15th Triennial World Congress, Barcelona, Spain

RIPPLE-SUPPRESSED MULTIRATE ADAPTIVE CONTROL

Mitsuaki Ishitobi, Masaki Kawanaka and Hiroaki Nishi

Department of Mechanical Engineering and Materials Science
Kumamoto University
2-39-1 Kurokami, Kumamoto 860-8555, JAPAN
E-mail: mishi@kumamoto-u.ac.jp
Phone and Fax: +81-96-342-3777

Abstract: This paper deals with adaptive control of linear time-invariant systems with unknown parameters and with two sampling rates: a slower one for the output and a faster one for the input. It is known that intersample ripples often arise in the outputs of the closed-loop multirate systems although multirate control has interesting advantages. In this paper, a ripple-suppressed multirate adaptive control scheme is proposed. A simulation example is given to show the effectiveness of the presented algorithm. *Copyright © 2002 IFAC*

Keywords: multirate, model reference adaptive control, digital control, tracking

1. INTRODUCTION

Due to the rapid development in the technology of very large-scale integration (VLSI), digital controllers are increasingly replacing analog controllers in the design of many control systems. It is well known that the sampling rate is a critical design parameter in the digital computer control of continuous-time systems. Control system performance demands a fast rate of sampling while sensor constraints restrict the achievable sampling frequencies. In many practical cases, however, single sampling rate systems, where the measurement sampling rate is the same as the control update rate, are often designed and implemented since they are simpler than multirate sampling systems, whereby control updates are executed at faster rate than output samples are taken. Recently, attention has been focused gradually on multirate sampling control (Araki and Yamamoto, 1986; Al-Rahmani and Franklin, 1992). Adaptive versions of multirate control have been also studied (Scattolini, 1988; Zhang and Tomizuka, 1988; Feliu *et al.*, 1990; Lu *et al.*, 1990, 1992; Albertos *et al.*, 1996; Ishitobi, 1997). It is shown that intersample ripples often arise in the outputs of the closed-loop multirate systems although multirate control has interesting advantages (Albertos *et al.*, 1996; Tangirala *et al.*, 2001). Lu *et al.* (1990, 1992) presented a ripple-suppressed multirate adaptive control scheme. Their algorithm, however, cannot be applied to unstable plants because the parameter identification and the output estimation are formulated using the output error method, and the passivity of a plant is assumed.

This paper proposes a ripple-suppressed multirate adaptive control scheme applicable to unstable plants. The key ideas are that the equation error method is used in the parameter identification and the output estimation, and that the control input is constructed by the measured outputs, the estimated outputs and past inputs. The estimated outputs are obtained through a state observer and a filter. A numerical example is given to show the effectiveness of the proposed method.

2. PROBLEM STATEMENT

Consider a linear time-invariant and sampled-data system

$$A_1(q^{-1})y(t) = q^{-d}B_1(q^{-1})u(t) \quad (1)$$

where q^{-1} is the unit delay operator, d represents a pure time delay, $\{u(t)\}, \{y(t)\}$ denote the input and output respectively and $A_1(q^{-1})$, $B_1(q^{-1})$ are given by

$$\begin{aligned} A_1(q^{-1}) &= 1 + a_{11}q^{-1} + \cdots + a_{1n}q^{-n} \\ &= (1-\alpha_1 q^{-1})\cdots(1-\alpha_n q^{-1}) \quad (2) \\ B_1(q^{-1}) &= b_{10} + b_{11}q^{-1} + \cdots + b_{1m}q^{-m} \\ & \qquad b_{10} \neq 0 \quad (3) \end{aligned}$$

The system (1) is assumed to be a minimal representation of the plant. Hence, the polynomials $A_1(q^{-1})$ and $B_1(q^{-1})$ are coprime. Further assume that the available on-line measurement data are $u(t)$ and $y(iJ)$, where $t = 0, 1, 2, \cdots$, $i = 0, 1, 2, \cdots$. The number J is any finite integer larger than or equal to two. Note that $\{y(iJ+k),\ k = 1, 2, \cdots, J-1\}$ are not available from the measured data.

Moreover, the following assumptions are imposed throughout this paper.

(a) The time delay d is known.
(b) The orders of the polynomials $A_1(q^{-1})$ and $B_1(q^{-1})$ are known. However, parameters a_{1i} and b_{1i} are unknown.
(c) The polynomial $B_1(q^{-1})$ is stable.

