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OPTIMAL CONTROL OF AN ALTERNATING AEROBIC-ANOXIC WASTEWATER TREATMENT PLANT

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Abstract: This paper reports on a new approach in modelling and controlling an alternating aerobic-anoxic wastewater treatment plant. The plant is controlled using a model predictive approach adapted for the specific problem. The model embedded in the control structure is a linear time-variant approximation to ASM No.1. The predictive structure only considers the zero frequency component of controlled variables for the optimal duty cycle value calculation. Simulations of the controller behaviour are presented considering no plant-model mismatch.

Keywords: water pollution, model, optimisation, frequency, simulation.

1. INTRODUCTION

Activated sludge wastewater treatment plants are the most common type of plants for wastewater treatment. The process involves the biochemical reduction of carbonaceous material and other toxic compounds commonly found in municipal and industrial wastewater. Within the process, numerous biochemical reactions occur, most of them with highly non-linear dynamics.

Due to this, the models that represent the complete system dynamic behaviour are complex and full of uncertainties, though some reduced models have proved to predict the system behaviour at some extent (Jeppsson, 1995; Anderson *et al., 2000)*. In addition, besides the flow schemes, which depend on the plant configuration, the only directly controllable variable is the injected dissolved oxygen into the wastewater.

Several control approaches have been proposed, simulated and tested to achieve control goals. One of the most recent control structures under research is the alternating aerobic-anoxic approach. Under this control scheme, switching the aeration system on and off indirectly controls the effluent characteristics.

Some theoretical and experimental studies developed by Kim *et al*, (2000), and Puta *et al*, (1999) find the optimal switching times in the sense of minimising a cost function using a mathematical model of the plant. In particular, Puta *et al., (1999)* includes the energy consumption as part of the function, therefore it considers a monetary factor.

The control approach discussed in this paper, uses a linear time variant modification of the model developed by Anderson *et al,* (2000), and finds the optimal switching time under a zero frequency signal tracking. Simulation results are presented for the case of no plant-model mismatch in a sequential batch reactor (SBR) with constant flow, and influent composition.

The paper is organised as follows. Section 2 describes the wastewater system and the modelling involved. The controller structure is described in Section 3. Finally, simulation results are presented in Section 4, and conclusions are drawn in Section 5.

2. WASTEWATER SYSTEM DESCRIPTION AND MODELLING

Modelling of activated sludge wastewater treatment plants is a very complex task, due to its severe non-linear behaviour. Several models to represent activated sludge wastewater plants have been proposed. A good overview of different models can be found in Chotkowski *et al*, (2001) and Brdys and Zhang (1999).

Of all the different models, one in particular has been accepted within the scientific community as a standard for scientific research. This model, the Activated Sludge Model No.1 (Henze, *et al.*, 1987), collects most of the biochemical reaction dynamics involved in the process, and therefore is considered a benchmark for the development of application specific models for wastewater treatment plants.

The ASM No.1 is a very complex mathematical structure, which consists of 13 non-linear differential equations with 19 parameters (in its original version), most of them with a high degree of uncertainty. Because of this reason the applicability of the ASM No.1 is restricted to benchmarking for simpler models and research.

In an activated sludge process different biochemical reactions occur. These processes can be classified into two different categories: aerobic and anoxic reactions. Aerobic reactions make use of the oxygen dissolved in the water body and the two main reactions are the oxidation of the carbonaceous material and nitrification. Under anoxic conditions, denitrification reactions are predominant. These reactions make use of nitrate as the oxidation agent instead of oxygen to produce free nitrogen and other compounds.

Anderson *et al*, (2000) presented two linear approximations to the ASM No.1, one for each phase: aerobic and anoxic. These models are obtained by approximating the half saturation non-linear terms to linear terms. The model is also of reduced order, which is accomplished by not including: soluble inert organic matter (S_I) and particulate inert organic matter (X_I), which are decoupled from the system; dissolved oxygen (S_o), since it is assumed to be controlled; alkalinity (S_{alk}), since denitrification can partially recover some alkalinity consumed through nitrification; and growth of particulate products (X_P), which does not interact with the other variables. In addition, dissolved oxygen is considered not to be a limiting factor during the aerobic process and to be totally absent during the anoxic phase. The final system is composed by two state space representations, one for each phase, of eighth order, with the following form:

$$\begin{aligned} \dot{x}(t) &= A_e \cdot x(t) + D_e \cdot x_{inf} \\ \dot{x}(t) &= A_a \cdot x(t) + D_a \cdot x_{inf} \end{aligned} \quad (1)$$

