15th Triennial World Congress, Barcelona, Spain

SIMPLE ROBUST CONTROLLERS: DESIGN, TUNING AND ANALYSIS

Roman Prokop, Petr Husták, Zdenka Prokopová

Institute of Information Technology, Tomas Bata University in Zlin
Nám. TGM 275, 762 72 Zlín, Czech Republic
tel., fax: ++420 67 754 3333
E-mail: prokop@ft.utb.cz

Abstract: The contribution is focused on robust design and analysis of continuous-time controllers for SISO systems without and with time delays. Controllers are obtained via solutions of diophantine equations in the ring of proper and stable rational functions by Youla-Kučera parameterization. Uncertainty is studied through the infinity norm H_∞. A scalar parameter was proposed as a tuning knob for minimization of the sensitivity function and uncertainty conditions. Both, feedback a feedback and feedforward structures of the controlled system are considered. Disturbance rejection and attenuation can be also easily solved by the proposed methodology. *Copyright © 2002 IFAC*

Key words: Uncertainty, norms, robust control, PID controllers.

1. INTRODUCTION

Continuous-time controllers of PID type have been widely used in many industrial applications for decades. There are several features of their success, e.g. structure simplicity, reliability, robustness in performance, etc., see e.g. (Aström and Hägglund, 1995; Bennet, 2000; Rad and Lo, 1992). However, the choice of the individual weighting of the three actions, i.e. proportional, integral and derivative has been a problem. Moreover, the number of tuning parameters in more sophisticated PID modifications proposed in (Aström, *et al.*, 1992) is even higher. The classical tuning algorithms were derived from Ziegler and Nichols method (Aström and Hägglund, 1995). The method is based on the ultimate cycle technique and it is not suitable for unstable and time delay systems, see e.g. in (Kaya and Atherton, 1999, Ventkatashankar and Chidambaram, 1994). Moreover, robust controllers and plant uncertainty became requisite and popular discipline in control theory during the last decade. Robustness also influenced design and tuning of PID controllers (Morari and Zafiriou, 1989; Prokop and Corriou, 1997). The necessity of robust control was naturally developed by the situation when the nominal plant (used in control design) differs from the real (perturbed and controlled) one. A suitable tool for parameter uncertainty is the infinity norm H_∞. Hence, a polynomial description of transfer functions had to be replaced by another one. A convenient description adopted from (Vidyasagar, 1985; Kučera 1993; Doyle, *et al.*, 1992) is a factorization approach where transfer functions are expressed as a ratio of two Hurwitz stable and proper rational functions. Then, the conditions of robust stability can be easily formulated in algebraic parlance and all controllers are obtained and parameterized via linear diophantine equations in an appropriate ring.

For SISO systems of the first and second orders this approach yields a class of PID like controllers. The methodology is proposed and analysed in (Prokop and Corriou, 1997; Prokop, *et al.*, 1997; Prokop and Prokopová, 1998). The algebraic approach illustrated by an example gives nontraditional PID structures proposed in e.g. (Aström, *et al.*, 1992; Morari and Zafiriou, 1989) in a different way. The fractional approach for SISO controllers brings a scalar parameter $m > 0$ which influences the dynamic of the feedback system as well as the robustness and sensitivity of proposed controllers. The methodology is suitable for stable and unstable systems (Prokop, *et al.*, 2000) and for systems with time delay (Prokop and Mészáros, 1995).

2. SYSTEM DESCRIPTION OVER RINGS

Let $R_m(s)$ denote a ring of Hurwitz stable and proper rational functions having no poles in the region **Re** $s > -m$; $m \geq 0$. For m=0 the traditional ring $R_s(s)$ see (Vidyasagar, 1985; Kučera, 1993) is obtained. Any transfer function H(s) of a (continuous-time) linear system has been traditionally expressed as a ratio of two polynomials in s. For the purposes of this contribution it is necessary to express the transfer functions as a ratio of two elements of $R_m(s)$. It can be easily performed by dividing, both the polynomial denominator and numerator by the same stable polynomial of the order of the original denominator:

$$H(s) = \frac{b(s)}{a(s)} = \frac{\dfrac{b(s)}{(s+m)^n}}{\dfrac{a(s)}{(s+m)^n}} = \frac{B(s)}{A(s)} \qquad (1)$$

$$n = \max\{\deg b(s), \deg a(s)\},\ m > 0$$

The scalar positive parameter m>0 can be conveniently used as a tuning knob for control behavior. Systems with time delay are obtained in a similar way by appropriate approximation of the term $e^{-\Theta s}$ see e.g. (Prokop, *et al.*, 2000).

