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ℓ_1 SUBOPTIMAL ROBUST CONTROLLERS FOR MIMO PLANTS UNDER COPRIME FACTOR PERTURBATIONS

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Abstract: This paper addresses the problem of synthesis of suboptimal robust controllers in the ℓ_1 setting. For MIMO plants under coprime factor perturbations and an exogenous disturbance bounded in ℓ_∞, the worst case ℓ_∞ norm of the system output is shown to be a linear fractional function of the induced norms of system's transfer functions. Then computing the suboptimal controller is reduced to approximate solution of finite family of standard ℓ_1 optimization problems. The model validation and identification problems are briefly discussed for systems with weighted perturbations.

Keywords: Discrete-time systems, ℓ_1 control, Optimal control, Robust control

1. INTRODUCTION

The theory of robust control in the ℓ_1 setting deals with the ℓ_∞ signal spaces and systems under norm bounded perturbations and bounded exogenous disturbances. Basic results on stability robustness and performance robustness have been obtained in Dahleh and Ohta (1998), Khammash and Pearson (1993, 1991). By problem of synthesis of ℓ_1 optimal robust controller is meant an optimal problem where the cost function is the worst-possible ℓ_∞ norm of the output, over the class of allowable disturbances and perturbations. Since the optimal problem is a complicated nonconvex problem, an auxiliary problem of achieving a prescribed performance level is usually considered. The latter was reduced in Khammash and Pearson (1993) to minimizing the spectral radius of a certain matrix composed of the ℓ_1 norms of system's impulse responses. A general solution of this problem was given in Khammash *et al.* (1998). The solution was based on linear relaxation and searching over a mesh in a set of scaling diagonal matrices. In order to get an approximate solution of the auxiliary problem with an ε tolerance, the number of linear programming problems to be solved is in the order of $\left(\frac{C}{\varepsilon}\right)^{n+1}$ where n equals to the number of independent perturbations and the constant C depends on *a priori* upper and lower bounds on some variables.

In the present paper, several classes of MIMO systems under coprime factor perturbations are considered. The robust performance measure for these classes is shown to be a linear fractional function of the norms of the system's transfer functions so that the auxiliary problem of achieving a prescribed performance level is reducible to a standard mixed sensitivity problem of ℓ_1 optimization. Then solving the problem of minimization of the spectral radius becomes redundant and the algorithms proposed in Sokolov (2000, 2001c) for synthesis of suboptimal robust controllers for SISO systems can be applied to MIMO systems as well. In order to get a suboptimal robust controller with an ε tolerance, the number of linear programming problems to be solved is in the order of $-\log_2 \varepsilon$ (Sokolov, 2000,2001c).

Implementation of any results of robust control theories is impossible without knowledge of the nominal system and the weights (norms) of perturbations. This gives rise to problems of model validation and more complicated problems of identification (Poolla, *et al.*, 1994; Smith and Doyle, 1992). We show that the model validation problem for the considered systems is easily solvable on-line in the case of signals weighted diagonally at the inputs of perturbations while less general scalar weighting perturbations opens the door to solving identification problem. In particular, the methods of synthesis of adaptive ℓ_1 suboptimal robust controllers proposed in Sokolov (2001a, b) for SISO systems can be extended to MIMO systems.

Notation $|x|_\infty := \max_i |x_i|$ for a vector $x = (x_1, \ldots, x_n)^* \in \mathbb{R}^n$.
ℓ_∞^n – the space of real vector valued sequences $x = (x(0), x(1), \ldots)$, $x(k) \in \mathbb{R}^n$, with the norm $\|x\|_\infty := \sup_k |x(k)|_\infty$.
ℓ_1 – the space of absolutely summable sequences with the norm $\|x\|_1 := \sum_k |x(k)|$ for $x \in \ell_1$.
ℓ^n – the space of arbitrary vector valued sequences of dimension n, $\ell := \ell^1$.
A map $F : \ell^p \to \ell^q$ is said to be ℓ_∞-stable if it is causal, takes ℓ_∞^p into ℓ_∞^q and

$$\|F\| := \sup_{\substack{x \in \ell_\infty^p \\ x \neq 0}} \frac{\|F(x)\|_\infty}{\|x\|_\infty} < +\infty \, .$$