In equation (1), A_e and D_e denote the system matrix representation for the aerobic phase and, A_a and D_a are the system matrix representations for the anoxic phase. The variable $x(t)$ is the state vector of the system whose components are specified in equation (2), and x_{inf} is the corresponding vector of influent characteristics (concentrations) into the system, for each state variable. The last term in equation (1) should be considered as a disturbance, since there is no possible control over the influent characteristics.

$$x(t) = \left(S_s \;\; X_s \;\; X_{B,H} \;\; X_{B,A} \;\; S_{NH} \;\; S_{NO} \;\; S_{ND} \;\; X_{ND}\right)^T \quad (2)$$

Since the control principle of this type of structure demands the switching between the two models at a given frequency and duty cycle, an appropriate model representation for the system behaviour over the entire time is required. Equation (3) shows the proposed model representation.

$$\begin{aligned} \dot{x}(t) = &\left(g(t,\delta)\cdot A_e + (1-g(t,\delta))\cdot A_a\right)\cdot x(t) + \\ &\left(g(t,\delta)\cdot D_e + (1-g(t,\delta))\cdot D_a\right)\cdot x_{inf} \end{aligned} \quad (3)$$

In equation (3), the switching function $g(t,\delta)$ represents a train of width modulated pulses of unit amplitude as shown in Fig 1. When the switching function is unity the aerobic phase is said to be ON, and when it is zero the aerobic phase is said to be OFF. The switching function depends on time and duty cycle 'δ', which is defined as the relation between the time the aerobic phase is ON (T_{ON}), and the switching period ($T_{switching}$), as presented in equation (4).

$$\delta = \frac{T_{ON}}{T_{switching}} \quad (4)$$

For mathematical simulation, $g(t,\delta)$ has been expanded into a finite Fourier series, where the Gibbs phenomenon has been eliminated by use of a saturation function in the convergence point of discontinuities. Equation (5) denotes the truncated Fourier Series expansion of $g(t,\delta)$.

$$\begin{aligned} g(t,\delta) = \delta + \sum_{n=1}^{N} \frac{1}{n\pi} &\left[\sin(2n\pi\delta)\cdot\cos(2n\pi f_o) + \right. \\ &\left.(1-\cos(2n\pi\delta))\cdot\sin(2n\pi f_o)\right] \end{aligned} \quad (5)$$

Equation (3) can be rearranged to give equation (6).

$$\begin{aligned} \dot{x}(t) = &\left(A_1\cdot g(t,\delta) + A_2\right)\cdot x(t) + \\ &\left(D_1\cdot g(t,\delta) + D_2\right)\cdot x_{inf} \end{aligned} \quad (6)$$

where $A_1=A_e-A_a$, $A_2=A_a$, $D_1=D_e-D_a$, and $D_2=D_a$.

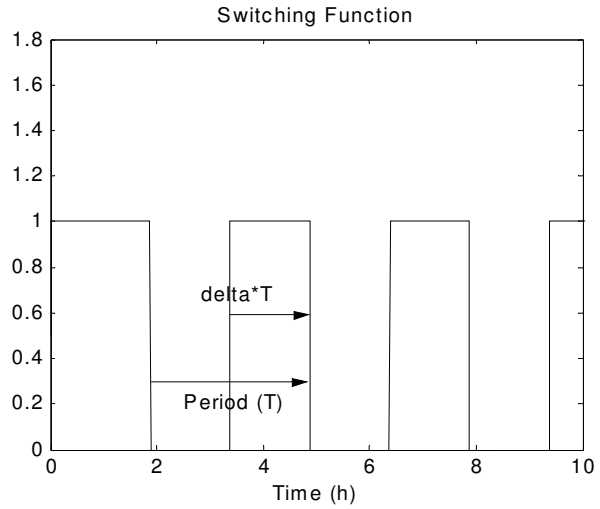

Fig. 1. Switching Function.

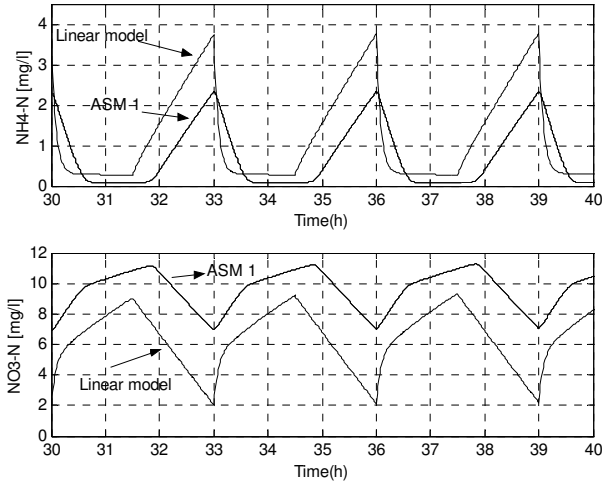

Fig. 2. Linear Time-Variant model and ASM No.1.