The H_∞ norm in these rings is defined by

$$\|H\| = \sup_{\operatorname{Re} s \geq 0} |H(s)| = \sup_{\omega \in E} |H(j\omega)|$$

$$\|H_1 H_2\| = \left\| \begin{matrix} H_1 \\ H_2 \end{matrix} \right\| = \sup_{\operatorname{Re} s \geq 0} \{|H_1(s)|^2 + |H_2(s)|^2\}^{\frac{1}{2}} \qquad (2)$$

This (called infinity) norm is the radius of the smallest circle containing the Nyquist plot of the transfer function and it is a convenient tool for the evaluation of uncertainty.

Almost all mathematical models differ from physical systems. Let $H(s) = \dfrac{B(s)}{A(s)}$ be a nominal plant and consider a family of perturbed systems $H'(s) = \dfrac{B'(s)}{A'(s)}$ given by inequalities

$$\begin{aligned} &\|A - A'\| \leq \varepsilon_1, \quad \|B - B'\| \leq \varepsilon_2 \\ or \quad &\|A - A' \quad B - B'\| \leq \varepsilon \end{aligned} \qquad (3)$$

where ε_1, ε_2, ε are positive constants.

3. CONTROL DESIGN IN $R_m(s)$

The basic two degree-of-freedom (2DOF) control system is depicted in Fig.1.

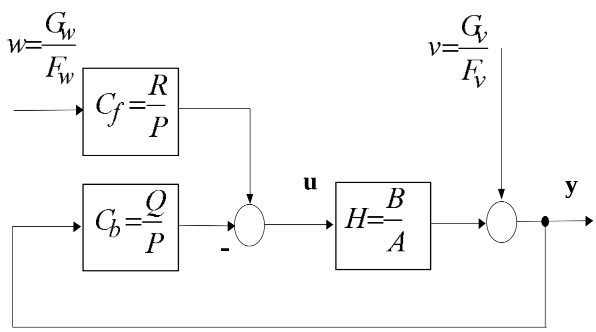

Fig. 1. Two-degrees of freedom control structure.

Note that a traditional one degree-of-freedom (1DOF) feedback controller operating on the tracking error is obtained for Q = R. External signals $w = \dfrac{G_w}{F_w}$ and $v = \dfrac{G_v}{F_v}$ represent the reference and disturbance signal, respectively. The most frequent case is a stepwise reference and a harmonic disturbance. Their denominators of transfer functions are then $F_w = \dfrac{s}{s+m}$ and $F_v = \dfrac{s^2+\omega^2}{(s+m)^2}$, respectively.

The basic task is now to ensure internal stability of the system in Fig.1. All stabilizing feedback controllers are given by all solutions of the linear diophantine equation (Vidyasagar, 1985; Kučera, 1993):

$$A\,P + B\,Q = 1 \qquad (4)$$

with the solvability condition that A and B are coprime. In the Youla- Kučera parameterization form, the feedback controllers are expressed by the fraction:

$$\frac{Q}{P}=\frac{Q_0-AT}{P_0+BT} \quad (5)$$

where P_0, $Q_0 \in R_m(s)$ is any particular solution of (4) and T is free in $R_m(s)$.

From the practical point of view, it is often desirable to ensure more than stability. Probably the most frequent problem of importance is that of reference tracking. Now, it is necessary to solve both structures 1DOF and 2DOF separately. Further, the following equations for 2DOF hold:

$$\begin{aligned} y &= \frac{AP}{AP+BQ}\frac{G_v}{F_v}+\frac{BR}{AP+BQ}\frac{G_w}{F_w} \\ e &= w-y=\left(1-\frac{BR}{AP+BQ}\right)\frac{G_w}{F_w}-\frac{AP}{AP+BQ}\frac{G_v}{F_v} \end{aligned} \quad (6)$$

and for 1DOF structure the last relation gives the form:

$$e=\frac{AP}{AP+BQ}\frac{G_w}{F_w}-\frac{AP}{AP+BQ}\frac{G_v}{F_v} \quad (7)$$

Since eq.(4) holds, the control error takes the form for the 2DOF:

$$e=(1-BR)\frac{G_w}{F_w}-AP\frac{G_v}{F_v} \quad (8)$$

and for the 1DOF structure:

$$e=AP\frac{G_w}{F_w}-AP\frac{G_v}{F_v} \quad (9)$$

For asymptotic tracking then follows:

a) F_w divides AP for 1DOF (10)

b) F_w divides 1-BR for 2DOF (11)

The last condition and the 2DOF structure give the second diophantine equation in the form:

$$F_wS+BR=1 \quad (12)$$

For the 1DOF structure, the asymptotic tracking problem leads to the condition of divisibility that F_w divides the product AP or $AP=F_wF_0$.

