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IDENTIFICATION OF A CLASS OF DYNAMICS UNDER PERSPECTIVE OBSERVATION

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Abstract: In this paper, we consider the problem of estimating the state of a class of perspective systems. The problem can be converted into the observation of a dynamical system with nonlinearities. A new discontinuous state observer, which is motivated by the sliding mode control method and adaptive techniques, is proposed for the obtained dynamical system. The attraction of the new method is that the algorithm is very simple and easy to be implemented, and it is robust to measurement noises. Further, minor *a priori* knowledge of the system is required in the new formulation. Simulation results show the superiority of the new method to the traditional ones.

Keywords: Nonlinear observer; Perspective observation; Identification of non-linear system; Discontinuous observer; Sliding mode method

1. INTRODUCTION

In the study of machine vision, observing the position of a moving object in the space by the image data with the aid of a CCD camera has been studied in the past years (Ayache, 1991; Ghosh, *et al.*, 2000; Jankovic, *et al*, 1995; Sridhar, *et al.*, 1993). A very typical method is the application of the extended Kalman filter (EKF) (Matthies, *et al.*, 1989; Sridhar *et al.*, 1993). Even though the convergence conditions of EKF have been recently established both as observer and filter (Boutayeb, *et al.*, 1997; Reif, *et al.*, 1999), it is well known that the EKF may fail in some real applications. A fatal shortcoming of EKF is that the algorithm is very complicated and can hardly be implemented practically with real image data. Further, the *a priori* knowledge about the noise is required. To overcome these difficulties, Jankovic *et al.* (1995) proposed a new recursive formulation called “identifier based observer” (IBO) based on a parameter identifier considered in model reference adaptive control (Narendra, *et al.*, 1989). The proposed IBO is guaranteed to converge in an arbitrarily large (but bounded) set of initial conditions, and since the convergence is exponential it is believed that the performance of IBO is reliable, robust and would quickly compute the position on real data (Jankovic, *et al.*, 1995). It should be noted that the *a priori* information about the upper bound of the state is required in the formulation of IBO, and the performance of IBO is similar to that of the EKF.

In this paper, we consider the state identification problem for the perspective system, where the motion parameters are assumed known. The formulated problem can be converted into the observation of a dynamical system with nonlinearities. To identify this class of nonlinear system, the method proposed by the authors (Chen, *et al.*, 2000) may be helpful in the formulation of the observer. In this paper, a new identification method is proposed to identify the space position of a moving object, where the upper bound of the state is adaptively updated online. The proposed method is a combination of the sliding mode method, adaptive method, and discontinuous observer techniques (Jankovic, *et al.*, 1995; Utkin, 1992). The discontinuous input in the sliding mode method is modified by a differentiable approach, and the “equivalent control” method is clarified theoretically. The attraction of the new method lies in that the algorithm is very simple, easy to be implemented practically, and robust to measurement noises. Further, the proposed method requires minor *a priori* knowledge about the system and can cope with a much more general class of perspective systems. Simulation results show that the new method is superior to the traditional ones.

The organization of the paper is as follows: Section 2 formulates the problem. In section 3, the robust observer is proposed. In section 4, examples are given to illustrate the new algorithm, and to compare the performance with the traditional methods.

2. PROBLEM STATEMENT

Consider the movement of the object described by

$$\frac{d}{dt}\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} = \begin{bmatrix} a_{11}(t) & a_{12}(t) & a_{13}(t) \\ a_{21}(t) & a_{22}(t) & a_{23}(t) \\ a_{31}(t) & a_{32}(t) & a_{33}(t) \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} b_1(t) \\ b_2(t) \\ b_3(t) \end{bmatrix} \quad (1)$$

where $x(t) = [x_1(t),\ x_2(t),\ x_3(t)]^T$ is the position in the space, $a_{ij}(t)(i,j=1,2,3)$ and $b_i(t)(i=1,2,3)$ are the motion parameters. It is supposed that the observed position in the image plane is defined by

$$y(t) = [y_1(t), \quad y_2(t)] = \left[\frac{x_1}{x_3}, \quad \frac{x_2}{x_3}\right]. \quad (2)$$

The perspective system is composed of equations (1) and (2).

In this paper, we make the following assumptions.

(A1). The parameters $a_{ij}(t)(i,j=1,2,3)$ and $b_i(t)(i=1,2,3)$ are known bounded functions of time t. $b_i(t)(i=1,2,3)$ are piecewise differentiable, and have bounded derivatives (at the undifferentiable points, we mean the left and right side derivatives).

(A2). $x_3(t)$ meets the condition $x_3(t) > \eta > 0$, where η is a constant.

(A3). $y(t)$ is bounded.