By substituting equation (5) into (6) and considering that the expressions contained within the brackets are time dependent, equation (6) can be rewritten as a time variant system presented in (7). This equation, is an approximation, since $g(t,\delta)$ is a truncated series.

$$\begin{aligned} \dot{x}(t) &\approx A(t,\delta)\cdot x(t) + D(t,\delta)\cdot x_{inf} \\ y(t) &= C\cdot x(t) \end{aligned} \qquad (7)$$

2.1 Model Verification.

In order to verify the validity of this model, simulations have been performed using the parameter data presented in the original paper of Anderson *et al*, (2000), and comparing it with a simulation of the full ASM No.1 also using Anderson *et al's. (2000)*, parameter data. The simulations where performed using MATLAB for a switching period of 3 hours and a duty cycle of 50% (δ=0.5). Results of these two simulations are presented in Fig.2. In the linear model, the number of terms used for equation (5) was of N=5.

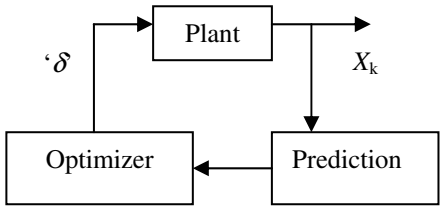

Fig. 3. Controller Structure.

The simulations presented in Fig. 2 are identical to the ones presented in Anderson *et al.*, (2000). It can be clearly observed that there is a model mismatch, but the trends are the same. The model mismatch can be corrected by using an off-line or on-line parameter estimation as in Jeppsson, (1996).

3. CONTROLLER ARCHITECTURE

The controller architecture follows the line of traditional GPC, with some modifications due to the cyclic nature of the embedded model. A block diagram of the controller is presented in Fig. 3.

3.1 Prediction.

The alternating aerobic-anoxic (AAA) wastewater time variant system, described by equation (7), can be sampled at a specific frequency, and therefore be transformed into a discrete system as presented in equation (8).

$$\begin{aligned} x[k+1] &= A[k,\delta]\cdot x[k] + D[k,\delta]\cdot x_{inf} \\ y[k] &= C\cdot x[k] \end{aligned} \qquad (8)$$

Using a recursive approach to calculate the predicted state of the system based on measurements of the state at sampling time k, it can be easily shown that the predictions at any future sampling instant $k+n$ and up to the prediction horizon Hp can be calculated using equation (9).

It should be clear, that full state measurement is assumed. This is an assumption which is difficult to overcome. Possible solutions to this problem are the use of soft-sensors using estimation algorithms. Some research work has been carried out in this field, which usually concludes that the main limitation is the identifiability of the process (model). Some examples of proposed observers can be found in Katebi (2001), Jeppsson (1995), Arnold and Dietze (2001), and Lindberg (1997).

$$
\begin{aligned}
\hat{x}[k+1\,|\,k] &= A[k,\delta]\cdot x[k] + D[k,\delta]\cdot x_{inf} \\
\hat{x}[k+2\,|\,k] &= A[k+1,\delta]\cdot \hat{x}[k+1\,|\,k] + D[k+1,\delta]\cdot x_{inf} \\
&= A[k+1,\delta]\cdot A[k,\delta]\cdot x[k] + \\
&\quad \left(A[k+1,\delta]\cdot D[k] + D[k+1]\right)\cdot x_{inf} \\
&\vdots \\
\hat{x}[k+n\,|\,k] &= \prod_{i=1}^{n} A[k+n-i]\cdot x[k] + \\
&\quad \sum_{j=1}^{n}\left(\prod_{i=1}^{n-j} A[k+n-i]\cdot D[k-1+j]\right)\cdot x_{inf} \\
&\vdots \\
\hat{x}[k+Hp\,|\,k] &= \prod_{i=1}^{Hp} A[k+Hp-i]\cdot x[k] + \\
&\quad \sum_{j=1}^{Hp}\left(\prod_{i=1}^{Hp-j} A[k+Hp-i]\cdot D[k-1+j]\right)\cdot x_{inf}
\end{aligned}
\tag{9}
$$