The next step is aimed to the disturbance rejection problem. Both structures can be solved in a unified way since the transfer functions from v to e are the same. The problem then gives a second condition of divisibility that the product AP has to be divisible by F_v, or $AP=F_vF_1$. More precisely F_v must divide the multiple AP. When define relatively prime elements A_0, F_{v0} in $R_m(s)$ by

$$\frac{A}{F_v}=\frac{A_0}{F_{v0}} \quad (13)$$

then the feedback controller is given by

$$C_b=\frac{Q}{P}=\frac{Q}{P_1F_{v0}} \quad (14)$$

where P_1, Q is any solution of the equation

$$AF_{v0}P_1+BQ=1 \quad (15)$$

In the case of (15) all stabilizing controllers are also given in the parametric form similar of (5):

$$\frac{Q}{P}=\frac{Q_0-AF_{v0}T}{P_1+BT} \quad (16)$$

For robust control, it is necessary to choose a part of all stabilizing controllers (4), (5) which stabilize perturbed plants (2). The answer can be found in (Vidyasagar, 1985). For perturbed plants (2) choose such P, Q in (4), (5) which fulfill the conditions

$$\varepsilon_1\,\|P\|+\varepsilon_2\,\|Q\|<1 \quad (17.a)$$

$$\text{or}\quad \varepsilon\,\left\|\begin{matrix}P\\Q\end{matrix}\right\|<1 \quad (17.b)$$

where P, Q are any solution of (5) or (15). The first condition of (17) is a sufficient one, the second is a necessary and sufficient one see (Kučera 1993; Vidyasagar, 1985). For a deeper insight into robustness the notion of the sensitivity function:

$$\in=\frac{y}{v}=AP \quad (18)$$

can be also used in the sense of (Doyle, *et al.*, 1992). For the mentioned SISO systems, sensitivity function $\in$ is a nonlinear function of $m>0$ and it can be minimized by a simple scalar optimization method. In this way, the „most robust“ controller of given structure can be obtained.

The proposed robust design can be also used for time delay systems. Let the transfer function of a controlled plant be:

$$G_1(s)=\frac{Ke^{-\tau s}}{s+\alpha} \quad (19)$$

Then the linear approximation of the time delay term has to be performed. It can be done in several methods. The first one and the simplest is neglect the delay $e^{-\tau s}$. Then the time delay is considered as a perturbation of the nominal transfer function. Next two approximations are based on the Taylor series approximation of $e^{-\tau s}$ in the numerator or in the denominator of the time delay term $e^{-\tau s} \approx (1-\tau s) \approx (1+\tau s)^{-1}$. The last model can be obtained by Padé approximation. All approximate transfer functions then can be covered by the second order in the form:

$$G(s) = \frac{b_1 s + b_0}{s^2 + a_1 s + a_0} = \frac{\dfrac{b_1 s + b_0}{(s+m)^2}}{\dfrac{s^2 + a_1 s + a_0}{(s+m)^2}} = \frac{B(s)}{A(s)} \quad (20)$$

for $m > 0$. See (Prokop and Meszaros, 1996) or (Prokop, *et al.*, 2000) for details.

4. PID CONTROLLERS AND TUNING

In this section, a derivation of a class of PID controllers will be shown through the scalar parameter m. Suppose a second order system by (19). Further, the step-wise reference with $F_w = \dfrac{s}{s+m}$ is assumed. Then the equation (4) takes the form:

$$\begin{aligned}&(s^2 + a_1 s + a_0)(p_1 s + p_0) + \\ &(b_1 s + b_0)(q_1 s + q_0) = (s+m)^3\end{aligned} \quad (21)$$

After comparing left and right hand side terms of (20) the general solution of (4) can be expressed.