The objective is to cause the output $\{y(t)\}$ to track the desired output $\{y_r(t)\}$.

3. ADAPTIVE CONTROL SYSTEM

The first step is to transform the model (1) into a form which can be identified from the available measured data sequence.

Multiplying both sides of (1) by

$$C(q^{-1}) = \prod_{i=1}^{n}\big(1 + \alpha_i q^{-1} + \cdots + \alpha_i^{J-1}q^{1-J}\big) \quad (4)$$

provides an equivalent non-minimal form (Lu *et al.*, 1990) of (1)

$$A(q^{-J})y(t) = q^{-d}B(q^{-1})u(t) \quad (5)$$

where

$$\begin{aligned} A(q^{-J}) &= A_1(q^{-1})C(q^{-1}) \\ &= 1 + a_1 q^{-J} + \cdots + a_n q^{-nJ} \quad (6) \end{aligned}$$

$$\begin{aligned} B(q^{-1}) &= B_1(q^{-1})C(q^{-1}) \\ &= b_0 + \cdots + b_{m+nJ-n}q^{-m-nJ+n} \quad (7) \end{aligned}$$

The equivalent non-minimal model (5) is convenient for parameter identification and output estimation in multirate sampling control.

Alternatively, the input/output relationship can be described in a regression vector form

$$y(t) = \boldsymbol{\phi}(t-1)^T\boldsymbol{\theta} \quad (8)$$

where

$$\begin{aligned} &\boldsymbol{\phi}(t-1)^T \\ &\quad = [-y(t-J), \cdots, -y(t-nJ), u(t-d), \\ &\qquad \cdots, u(t-d-m-nJ+n)] \quad (9) \\ &\boldsymbol{\theta}^T = [a_1, \cdots, a_n, b_0, \cdots, b_{m+nJ-n}] \quad (10) \end{aligned}$$

If the estimate of $\boldsymbol{\theta}$ is denoted by

$$\begin{aligned} \widehat{\boldsymbol{\theta}}(t)^T = [&\widehat{a}_1(t), \cdots, \widehat{a}_n(t), \\ &\widehat{b}_0(t), \cdots, \widehat{b}_{m+nJ-n}(t)] \quad (11) \end{aligned}$$

then the parameter adaptation algorithm is given by

$$\begin{aligned} \widehat{\boldsymbol{\theta}}(iJ) &= \widehat{\boldsymbol{\theta}}(iJ-J) \\ &\quad + \frac{P(iJ)\boldsymbol{\phi}(iJ-1)[y(iJ)-\overline{y}(iJ)]}{\lambda + \boldsymbol{\phi}(iJ-1)^T P(iJ)\boldsymbol{\phi}(iJ-1)} \quad (12) \\ P^{-1}(iJ+J) &= \lambda P^{-1}(iJ) \\ &\quad + \boldsymbol{\phi}(iJ-1)\boldsymbol{\phi}(iJ-1)^T \quad (13) \\ \overline{y}(iJ) &= \boldsymbol{\phi}(iJ-1)^T\widehat{\boldsymbol{\theta}}(iJ-J) \quad (14) \\ 0 < \lambda &\leq 1,\ P^{-1}(0) > 0,\ \widehat{b}_0(0) \neq 0 \quad (15) \\ \widehat{\boldsymbol{\theta}}(iJ+k) &= \widehat{\boldsymbol{\theta}}(iJ), \quad k = 1, \cdots, J-1 \quad (16) \end{aligned}$$

Note that $\widehat{\boldsymbol{\theta}}(t)$ is not updated at the $J-1$ inter-sampling instants of the outputs $y(iJ)$, and it is not necessary to calculate the parameter identification law (12)-(15).

Now define the parameter vector of the minimal representation model (1) by

$$\boldsymbol{\zeta}^T = [a_{11}, \cdots, a_{1n}, b_{10}, \cdots, b_{1m}] \quad (17)$$

and its estimated vector by

$$\begin{aligned} \widehat{\boldsymbol{\zeta}}(t)^T = [&\widehat{a}_{11}(t), \cdots, \widehat{a}_{1n}(t), \\ &\widehat{b}_{10}(t), \cdots, \widehat{b}_{1m}(t)] \quad (18) \end{aligned}$$

then $\widehat{\boldsymbol{\zeta}}(t)$ is calculated as follows.