Equation (9) can be arranged into a matrix representation, and output predictions can be calculated as presented in equation (10).

$$
\begin{pmatrix} \hat{x}[k+1\,|\,k] \\ \vdots \\ \hat{x}[k+Hp\,|\,k] \end{pmatrix} = \begin{pmatrix} A[k,\delta] \\ \vdots \\ \prod_{i=1}^{Hp} A[k+Hp-i] \end{pmatrix}\cdot \hat{x}[k] + \begin{pmatrix} D[k,\delta] \\ \vdots \\ \sum_{j=1}^{Hp}\left(\prod_{i=1}^{Hp-j} A[k+Hp-i]\cdot D[k-1+j]\right) \end{pmatrix}\cdot x_{inf} \tag{10}
$$

$$
\begin{pmatrix} \hat{y}[k+1\,|\,k] \\ \vdots \\ \hat{y}[k+Hp\,|\,k] \end{pmatrix} = \begin{pmatrix} C & 0 & \cdots & 0 \\ 0 & C & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & C \end{pmatrix}\cdot \begin{pmatrix} \hat{x}[k+1\,|\,k] \\ \vdots \\ \hat{x}[k+Hp\,|\,k] \end{pmatrix}
$$

This, in an abbreviated notation, can be written as in equation (11).

$$
\begin{aligned}
\hat{X} &= A[k,n]\cdot x[k] + D[k,n]\cdot x_{inf} \\
\hat{Y} &= C\cdot \hat{X}
\end{aligned}
\tag{11}
$$

where n varies between 1 and Hp, and denotes the row element numbering.

3.2 Cost Function and Optimisation.

The selection of an appropriate cost function depends on many factors. However, the approach developed in this paper makes use of the quadratic error of the average value (zero frequency component) of the predictions over a complete aerobic-anoxic cycle and the set point or a reference trajectory to approach the set point. Only the unconstrained case is analysed.

In order to calculate the output predictions of the AAA system the algorithm described by equation (10) is used. The average value of each state variable arranged in the vector representation presented in equation (2) is calculated by the average of the predictions over the horizon *Hp*. Equation (12) shows how the average (zero frequency component) of the discrete vector signal is calculated.

$$
\bar{y} = \frac{1}{Hp}\sum_{i=1}^{Hp} \hat{y}[k+i\,|\,k] \tag{12}
$$

where $\hat{y}[k+i]$ and $\bar{y} \in \Re^8$. Therefore, using equation (12), the cost function is defined as follows.

$$
V = (\bar{y}-\gamma)^T \cdot Q \cdot (\bar{y}-\gamma) \tag{13}
$$

where γ could be a reference trajectory to approach the set point, which is updated on each prediction cycle, or the set point; and Q is a weights matrix of adequate dimensions which can be time dependent and used to include penalizing functions. For this case Q has been considered to be the identity matrix I.

An additional term can be added to equation (13), to penalize steep changes in the control input, and represent a minimum energy consumption approach. The final cost function is as presented in equation (14). In this equation δ is the parameter to be optimised at prediction cycle k+1, while δ_k is the optimal δ value found at prediction cycle k.

$$
V = (\bar{y}-\gamma)^t \cdot (\bar{y}-\gamma) + (\delta-\delta_k)^2 \tag{14}
$$

Finally, the search for the optimum value of the duty cycle δ that minimizes equation (14) can be done using several numeric methods, since it is difficult to find a closed analytical form representation for the gradient.

4. SIMULATION RESULTS

Two types of simulations have been performed. The first one considers a reference trajectory to approach the set point, and the second uses the set point directly. For the reference trajectory case, a time constant of 12 hours for the reference trajectory, with a sampling time of 3 hours has been chosen. The prediction horizon is of one cycle (3 hours) for both cases. The reference trajectory at cycle n can then be calculated as (Maciejowski, 2001):

$$
\gamma_{n+1} = s - e^{-(n+1)\cdot\frac{T_s}{Tref}}\cdot e_k \tag{16}
$$

where e_k is the error between the plant output and the set point s at prediction instant k.

The set point for S_{NH} and S_{NO} are the ones presented in Table 1. The system initial conditions are the influent characteristics, presented in Anderson *et al*., (2000).

Table 1 System Set Point

S_{NH}	1.26 [mg/l]
S_{NO}	5.31 [mg/l]

Simulation results, for the case in which a reference trajectory is used are presented in Fig. 4, and Fig. 5 shows simulations for the case in which the set point is used directly.