A linear causal time-invariant system $H : \ell^p \to \ell^q$ is defined by the convolution

$$Hx(t) := \sum_{k=0}^{t} H(k)x(t-k) \, , \quad H(k) := (H_{ij}(k)),$$

where the same notation H is used for the $q \times p$ matrix of impulse responses $H_{ij} \in \ell$. If $H_{ij} \in \ell_1$ for all i, j then the system H is ℓ_∞-stable and the induced norm of H is

$$\|H\| = \max_i \sum_j \|H_{ij}\|_1 \, .$$

The matrix function $H(\lambda) := \sum_{k=0}^\infty H(k)\lambda^k$ of the complex variable λ is called the transfer matrix of the system H and $\|H(\lambda)\| := \|H\|$.

2. ℓ_1 OPTIMAL ROBUST CONTROL PROBLEM

Consider the system in Fig. 1 where $H = H(G, K)$ is the matrix of impulse responses of a linear time-invariant causal nominal system comprised of a nominal plant G and a controller K, $z \in \ell^{n_z}$ – the regulated output, $w \in \ell_\infty^{n_w}$ – the exogenous disturbance, Δ – the linear time-variant or nonlinear perturbation,

$$\Delta \in \underline{\Delta} := \{\Delta = diag(\Delta_1, \cdots, \Delta_n) \mid$$
$$\Delta_i : \ell_\infty^{q_i} \to \ell_\infty^{p_i} \text{ is strictly causal, } \|\Delta_i\| \le 1\} \, .$$

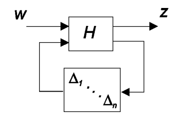

Fig. 1. System with structured uncertainty

The system in Fig. 1 is said to be robustly stable against the class of perturbations $\underline{\Delta}$, if for any $\Delta \in \underline{\Delta}$ the mapping of w into z is ℓ_∞-stable (Khammash and Pearson, 1991).

The problem of synthesis of ℓ_1 optimal robust controller is stated as follows:

$$J(H) := \sup_{\Delta \in \underline{\Delta}} \sup_{\|w\|_\infty \le 1} \|z\|_\infty \longrightarrow \inf_K \, . \quad (1)$$

Under zero perturbation ($\Delta = 0$) problem (1) is the standard problem of synthesis of ℓ_1 optimal controller (Barabanov, 1996; Dahleh and Diaz-Bobillo, 1995).

Represent the matrix H of impulse responses in the block form

$$H = H(G, K) = \begin{bmatrix} H_{00} & \cdots & H_{0n} \\ \vdots & & \vdots \\ H_{n0} & \cdots & H_{nn} \end{bmatrix}$$

where the dimensions of the blocks are associated with the dimensions of $n+1$ input and $n+1$ output signals. In particular, H_{00} is a $n_z \times n_w$ matrix and H_{ij} is a $q_i \times p_j$ matrix, $i, j \ge 1$.

Let the nominal plant G be stabilizable and K be some stabilizing controller so that the nominal system H is ℓ_∞-stable. For simplicity of presentation assume for a while that all signals in the system are scalar and define

$$\hat{H} := \begin{bmatrix} \|H_{00}\|_1 & \cdots & \|H_{0n}\|_1 \\ \vdots & & \vdots \\ \|H_{n0}\|_1 & \cdots & \|H_{nn}\|_1 \end{bmatrix} .$$

Represent the matrix $\hat{H}$ in the block form

$$\hat{H} = \begin{bmatrix} \hat{H}_{00} & \hat{H}_{01} \\ \hat{H}_{10} & \hat{H}_{11} \end{bmatrix} , \quad \hat{H}_{00} - 1 \times 1 \, , \quad \hat{H}_{11} - n \times n \, .$$