At first, the update algorithm of $\widehat{\boldsymbol{\zeta}}(t)$ at the instants of the output sampling $t = 0, J, 2J, \cdots$, is explained. From (6) and (7), it is obvious that

$$A(q^{-J})B_1(q^{-1}) = A_1(q^{-1})B(q^{-1}) \quad (19)$$

Hence, using the relation

$$\widehat{A}(t,q^{-J})\widehat{B}_1(t,q^{-1})=\widehat{A}_1(t,q^{-1})\widehat{B}(t,q^{-1}) \quad (20)$$

the estimated parameter vector $\widehat{\boldsymbol{\zeta}}(iJ)$ of the minimal model is expressed by the identified parameters $\widehat{a}_i(iJ)$ and $\widehat{b}_i(iJ)$ of the non-minimal model (8)

$$M(iJ)\widehat{\boldsymbol{\zeta}}(iJ) = \widehat{\boldsymbol{\eta}}(iJ) \quad (21)$$

where

$$M\ (iJ)$$

$$= \begin{array}{c} \quad n \qquad\qquad\qquad m+1 \\ \left[\begin{array}{cccccc} 0 & & 0 & 1 & & 0 \\ -\widehat{b}_0 & \ddots & & \widehat{a}_1 & \ddots & \\ -\widehat{b}_1 & \ddots & 0 & \vdots & \ddots & 1 \\ \vdots & \ddots & -\widehat{b}_0 & \widehat{a}_{nJ} & \ddots & \widehat{a}_1 \\ -\widehat{b}_l & \ddots & -\widehat{b}_1 & & \ddots & \vdots \\ & \ddots & \vdots & & & \widehat{a}_{nJ} \\ 0 & & -\widehat{b}_l & & 0 & \end{array}\right] \end{array} \quad (22)$$

$$l = m + nJ - n$$

$$\widehat{a}_{\ j}(iJ) = \left\{ \begin{array}{ll} \widehat{a}_{j/J}(iJ), & j = kJ,\ k = 1,\cdots,n \\ 0, & \text{otherwise} \end{array} \right. \quad (23)$$

$$\widehat{\boldsymbol{\eta}}\ (iJ)^T = [\widehat{b}_0(iJ),\cdots,\widehat{b}_{m+nJ-n}(iJ), 0,\cdots,0] \quad (24)$$

Since $M(iJ)$ is not a square matrix, the pseudo inverse matrix is used to obtain $\widehat{\boldsymbol{\zeta}}(iJ)$

$$\widehat{\boldsymbol{\zeta}}(iJ) = [M(iJ)^T M(iJ)]^{-1} M(iJ)^T \widehat{\boldsymbol{\eta}}(iJ) \quad (25)$$

At the instants of the input sampling between the output sampling, the parameter update is not executed as follows.

$$\widehat{\boldsymbol{\zeta}}(iJ+k) = \widehat{\boldsymbol{\zeta}}(iJ), \quad k = 1,\cdots,J-1 \quad (26)$$

Next, the output estimate is obtained through a state observer and a filter.

i) State estimate at the output measurement instants ($t = J,\ 2J,\ \cdots$)

The state space equation of the plant (1) can be expressed as

$$\left\{ \begin{array}{ll} \boldsymbol{x}(t+1) & = \Phi\boldsymbol{x}(t) + \boldsymbol{\psi}u(t) \\ y(t) & = \boldsymbol{c}^T\boldsymbol{x}(t),\ \boldsymbol{c}^T = [1\ 0\cdots 0] \end{array} \right. \quad (27)$$

Since the output sampling interval is longer than the input by J times, the state space equation with the measurable outputs and inputs is given by

$$\left\{ \begin{array}{ll} \boldsymbol{x}(t) & = \Phi^J\boldsymbol{x}(t-J) + \boldsymbol{\psi}u(t-1) \\ & \quad +\Phi\boldsymbol{\psi}u(t-2) + \cdots \\ & \quad +\Phi^{J-1}\boldsymbol{\psi}u(t-J) \\ y(t) & = \boldsymbol{c}^T\boldsymbol{x}(t) \end{array} \right. \quad (28)$$