$$\begin{aligned}P &= \frac{s + p_0}{s+m} + \frac{b_1 s + b_0}{(s+m)^2} T; \\ Q &= \frac{q_1 s + q_0}{s+m} - \frac{s^2 + a_1 s + a_0}{(s+m)^2} T;\end{aligned} \quad (22)$$

where

$$d_1 = 3m^2 - a_0 - 3a_1 m + a_1^2 - \frac{b_1}{b_0} m^3 + \frac{b_1}{b_0} a_0 (3m - a_1)$$

$$q_1 = \frac{d_1}{b_0 + \dfrac{b_1^2}{b_0} a_0 - a_1 b_1}$$

$$p_0 = 3m - a_1 - b_1 q_1$$

$$q_0 = \frac{m^3 - a_0 p_0}{b_0}$$

were obtained by straightforward calculations.
The divisibility condition $F_w \backslash P$ is achieved for $T = t_0 = -\dfrac{p_0 m}{b_0}$ and the final solution is:

$$\widetilde{P} = \frac{s^2 + \widetilde{p}_0 s}{(s+m)^2}; \quad \widetilde{Q} = \frac{\widetilde{q}_2 s^2 + \widetilde{q}_1 s + \widetilde{q}_0}{(s+m)^2} \quad (23)$$

where

$$\begin{aligned}\widetilde{p}_0 &= m + p_0 + b_1 t_0;\ \widetilde{q}_2 = q_1 - t_0; \\ \widetilde{q}_1 &= q_0 + q_1 m - a_1 t_0;\ \widetilde{q}_0 = q_0 m - a_0 t_0\end{aligned} \quad (24)$$

and the transfer function for $\dfrac{Q}{P}$ is given:

$$\frac{Q}{P} = \frac{\widetilde{q}_2 s^2 + \widetilde{q}_1 s + \widetilde{q}_0}{s(s + p_0)} \quad (25)$$

It is clear that (24) corresponds with the realistic PID controller, see (Aström, et al., 1992)

$$C_{PID}(s) = K\left(1 + \frac{1}{T_I s} + \frac{T_D s}{Ns + 1}\right) \quad (26)$$

<u>Remarks:</u>
1. Equation (12) for a feedforward part can be solved in a similar way. The final controller contains the setpoint weighting, see (Aström and Hägglund, 1995).
2. As follows from (22) - (24) the scalar m>0 is incorporated into controller parameters as an independent and nonlinear variable. This "tuning knob" influences stability of the closed-loop system as well as the robustness and behavior.
3. The sensitivity function as well as the left hand sides of inequalities (17), (18) are non-linear functions of this scalar parameter m>0.

5. SIMULATIONS AND ANALYSIS

<u>Example 1:</u> Stable second order system
The nominal stable process is

$$G(s) = \frac{1}{s^2 + s + 1} = \frac{B}{A} \quad (27)$$

and the perturbation is caused by time delay:

$$G'(s) = \frac{e^{-0.5s}}{s^2 + s + 1} = \frac{B'}{A'} \quad (28)$$

The 1DOF controller was calculated by (22)-(24). Fig.2 illustrates the left side of the necessary condition (17.a) as a nonlinear function $\eta(m)$ of m>0 with the minimal value m_0 = 0.69. The interval for $\|\eta(m) \le 1\|$ is <0.19; 1.34>.

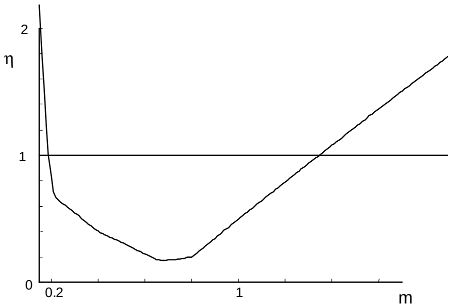

Fig. 2. Condition (17.a) as a function a of $m > 0$ for nominal plant (27).

Fig.3 shows the control responses for the minimal value m_0 =0.69. This figure demonstrates that the most robust behavior would not be optimal from the user point of view. Fig.4 illustrates the nominal and perturbed response for another value of the interval <0.19; 1.34> and this response does not exhibit the nonminimal behavior.

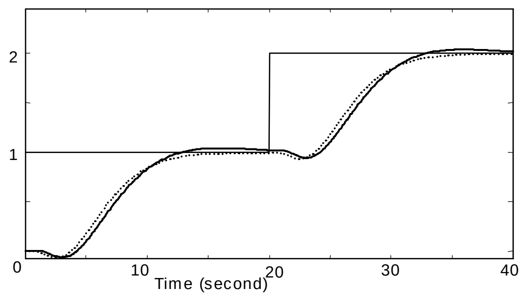

Fig. 3. 1DOF response for m=0.69(dotted - nominal, solid - perturbed).