It was shown in Khammash and Pearson (1991) that the system in Fig. 1 is robustly stable against the class $\underline{\Delta}$ if and only if $\rho(\hat{H}_{11}) < 1$ where $\rho(\cdot)$ denotes the spectral radius of a matrix. An explicit formula for the robust performance measure $J(H)$ was obtained in Khammash (1997):

$$J(H) = \mathcal{F}(\hat{H})$$

where

$$\mathcal{F}(\hat{H}) := \hat{H}_{00} + \hat{H}_{01}(I - \hat{H}_{11})^{-1}\hat{H}_{10} \, . \quad (2)$$

One can see that (1) is a complicated nonconvex problem of mathematical programming. A standard way of its approximate solution is in solving an auxiliary problem

$$J(H) \leq \gamma\,, \quad \gamma > 0\,, \tag{3}$$

with subsequent searching for a near to optimal value of the parameter γ. The auxiliary problem can be reduced to the problem $J(H) \leq 1$ by scaling the signal z. It was shown in Khammash and Pearson (1991) that

$$(\ \forall \Delta \in \underline{\mathbf{\Delta}} \quad \sup_{\|w\|_\infty \leq 1} \quad \|z\|_\infty < 1\ )$$

if and only if

$$\rho(\hat{H}) < 1\,.$$

Based on this result, the *robustness synthesis problem* was stated in Khammash and Pearson (1993) as follows

$$\inf_K \ \rho(\hat{H})\,. \tag{4}$$

Let $H = H(Q) = T_1 - T_2 Q T_3$ be the standard parameterization of stable systems using the Youla parameter Q (Dahlen and Diaz-Bobillo, 1995). It was shown in Khammash and Pearson (1993) that problem (4) is equivalent to

$$\inf_{D \in \mathbf{D}} \ \inf_Q \ \|D^{-1} H(Q) D\|\,, \tag{5}$$

$$\mathbf{D} := \{\ D \mid D = diag(d_0, \cdots, d_{n+1}),\ d_i > 0\ \}\,.$$

An approximate solution of problem (5) proposed in Khammash *et al.* (1998) is based on linear relaxation, gridding, and introducing *a priori* lower and upper bounds for variables, the scaling coefficients d_i, and the cost function. To obtain an approximate solution of (5) with an ε accuracy under additional restriction on the number of variables, the number of linear programming problems to be solved is in the order of

$$\left(\frac{C}{\varepsilon}\right)^{n+1}$$

where the constant C depends on the above-mentioned lower and upper bounds.

In the next section, it will be shown that the auxiliary problem (3) for MIMO plants under coprime factor perturbations is reducible directly to known problems of ℓ_1 optimization so that solving computer consuming problem (5) becomes redundant.

3. MIMO PLANT UNDER COPRIME FACTOR PERTURBATIONS

Consider the closed loop control system

$$\tilde{M}(q^{-1}) z = \tilde{N}(q^{-1}) u + v\,, \quad u = Kz \tag{6}$$

where $\tilde{M}(q^{-1})$ and $\tilde{N}(q^{-1})$ are $n_z \times n_z$ and $n_z \times n_u$ left coprime polynomial matrices in the shift operator q^{-1} ($q^{-1}x(t) := x(t-1)$) and K is a rational matrix of controller. The transfer matrix of the plant G is $G(\lambda) = \tilde{M}^{-1}(\lambda)\tilde{N}(\lambda)$ ($\det \tilde{M}(0) \neq 0$). The total disturbance v in the plant G is of the form

$$v := \Delta_1 z + \Delta_2 u + w \tag{7}$$

where Δ_1 and Δ_2 are the perturbations and w is the exogenous disturbance. In view of the equality

$$(\tilde{M} - \Delta_1) z = (\tilde{N} + \Delta_2) u + w \tag{8}$$

the perturbations Δ_1 and Δ_2 can be considered as independent perturbations in the output and control, respectively.

The initial conditions in system (6) are assumed to be zero. The formulae for $J(H)$ derived below hold for nonzero initial conditions as well if the cost function $J(H)$ is replaced by a steady-state robust performance measure as follows. In (1), the seminorm

$$\limsup_{t \to +\infty} |z(t)|_\infty$$

substitutes for the norm $\|z\|_\infty$ and the class of perturbations $\underline{\mathbf{\Delta}}$ is additionally restricted by fading or finite memory perturbations (Sokolov, 2001a).

