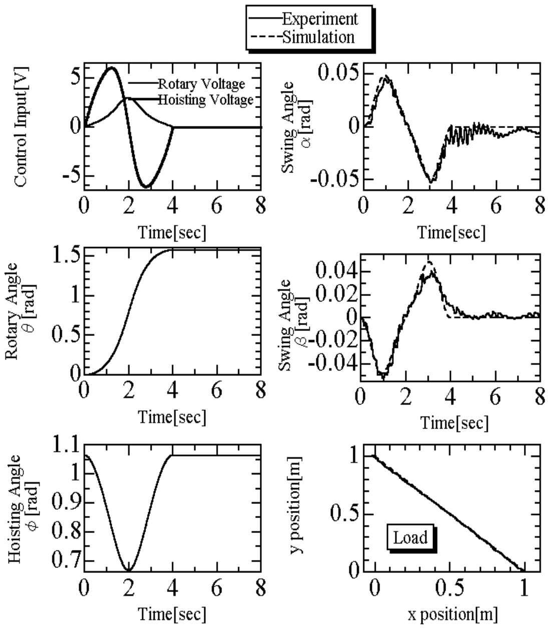

Fig. 7. Experimental results of linear transfer with the optimal velocity

velocity and the optimal velocity respectively. It is clear that the locus of the load on the x-y plane in Fig. 6 is a straight line and, at the end point of transfer, the load retains some residual vibration. But in Fig. 7 the locus of the load is a straight line and the sway of the load scarcely remains at the end point of transfer. The actual rotary angle and hoisting angle in the experiments are almost identical with those in the simulation results in the two figures, and the swing angle is qualitatively the same as that in the simulation. The actual calculation declares that the cost function converges after only 18 repeated calculations (for rotary transfer, the number is 262), and that it takes about 3.63 seconds (for rotary transfer it takes 69.21 seconds). These results demonstrate that, for actual application, the LTT method is more effective.

4. CONCLUSION

Nonlinear optimal control using the linear transfer transformation (LTT) model was presented to control the sway of a rotary crane. The experimental results agreed well with the simulation results. Both the validity of the model and the effectiveness of the control by the LTT model were also demonstrated. Compared with the conventional approach using the rotary crane model represented in three-dimensional space, it was clarified that the calculation time of optimization was faster in the proposed approach using the LTT model embedded in two-dimensional space, because centrifugal force did not exist for the calculation of optimal control.

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