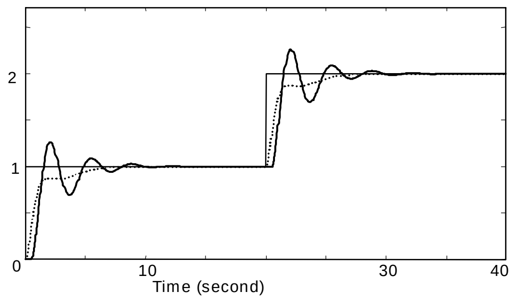

Fig. 4. 1DOF response for m=1.3 (dotted - nominal, solid - perturbed).

Example 2: Integrator plus time delay process

Integrator plus dead time model was found to be a suitable model for a number of technological plants (Kookos, *et al*. 1999). In this example, the integrator with transfer function:

$$H(s) = \frac{e^{-2s}}{s} \tag{29}$$

was controlled. Fig. 5 shows the control response with neglecting of the time delay term and PI controller. Fig. 6 represents the PID control in which the time delay term is approximated by a first order Taylor denominator expansion. as well as the Nyquist plot of the open loop. The robustness of the resulting control system is clear from the distance of the Nyquist curve from the critical point -1.

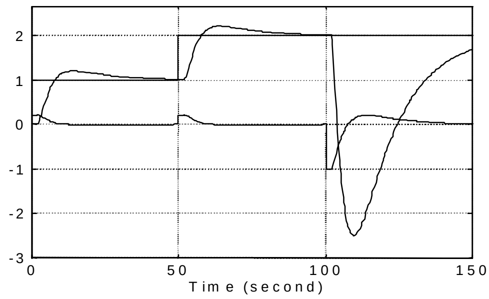

Fig. 5. Control response of the integrator with dead time (29) with neglecting of the dead-time (m=0.1)

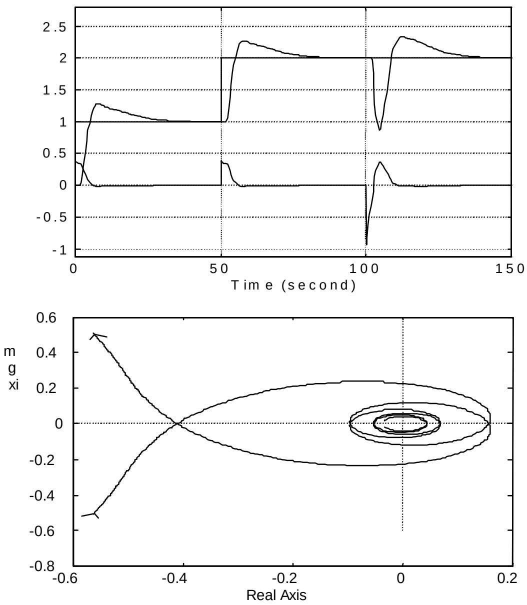

Fig. 6. Control response of the integrator with time delay (29) and the open loop Nyquist plot with the denominator approximation ($m=0.3$)

6. PROGRAM IMPLEMENTATION

A MATLAB-package with simulation support in SIMULINK was developed for nominal plants of the first and second orders with or without dead-time. These transfer functions in the mentioned control design cover a class of generalized PID-like control structures. The program system enables design and simulation of a wide spectrum of robust control problems.

First, a nominal plant of a desired structure with its transfer function and dead time has to be defined. A transfer function of the perturbed plant can be set up. The default choice is the identity of nominal and perturbed systems. Then, a control structure (1DOF or 2DOF ones) is chosen. Fig. 7 shows the main menu of the program system.

Further, there are three main options for the control design. The simplest case calculates controller parameters for a given $m>0$ according to the above mentioned methodology. The second possibility obtains the particular m_0 by scalar minimization of the norm of the sensitivity function. In this way, the „most robust“ regulator for a given nominal plant is obtained. The third option with respect to perturbations generates a scalar optimization of robust stability conditions in the sense of (17).