Introduce the Youla parameterization of all stable transfer functions G_{zv} and G_{uv} from the total disturbance v to the output z and control u, respectively. Let $G = NM^{-1}$ be the right coprime factorization of G and polynomial matrices X and Y be solutions to the Bezout equation

$$\tilde{M}X - \tilde{N}Y = I\,.$$

Then all stable transfer functions G_{zv} and G_{uv} associated with stabilizing rational controllers K are of the form (Barabanov, 1995)

$$G_{zv} = X - NQ\,, \quad G_{uv} = Y - MQ$$

where Q is the Youla parameter (that is, arbitrary stable rational $n_u \times n_z$ matrix).

A representation of the closed loop system (6) and (7) associated with the Fig. 1 is of the form

$$\begin{bmatrix} z \\ z \\ u \end{bmatrix} = \begin{bmatrix} G_{zv} & G_{zv} & G_{zv} \\ G_{zv} & G_{zv} & G_{zv} \\ G_{uv} & G_{uv} & G_{uv} \end{bmatrix} \begin{bmatrix} w \\ \xi_1 \\ \xi_2 \end{bmatrix}\,, \tag{9}$$

$$\begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} = \begin{bmatrix} \Delta_1 & 0 \\ 0 & \Delta_2 \end{bmatrix} \begin{bmatrix} z \\ u \end{bmatrix}$$

so that the system has 2-block uncertainty: $n = 2$. Owing to a special structure of the system matrix H, the robust performance measure $J(H)$ becomes a linear fractional function of the norms of the transfer functions G_{zv} and G_{uv}.

Theorem 1. The system (6) and (7) is robustly stable against the class $\underline{\Delta}$ if and only if

$$\|G_{zv}\| + \|G_{uv}\| < 1 . \tag{10}$$

If the system is robustly stable, then

$$J(H) = \frac{\|G_{zv}\|}{1 - \|G_{zv}\| - \|G_{uv}\|} . \tag{11}$$

The proof is omitted.

Remark. The robust stability condition (10) was first obtained in Dahleh (1992) where an optimal problem for robust controller synthesis was stated as minimizing the left hand side of (10). This is equivalent to minimizing the denominator of the robust performance function (11).

It follows from Theorem 1 that for any $\gamma > 0$ the auxiliary problem $J(H) \leq \gamma$ is equivalent to the problem

$$\left(\frac{1}{\gamma} + 1\right) \|G_{zv}\| + \|G_{uv}\| \leq 1 . \tag{12}$$

Problem (12) is a standard mixed sensitivity problem of ℓ_1 optimization and can be solved approximately by known methods (Dahleh and Ohta, 1995; Khammash 2000). Then the synthesis of ℓ_1 suboptimal robust controller can be realized similarly to Sokolov (2000).

The uncertainty in the plant (8) is structured. In the literature on robust adaptive control, systems with unstructured uncertainty ($n = 1$) have received wider acceptance. In this case the total disturbance in the plant is of the form

$$v = \Delta \begin{bmatrix} z \\ u \end{bmatrix} + w \tag{13}$$

and the following theorem holds.

Theorem 2. The system (6) and (13) is robustly stable against the class $\underline{\Delta}$ if and only if

$$\max\{\|G_{zv}\| , \|G_{uv}\|\} < 1 .$$

If the system is robustly stable, then

$$J(H) = \frac{\|G_{zv}\|}{1 - \max\{\|G_{zv}\| , \|G_{uv}\|\}} .$$

The proof is omitted.

It follows from Theorem 2 that for any $\gamma > 0$ the auxiliary problem $J(H) \leq \gamma$ is equivalent to the problem

$$\frac{1}{\gamma}\|G_{zv}\| + \max\{\|G_{zv}\|, \|G_{uv}\|\} \leq 1 . \tag{14}$$

Although problem (14) is not a standard problem of ℓ_1 optimization, its approximate solution can be obtained by a simple extension of the scaled-Q method proposed in Khammash (2000) and the synthesis of ℓ_1 suboptimal robust controller can be realized similarly to Sokolov (2001c).