Remark 1: It is easy to see that assumptions (A2) and (A3) are reasonable by referring to the practical systems.

The purpose of this paper is to estimate the position $x(t)$ by using the observed information $y_1(t)$ and $y_2(t)$ in the image plane.

3. FOMULATION OF THE OBSERVER

Define

$$y_3(t) = \frac{1}{x_3(t)}. \quad (3)$$

Then, equation (1) can be rewritten as

$$\begin{cases} \dot{y}_1(t) = a_{13} + (a_{11} - a_{33})y_1 + a_{12}y_2 - a_{31}y_1^2 \\ \qquad - a_{32}y_1y_2 + (b_1 - b_3y_1)y_3 \\ \dot{y}_2(t) = a_{23} + a_{21}y_1 + (a_{22} - a_{33})y_2 - a_{31}y_1y_2 \\ \qquad - a_{32}y_2^2 + (b_2 - b_3y_2)y_3 \\ \dot{y}_3(t) = -(a_{31}y_1 + a_{32}y_2 + a_{33})y_3 - b_3y_3^2 \end{cases} \quad (4)$$

It is obvious that the position of the object in the space can be calculated as

$$x_1(t) = \frac{y_1(t)}{y_3(t)}, \quad x_2(t) = \frac{y_2(t)}{y_3(t)}, \quad x_3(t) = \frac{1}{y_3(t)} \quad (5)$$

if $y_3(t)$ is available. So, the remaining task is to estimate $y_3(t)$.

In the following, the observer of system (4) is formulated. We consider the system described by

$$\begin{cases} \dot{\hat{y}}_1(t) = a_{13} + (a_{11} - a_{33})y_1 + a_{12}y_2 - a_{31}y_1^2 \\ \qquad - a_{32}y_1y_2 + (b_1 - b_3y_1)\hat{y}_3 + \hat{\lambda}_1(t)\dfrac{e_1}{|e_1| + \delta_1} \\ \dot{\hat{y}}_2(t) = a_{23} + a_{21}y_1 + (a_{22} - a_{33})y_2 - a_{31}y_1y_2 \\ \qquad - a_{32}y_2^2 + (b_2 - b_3y_2)\hat{y}_3 + \hat{\lambda}_2(t)\dfrac{e_2}{|e_2| + \delta_2} \\ \dot{\hat{y}}_3(t) = -(a_{31}y_1 + a_{32}y_2 + a_{33})\hat{y}_3 - b_3\hat{y}_3^2 \\ \qquad + \alpha\phi^T(t)w(t), \\ \hat{y}_3(t_i + 0) = M \cdot sign(\hat{y}_3(t_i - 0)) \end{cases} \quad (6)$$

where the initial condition is determined as

$$\hat{y}_1(0) = y_1(0), \quad \hat{y}_2(0) = y_2(0), \quad \hat{y}_3(0) = \hat{y}_{30}, \quad (7)$$

$\hat{y}_{30}$ is a positive constant; t_i is defined as

$$t_i = \min\{t : t > t_{i-1} \ and \ |\hat{y}_3(t)| \ge 2M\}, \quad (8)$$

and $t_0 = 0$; $M > 0$ is a large constant; $e_1(t)$ and $e_2(t)$ are respectively defined as

$$e_1 = y_1 - \hat{y}_1, \quad e_2 = y_2 - \hat{y}_2; \quad (9)$$

$\delta_i > 0 (i=1,2)$ are design parameters; $\phi(t)$ and $w(t)$ are respectively defined as

$$\phi(t) = [(b_1 - b_3y_1),\ (b_2 - b_3y_2)]^T, \quad (10)$$

$$w(t) = \left[\hat{\lambda}_1(t)\frac{e_1}{|e_1| + \delta_1},\ \hat{\lambda}_2(t)\frac{e_2}{|e_2| + \delta_2}\right]^T; \quad (11)$$

$\hat{\lambda}_1(t)$ and $\hat{\lambda}_2(t)$ are respectively defined as

$$\dot{\hat{\lambda}}_1(t) = \begin{cases} 2\alpha_1|e_1| & if\ |e_1| > 2\delta_1 \\ 0 & otherwise \end{cases}, \quad (12)$$

$$\dot{\hat{\lambda}}_2(t) = \begin{cases} 2\alpha_2|e_2| & if\ |e_2| > 2\delta_2 \\ 0 & otherwise \end{cases}, \quad (13)$$

α, α_1 and α_2 are positive constants, $\hat{\lambda}_1(0)$ and $\hat{\lambda}_2(0)$ can be any positive constants.

Remark 2: t_i defined in (8) are the discontinuous points of the system (6). By observing (6) and (8), it can be easily seen that $\hat{y}_3(t)$ is bounded by $|\hat{y}_3(t)| \le 2M$.