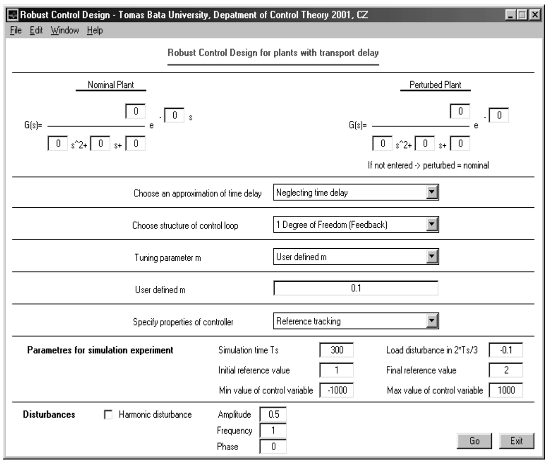

Fig. 7. The main menu window in MATLAB support for automatic tuning.

The simulation of the perturbed plant with the regulator computed for the nominal transfer function is performed in the standard Simulink environment. Various simulation parameters (simulation horizon, reference, load and output disturbances, input constraints, etc.) can also be defined by the user.

7. CONCLUSIONS

A design method based on fractional representation was developed for SISO continuous-time systems generally with time delay. Resulting control laws for first and second order systems give a class of generalized PI and PID structures. The proposed method enables to tune and influence the robustness and control behavior by a single scalar parameter $m>0$. The tuning parameter can be chosen arbitrarily or it is a result of the robust and sensitivity optimization. The proposed methodology is supported by a Matlab + Simulink program system for automatic design and simulation.

ACKNOWLEDGEMENTS

This work was supported by the grant of Ministry of Education of the Czech Republic No. MSM 281 100 001 and by the Europoly project INCO Copernicus No. CP 977 010.

REFERENCES

Aström, K.J., C.C. Hang, P. Persson, and W.K. Ho (1992). Towards intelligent PID control. *Automatica*, **Vol. 28**, **No.1**, pp. 1-9.

Aström, K.J. and T. Hägglund (1995). *PID Controllers: Theory, Design and Tuning.* Instrument Society of America, USA.

Bennet, S. (2000). The past of PID controllers. In: *Prepr. IFAC Workshop on Dig. Control*, pp.3-13, Terrassa, Spain,.

De Paor, A.M. and M.O´Malley (1989). Controllers of Ziegler Nichols type for unstable processes. , *Int. J. Control* , **Vol. 49**, pp. 1273-84.

Doyle, C.D., B.A. Francis and A.R. Tannenbaum (1992). *Feedback Control Theory*. Macmillan, New York.

Hwang, S.H., S.J. Shiu and M.L Lin (1997). PID control of unstable systems having time delay. In: *Prepr. IFAC ECC Bruxelles*, No.559.

Kaya, I. and D.P.Atherton (1999). A new PI-PD Smith predictor for control of processes with long dead time. In: *Prepr. 14th World Congress IFAC Beijing*, pp. 283-288.

Kučera, V. (1993). Diophantine equations in control - A survey, *Automatica*, **Vol. 29**, **No.6**, pp. 1361-1375.

Morari, M. and E. Zafiriou (1989). *Robust Process Control*. Prentice Hall, New Jersey.

Prokop, R. and A. Mészáros (1996). Design of robust controllers for SISO time delay systems. *Journal of Electrical Engineering*, **Vol. 47**, **No. 11-12**, pp. 287-294.

Prokop, R. and J.P. Corriou (1997). Design and analysis of simple robust controllers, *Int. J. Control* , **Vol. 66**, **No. 6**, pp. 905-921.

Prokop, R., P. Dostál and Z. Prokopová (2000). Robust linear controllers for unstable systems with time delay. In: *Proc. 3rd IFAC Symp. Robust Control Design*, Prague, pp. 171-176.

Prokop, R. and Z.Prokopová (1998): Robust Tuning of PID-like Controllers. In: Prepr. of 5th Symp. on LCA, Shenyang, Sept. 8-10, 1998, pp. TS1-25-30.

Rad, A.B. and W.L. Lo (1997). Predictive PI controller, *Int. J. Control* , **Vol. 63**, **No. 6**, pp. 953-975.

Venkatashankar, V. and M. Chidambaram (1994). Design of P and PI controllers for unstable firs-order plus time delay systems, *Int. J. Control*, **Vol. 60**, **No. 1**, pp. 137-144.

Vidyasagar, M. (1985). *Control system synthesis: a factorization approach.* MIT Press, Cambridge, M.A.

Watanabe, K. and M. Ito (1981). A process model control for linear systems with delay. *IEEE Trans. On AC*, **Vol. 26**, **No. 6**, pp. 1261-66.