For simplicity of presentation we considered so far perturbations with equal weights, that is, with equal maximal admissible norms. In the next section we consider systems with weighted perturbations keeping the linear fractional form of the robust performance function.

4. WEIGHTED PERTURBATIONS

4.1 *Systems allowing model validation*

Consider two classes of systems (6) with the following total disturbance v. In the case of structured uncertainty,

$$v = \Delta_1 D^z z + \Delta_2 D^u u + \delta_w w \tag{15}$$

where

$$D^z = diag\{d_1^z, \cdots, d_{n_z}^z\},$$
$$D^u = diag\{d_1^u, \cdots, d_{n_u}^u\},$$

and $\delta_w \in \mathbb{R}$. Without loss of generality one can assume d_i^z, d_j^u, and δ_w to be nonnegative.

In the case of unstructured uncertainty,

$$v = \Delta \begin{bmatrix} D^z z \\ D^u u \end{bmatrix} + \delta_w w . \tag{16}$$

Theorem 3.

$$J(H) = \frac{\delta_w \|G_{zv}\|}{1 - \|D^z G_{zv}\| - \|D^u G_{uv}\|}$$

for the system (6) and (15), and

$$J(H) = \frac{\delta_w \|G_{zv}\|}{1 - \max\{\|D^z G_{zv}\| , \|D^u G_{uv}\|\}}$$

for the system (6) and (16). Any of the systems is robustly stable iff the denominator of $J(H)$ associated with the system is positive.

The proof is omitted.

Remark. Note that perturbations in (15) and (16) are weighted at the inputs of the operator Δ. Therefore the weighting matrices D^z and D^u enter into the system matrix H as left factors. Since left multiplying is associated with rows operations of the transfer matrices G_{zv} and G_{uv}, the system matrix H keeps its special structure with repeated columns owing to which the robust performance measure $J(H)$ becomes a linear fractional function of the norms of the transfer matrices G_{zv} and G_{uv}.

It follows from Theorem 3 that for any $\gamma > 0$ the auxiliary problem $J(H) \leq \gamma$ for the system (6) and (15) is equivalent to the problem

$$\frac{\delta_w}{\gamma}\|G_{zv}\| + \|D^z G_{zv}\| + \|D^u G_{uv}\| \leq 1 \, , \tag{17}$$

and for the system (6) and (16) – to the problem

$$\frac{\delta_w}{\gamma}\|G_{zv}\| + \max\{\|D^z G_{zv}\|, \|D^u G_{uv}\|\} \leq 1 \tag{18}$$

Then the synthesis of ℓ_1 suboptimal robust controllers can be realized similarly to Sokolov (2000, 2001c).

Now we proceed to the problem of model validation. Let $z_0^t = (z(0), \cdots, z(t))$ be the measured outputs of some physical system subjected to the control actions $u_0^t = (u(0), \cdots, u(t))$. A model $G = \tilde{M}^{-1}\tilde{N}$ and weights D^z, D^u, and δ_w are said to be *not invalidated* by the observed input-output data z_0^t, u_0^t if there exists $\Delta \in \underline{\Delta}$ and w, $\|w\|_\infty \leq 1$, such that equations (6) and (15) (or (16) in the case of the hypothesis of unstructured uncertainty) hold on the time interval $[0, t]$.

Theorem 4. A model $G = \tilde{M}^{-1}\tilde{N}$ and weights D^z, D^u, and δ_w are not invalidated by the observed input-output data z_0^t, u_0^t if and only if

$$|(\tilde{M}z)(\tau) - (\tilde{N}u)(\tau)|_\infty \leq \delta_w + \tag{19}$$

$$\max_{s<\tau} |D^z z(s)|_\infty + \max_{s<\tau} |D^u u(s)|_\infty$$

for all $\tau = 0, 1, \cdots, t$ in the case of the hypothesis of structured uncertainty (15) and

$$|(\tilde{M}z)(\tau) - (\tilde{N}u)(\tau)|_\infty \leq \delta_w + \tag{20}$$

$$\max_{s<\tau} \max\{|D^z z(s)|_\infty \, , |D^u u(s)|_\infty\}$$

for all $\tau = 0, 1, \cdots, t$ in the case of the hypothesis of unstructured uncertainty (16).