Hence, the state observer for (28) is designed by

$$\begin{aligned} \widehat{\boldsymbol{z}}(t) &= F\widehat{\boldsymbol{z}}(t-J) + \widehat{\boldsymbol{g}}(t)y(t-J) \\ &\quad +\widehat{\boldsymbol{\psi}}_1(t)u(t-1) + \cdots \\ &\quad +\widehat{\boldsymbol{\psi}}_J(t)u(t-J) \end{aligned} \quad (29)$$

$$F = \left[\begin{array}{cccc} -f_1 & 1 & \cdots & 0 \\ \vdots & \ddots & \ddots & 1 \\ -f_n & 0 & \cdots & 0 \end{array} \right]$$

$$\widehat{\boldsymbol{g}}(t) = \left[\begin{array}{c} f_1 - \widehat{a}_{J1} \\ \vdots \\ f_n - \widehat{a}_{Jn} \end{array} \right]$$

$$\widehat{\boldsymbol{\psi}}_i(t) = \widehat{H}_0(t)\widehat{\Phi}^{k-1}(t)\widehat{\boldsymbol{\psi}}(t),\ k = 1,\cdots,J$$

where F is a stable matrix, $\widehat{H}_0(t)$ is a non-singular matrix which transforms (28) to the observable canonical form, and $\widehat{a}_{Jk}(t)$ ($k = 1,\cdots,n$) is the coefficient of the term with the order $(n-k)$ for the characteristic polynomial of $\widehat{\Phi}^J(t)$ which is the estimate of Φ^J. Hence, the estimated state space variable is obtained as

$$\widehat{\boldsymbol{x}}(t) = \widehat{H}_0^{-1}(t)\widehat{\boldsymbol{z}}(t) \quad (30)$$

ii) State estimate at the intersampling instants of the output measurement ($t \neq J,\ 2J,\ \cdots$)

The state estimate is constructed by the following filters.

$$\left\{ \begin{array}{ll} \overline{\boldsymbol{x}}(t) & = \widehat{\Phi}(t-1)\widehat{\boldsymbol{x}}(t-1) \\ & \quad +\widehat{\boldsymbol{\psi}}(t-1)u(t-1) \\ & \quad t = J+1,\ 2J+1,\ \cdots \\ \overline{\boldsymbol{x}}(t) & = \widehat{\Phi}(t-1)\overline{\boldsymbol{x}}(t-1) \\ & \quad +\widehat{\boldsymbol{\psi}}(t-1)u(t-1) \\ & \quad t \neq J+1,\ 2J+1,\ \cdots \end{array} \right. \quad (31)$$

These filters generate the output estimate

$$\widehat{y}(t) = \boldsymbol{c}^T\overline{\boldsymbol{x}}(t),\ t \neq 0, J, 2J, \cdots \quad (32)$$

The input is calculated by the following steps. We give an asymptotically stable polynomial

$$A_r(q^{-1}) = 1 + d_{11}q^{-1}\cdots + d_{1h}q^{-h}, \quad h \leq n+d-1$$

Next, the polynomials $R(t,q^{-1})$ and $S(t,q^{-1})$ are determined by solving the following polynomial equation

$$A_r(q^{-1}) = \widehat{A}_1(t,q^{-1})R(t,q^{-1})$$

$$+q^{-d}S(t,q^{-1}) \qquad (33)$$

where

$$R(t,q^{-1}) = 1 + r_1(t)q^{-1} + \cdots + r_{d-1}(t)q^{-d+1} \qquad (34)$$

$$S(t,q^{-1}) = s_0(t) + s_1(t)q^{-1} + \cdots + s_{n-1}(t)q^{-n+1} \qquad (35)$$

Finally, the adaptive control law is given by

$$u(t) = \frac{1}{\widehat{b}_{10}(t)}[A_r(q^{-1})y_r(t+d) - S(t,q^{-1})\widetilde{y}(t) - \{\widehat{B}_1(t,q^{-1})R(t,q^{-1}) - \widehat{b}_{10}(t)\}u(t)] \qquad (36)$$

where

$$\widetilde{y}(t) = \begin{cases} y(t) & t = 0, J, 2J, \cdots \\ \widehat{y}(t) & t \neq 0, J, 2J, \cdots \end{cases}$$

It is worth noting that the output estimates $\widehat{y}(t)$ at the input sampling instants ($t \neq 0, J, 2J, \cdots$) are used in the input (36) because real values of the output are not measured between the output sampling instants.