It is interesting to observe that in the case in which a reference trajectory is used, the optimal control input δ begins with a lower value than in the case of the use of the set point directly. The cost function also seems to converge to the minimal value faster when using the reference trajectory. The simulations also show that once the system is near the setpoint, the control input begins to oscillate around the optimal value. A possible explanation for this behaviour is that the control system (optimiser) is not able to keep both controlled variables at the same time in the exact setpoints, but in a near neighbourhood.

An important limitation of this method is that the maximum output magnitude cannot be controlled directly. This means that at certain periods of time the output concentrations are higher than the permissible. Some possible formulations to solve this problem can be the use of constraints to limit the effluent concentrations and the calculation of an adequate setpoint.

There are several ways in which constraints can be included, but probably the use of the weight matrix *Q*, which in this case has been assumed to be the identity matrix (*I*), is the easiest. This matrix could include time-dependent penalization functions, which include the constraints in the quadratic cost function.

5. CONCLUSIONS

This paper presents a different approach to modelling an alternating wastewater treatment plant based on Anderson *et al.'s, (2000)* model, in which an approximate reduced-order linear model, has been derived. The model is considered to be an approximation due to the use of a truncated Fourier series. Furthermore, a predictive control algorithm, similar to GPC, is deducted. The algorithm minimizes a quadratic cost function, which includes the mean value of the predictions of the controlled variables and the change in the control input (i.e. duty cycle – 'δ).
Simulation results are presented for the unconstrained case, with no plant-model mismatch, and under controlled conditions (i.e. constant flow and influent composition). The simulations show that the algorithm converges to the proximity of the predetermined setpoint. Also, they show that the maximum values cannot be directly controlled, which means that in this case (i.e. unconstrained case) permissible limit violations are almost inevitable. This suggests that the inclusions of constraints for the controlled effluent variables should be investigated.

It was assumed in this work that full state information was available. However, this is not possible in a real wastewater treatment plant and the use of state observers-estimators is necessary. Additionally further simulations and studies should be carried out for the case in which there is a plant-model mismatch.

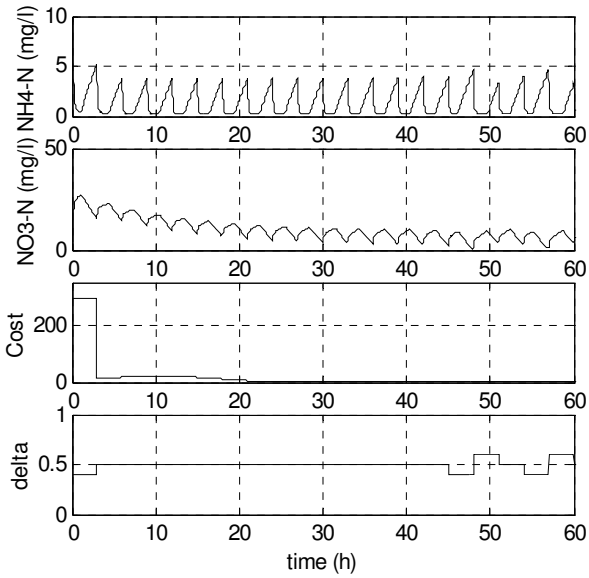

Fig. 4. System response with reference trajectory.

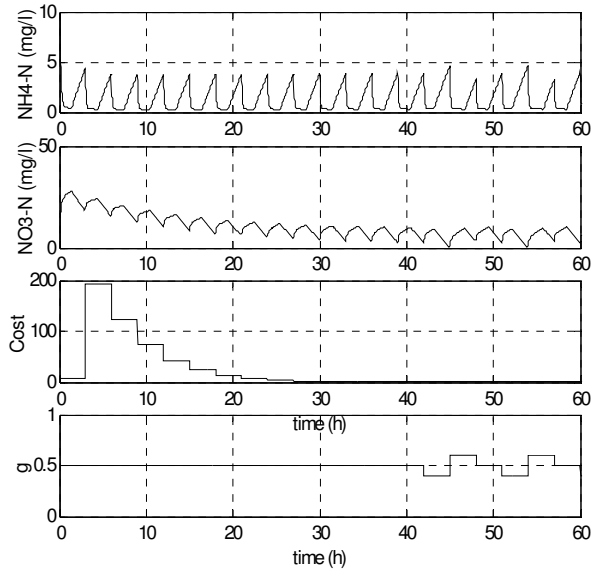

Fig. 5. System response with no reference trajectory.

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