The proof is omitted.

Remark. Since the maxima in the right hand sides of (19) and (20) can be computed recursively, the model validation problem is solvable on-line. Note that the case of unstructured uncertainty was discussed in Poolla *et al.* (1994) and is covered by more general Theorem 5.9 from Poolla *et al.* (1994).

4.2 *Systems allowing identification*

Consider two classes of systems (6) with the following total disturbance v. In the case of structured uncertainty,

$$v = \delta_z \Delta_1 z + \delta_u \Delta_2 u + \delta_w w \, , \tag{21}$$

where $\delta_z \geq 0$ and $\delta_u \geq 0$ are the weights of the perturbations in the output and control, respectively. In the case of unstructured uncertainty,

$$v = \delta_o \Delta \begin{bmatrix} z \\ u \end{bmatrix} + \delta_w w \, , \tag{22}$$

where $\delta_o \geq 0$ is the weight of the unstructured perturbation.

Theorem 5.

$$J(H) = \frac{\delta_w \|G_{zv}\|}{1 - \delta_z \|G_{zv}\| - \delta_u \|G_{uv}\|}$$

for the system (6) and (21), and

$$J(H) = \frac{\delta_w \|G_{zv}\|}{1 - \delta_o \max\{\|G_{zv}\| \, , \|G_{uv}\|\}}$$

for the system (6) and (22).

Theorem 5 is a special case of Theorem 3 associated with $D^z = \delta_z I$ and $D^u = \delta_u I$ in the case of structured uncertainty and $D^z = \delta_o I$ and $D^u = \delta_o I$ in the case of unstructured uncertainty. Then the auxiliary problems $J(H) \leq \gamma$ for the system (6) and (21) and the system (6) and (22) are reducible to problems wich are similar to (17) and (18).

For the system (6) and (21), inequalities (19) take the form

$$|(\tilde{M}z)(\tau) - (\tilde{N}u)(\tau)|_\infty \leq \delta_w + \tag{23}$$

$$\delta_z \max_{s<\tau} |z(s)|_\infty + \delta_u \max_{s<\tau} |u(s)|_\infty \, ,$$

$\tau = 0, 1, 2, \cdots$. For the system (6) and (22) inequalities (20) take the form

$$|(\tilde{M}z)(\tau) - (\tilde{N}u)(\tau)|_\infty \leq \delta_w + \tag{24}$$

$$\delta_o \max_{s<\tau} \max\{|z(s)|_\infty \, , |u(s)|_\infty\} \, ,$$

$\tau = 0, 1, 2, \cdots$. A possibility of estimation of unknown $\tilde{M}$, $\tilde{N}$, $\delta_w, \delta_y, \delta_u, \delta_o$ follows now from the fact that the coefficients of polynomials $\tilde{M}$ and $\tilde{N}$ and all of the weights become coefficients at known functions of the observed input-output data. Then inequalities (23) and (24) can be rewritten as linear inequalities with respect to the unknown coefficients and estimation schemes proposed in Sokolov (2001b) for SISO systems can be applied for the considered MIMO systems.

5. CONCLUSION

The problem of synthesis of ℓ_1 suboptimal robust controllers for MIMO plants under coprime factor perturbations and bounded exogenous disturbance has been considered. The robust performance measure was taken as the worst-possible

ℓ_∞ norm of the system output. It was shown that the auxiliary problem of achieving a prescribed performance level is reduced to the standard mixed sensitivity problem of ℓ_1 optimization. Since implementation of any results on robust control is impossible without knowledge of the nominal system and the weights of perturbations, the model validation and identification problems were briefly discussed. It was shown that in the case of signals weighted diagonally at the inputs of perturbations the model validation problem is solvable on-line while scalar weighting perturbations opens door to solving identification problem.

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