Remark 1: The parameter identifier (12)-(15) is an equation error method. An output error method for parameter estimation was proposed by Lu *et al.* (1990). It was shown that the output error method requires the stability of plants (Lu *et al.*, 1990).

4. CONVERGENCE ANALYSIS

The convergence properties of the multi-rate adaptive control system derived in the previous section are developed under the persistency of excitation (PE) condition.

The estimation model (5) is an overparameterized one. In general, parameter estimates converge to a linear hypersurface in an overparameterized model even if the PE condition is satisfied (Lu and Fisher, 1989; Heymann, 1988). However, since the monic common factor polynomial (4) is unique in the multi-rate sampling systems, the limiting linear hypersurface reduces to a point; the true parameter vector. In other words, the estimated parameter vector $\widehat{\boldsymbol{\theta}}(t)$ approaches the true parameter vector $\boldsymbol{\theta}$ under the PE condition (Lu and Fisher, 1989).

When the estimated parameter vector $\widehat{\boldsymbol{\theta}}(iJ)$ coincides with the true value $\boldsymbol{\theta}$, the matrix $\left[M(iJ)^T M(iJ)\right]$ is nonsingular since the polynomials $A_1(q^{-1})$ and $B_1(q^{-1})$ are coprime (Kawashima, 1993). Hence, the estimated parameter vector $\widehat{\boldsymbol{\zeta}}(iJ)$ of the minimal model is obtainable with probability 1 under the PE condition, and converges to the true value vector $\boldsymbol{\zeta}$ when $\widehat{\boldsymbol{\theta}}(iJ)$ tends to $\boldsymbol{\theta}$; that is, the polynomial equation (20) is solvable. If the matrix $\left[M(iJ)^T M(iJ)\right]$ is singular at some time iJ, the estimated vector $\widehat{\boldsymbol{\zeta}}(t)$ at the previous time $t = iJ - J$ can be used.

It is obvious that the state observer ((29) and (30)) makes $\widehat{\boldsymbol{x}}(iJ)$ to go to the true vector $\boldsymbol{x}(iJ)$ at the output sampling instants when the estimated parameters vector $\widehat{\boldsymbol{\zeta}}(iJ)$ of the minimal model (1) tends to the true one $\boldsymbol{\zeta}$.

Next, if the estimated state vector $\widehat{\boldsymbol{x}}(iJ)$ reaches the true vector $\boldsymbol{x}(iJ)$ at the output sampling instants, the estimated vector $\overline{\boldsymbol{x}}(t)$ of the filters at the intersampling instants of the outputs also tends to the true value $\boldsymbol{x}(t)$ because the first filter of (31) consists of the estimated state vector $\widehat{\boldsymbol{x}}(iJ)$.

Moreover, the estimated output $\widehat{y}(t)$ also goes to the true output $y(t)$ at the intersampling instants of the outputs $t \neq J,\ 2J,\ \cdots$, though they are not measured.

Finally the input leads to the relation

$$A_r(q^{-1})(y_r(t+d) - y(t+d)) \to 0 \qquad (37)$$

since the control law (36) is the same as the case of single-rate sampling control except that the output estimates are used instead of the true values of the output at the intersampling instants of the outputs. The equation (37) implies the achievement of the tracking since $A_r(q^{-1})$ is a stable polynomial.

As a result, we have the following theorem.

Theorem: The tracking is achieved and all signals are bounded for all time under the persistency of excitation (PE) condition.

Remark 2: It is not hard to understand that the tracking is achieved not only at the output measurement instants but also at the input update instants. Therefore, the ripple at the intersampling instants of the output measurement is suppressed.

5. A NUMERICAL EXAMPLE

This section shows simulation results obtained when the proposed multirate adaptive control algorithm is applied to a second order system. Consider a plant with a continuous-time transfer function expressed as

$$G(s) = \frac{2s+1}{(s+1)(s-2)} \qquad (38)$$

When a discrete-time system is composed of a zero-order hold, the continuous-time system $G(s)$ and a sampler in series, the sampled output $y(t)$ is related to the sampled input $u(t)$ by the following equation

$$A_1(q^{-1})y(t) = q^{-1}B_1(q^{-1})u(t) \tag{39}$$

where

$$A_1(q^{-1}) = 1 + a_{11}q^{-1} + a_{12}q^{-2} \tag{40}$$
$$B_1(q^{-1}) = b_{10} + b_{11}q^{-1} \tag{41}$$

If the sampling period T_s of the input is selected as 0.024, the corresponding discrete-time system has the following parameters

$$\begin{cases} a_{11} = -2.02545637 \\ a_{12} = 1.02429032 \\ b_{10} = 0.04888031 \\ b_{11} = -0.04829729 \end{cases} \tag{42}$$

Here, the polynomial $B_1(q^{-1})$ is stable.

Assume that the output sampling period is 0.12. In other words, the output $y(t)$ can be measured only once in every five control updates, i.e., $J = 5$.

Note that the adaptive control algorithm proposed by Lu *et al.* (1990) cannot be applied to the plant (39) since it is not stable.

The desired output is given by the output of the discrete-time system for a transfer function

$$G_r(s) = \frac{1}{2s+1} \tag{43}$$

The input of $G_r(s)$ is a rectangular wave signal with amplitude 1.0 and with period $tT_s = 20$ (833 steps) to satisfy the PE condition.

The initial values of the estimated parameters and the design parameters are chosen as

$$\begin{cases} \widehat{a}_1(0) = -2.158, & \widehat{a}_2(0) = 1.127 \\ \widehat{b}_0(0) = 0.04888, & \widehat{b}_1(0) = 0.05071 \\ \widehat{b}_2(0) = 0.05264, & \widehat{b}_3(0) = 0.05468 \\ \widehat{b}_4(0) = 0.05683, & \widehat{b}_5(0) = -0.04639 \\ \widehat{b}_6(0) = -0.04794, & \widehat{b}_7(0) = -0.04958 \\ \widehat{b}_8(0) = -0.05132, & \widehat{b}_9(0) = -0.05316 \end{cases} \tag{44}$$

$$P(0) = 1000I, \qquad \lambda = 0.99, \tag{45}$$
$$A_r(q^{-1}) = 1 - 0.8q^{-1} \tag{46}$$

The initial values of the output, the state observer and the filter are set as zero.

Figure 1 illustrates the output and the desired output. Figure 2 shows the input trajectory. The estimated parameters of the polynomials $A_1(q^{-1})$ and $B_1(q^{-1})$ are shown in Figs.3-6.

The output approaches the desired output.

6. CONCLUSIONS

This paper gives a ripple-suppressed multirate adaptive control algorithm. It is applicable to unstable systems. The proposed scheme is based on the equation error method in parameter identification. Further, the control input is constructed by the measured outputs, the estimated outputs and past inputs. In addition, the estimated outputs are obtained through a state observer and a filter. A numerical example is shown to indicate the effectiveness of the proposed method.

REFERENCES

Albertos, P., J. Salt and J. Tormero (1996). Dual-rate adaptive control, *Automatica*, **32**-7, 1027-1030.

Al-Rahmani, H.M. and G. F. Franklin (1992). Multirate control: A new approach, *Automatica*, **28**-1, 35-44.

Araki, M. and K. Yamamoto (1986). Multivariable multirate sampled-data systems, state-space description, transfer characteristics and nyquist criterion, *IEEE Trans. on Automat. Contr.*, **AC-31**-2, 145-154.

Feliu,V., J. A. Cerrada and C. Cerrada (1990). A method to design multirate controllers for plants sampled at a low rate, *IEEE Trans. on Automat. Contr.*, **AC-35**-1, 57-60.

Heymann, M. (1988). Persisitency of excitation results for structured nonminimal models, *IEEE Trans. on Automat. Contr.*, **AC-33**-1, 112-116.

Ishitobi, M. (1997). Multirate adaptive control of a servo motor system, *Proc. of 11th IFAC Symposium on System Identification*, 293-298.

Kawashima, H. (1993). Multirate sampling adaptive control, Master Thesis, Dept. of Mechanical Eng., Kumamoto Univ. (in Japanese).

Lu, W. and D. G. Fisher (1989). RLS parameter convergence with overparameterized models, *Syst. & Contr. Lett.*, **12**-2, 133-138.

Lu, W., D. G. Fisher and S. L. Shah (1990). Multirate constrained adaptive control, *Int. J. Control*, **51**-6, 1439-1456.

Lu, W. and D. G. Fisher (1992). Multirate adaptive inferential estimation, *IEE Proc., Pt. D*, **139**-2, 181-189.

Scattolini, R. (1988). Self-tuning control of systems with infrequent and delayed output sampling, *IEE Proc., Pt. D*, **135**-4, 213-221.

Tangirala, A. K., D. Li, R. S. Patwardhan, S. L. Shah and T. Chen (2001). Ripple-free conditions for lifted multirate control systems, *Automatica*, **37**-10, 1637-1645.

Zhang, Q. and M. Tomizuka (1988). Multirate sampling adaptive control and its application to thermal mixing systems, *Int. J. Control*, **47**-3, 735-744.

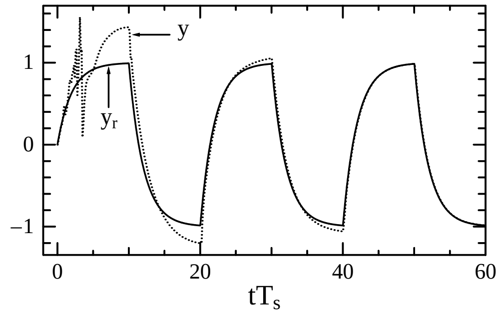

Fig. 1. The output comparison trajectories.

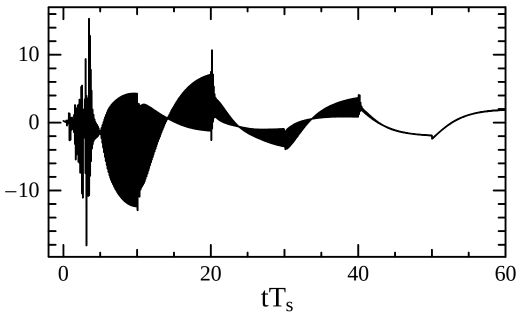

Fig. 2. The input trajectories.

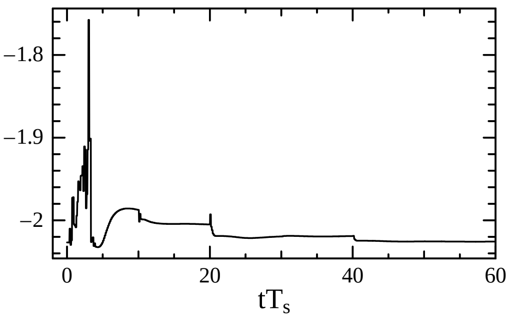

Fig. 3. The plant parameter trajectory, $\widehat{a}_{11}$

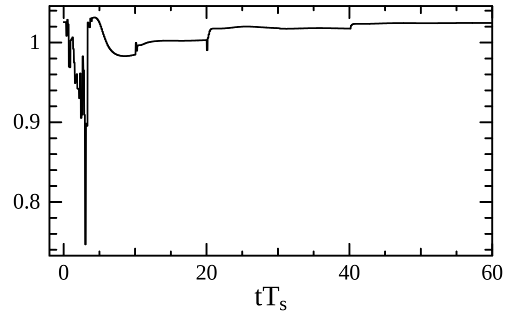

Fig. 4. The plant parameter trajectory, $\widehat{a}_{12}$

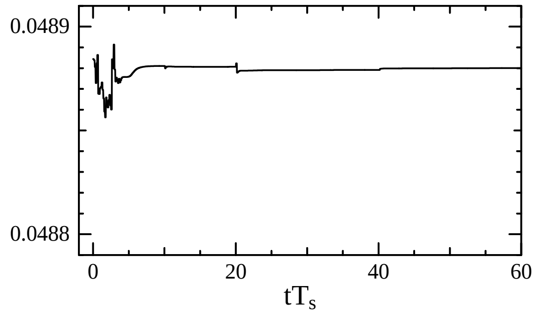

Fig. 5. The plant parameter trajectory, $\widehat{b}_{10}$

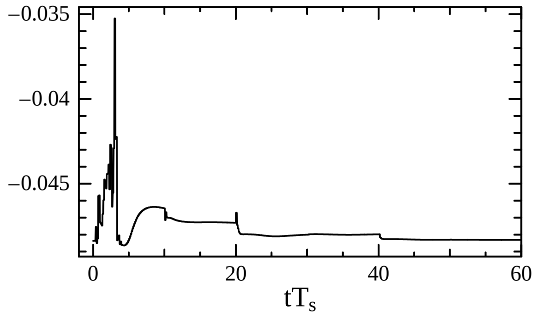

Fig. 6. The plant parameter trajectory, $\widehat{b}_{11}$