Now, combining (4) and (6) yields

$$\begin{cases} \dot{e}_1(t) = (b_1 - b_3y_1)e_3 - \hat{\lambda}_1(t)\dfrac{e_1}{|e_1| + \delta_1}, \quad e_1(0) = 0 \\ \dot{e}_2(t) = (b_2 - b_3y_2)e_3 - \hat{\lambda}_2(t)\dfrac{e_2}{|e_2| + \delta_2}, \quad e_2(0) = 0 \\ \dot{e}_3(t) = -(a_{31}y_1 + a_{32}y_2 + a_{33})e_3 - b_3(y_3 + \hat{y}_3)e_3 \\ \qquad - \alpha\phi^T(t)w(t) \end{cases} \quad (14)$$

where $e_3(t)$ is defined as

$$e_3 = y_3 - \hat{y}_3. \quad (15)$$

Remark 3: For $i = 1,2$, the terms $\hat{\lambda}_i(t)\dfrac{e_i}{|e_i| + \delta_i}$ are introduced to assure that $|e_i(t)|$ and $|\dot{e}_i(t)|$ are very small. This is motivated by the sliding mode method where the discontinuous form $\hat{\lambda}_i(t)\cdot sign(e_i)$ is modified as $\hat{\lambda}_i(t)\dfrac{e_i}{|e_i|+\delta_i}$. The upper bounds of $|(b_i - b_3 y_i)e_3|$ (their boundedness can be confirmed by assumptions (A1)-(A3) and Remark 2) are not needed in the new method, and are adaptively updated by (11) and (12).

Remark 4: For the system

$$\dot{z}(t) = v(t) - N\cdot sign(z(t)), \tag{16}$$

where $|v(t)| < N$, we can prove that $z(t)\to 0$ as $t\to\infty$, i.e. a sliding mode exists on $z(t)=0$. We can never prove that $\dot{z}(t)\to 0$ as $t\to\infty$. However, in the literature of traditional sliding mode control, $\dot{z}(t)$ is also considered as 0 when the sliding mode occurs, and regard $N\cdot sign(z(t))$ as the estimate of $v(t)$ (the so-called "equivalent control method") [12]. It is obvious that this approach is short of theoretical proof. This difficulty is overcome by our new approach (see also Remark 3 and Lemma 1).

About the constructed system (6), we have the next lemma to state the boundedness of $\hat{\lambda}_i(t)$, $|e_i(t)|$ and $|\dot{e}_i(t)|$, and to assure that $|e_i(t)|$ and $|\dot{e}_i(t)|$ can be controlled by the design parameters δ_1 and δ_2.

Lemma 1: For the constructed system (6), it can be proved that, for $i=1,2$, $|e_i(t)|$ and $\hat{\lambda}_i(t)$ are uniformly bounded, and there exist $T_i>0$ and $\varepsilon_i(\delta_i)>0$ such that $|e_i(t)|\le 2\delta_i$ and $|\dot{e}_i(t)|\le \varepsilon_i(\delta_i)$ as $t>T_i$, where $\varepsilon_i(\delta_i)\to 0$ as $\delta_i\to 0$.

Proof: The proof is composed of two steps.

Step 1 For $i=1,2$, $|e_i(t)|$ and $\lambda_i(t)$ are uniformly bounded, and there exist $t_i>0$ such that $|e_i(t)|\le 2\delta_i$ as $t>t_i$.

By remark 3, suppose the upper bound of $|(b_1 - b_3 y_1)e_3(t)|$ is λ_1, i.e.

$$|(b_1 - b_3 y_1)e_3(t)| < \lambda_1. \tag{17}$$

Now, consider the Lyapunov candidate

$$V_1(t) = (e_1(t))^2 + \frac{1}{\alpha_1}\left(\frac{1}{2}\hat{\lambda}_1(t) - \lambda_1\right)^2. \tag{18}$$

If $|e_1(t)| > 2\delta_1$, then differentiating $V_1(t)$ yields

$$\begin{aligned}
&\dot{V}_1(t)\\
&= 2e_1(t)(b_1 - b_3 y_1)e_3 - 2\hat{\lambda}_1(t)\frac{e_1^2}{|e_1|+\delta_1} + 2\left(0.5\hat{\lambda}_1(t) - \lambda_1\right)|e_1|\\
&= 2\left(e_1(t)(b_1 - b_3 y_1)e_3 - \lambda_1|e_1|\right) + 2\hat{\lambda}_1(t)\frac{|e_1|\delta_1}{|e_1|+\delta_1} - \hat{\lambda}_1(t)|e_1|\\
&\le -\hat{\lambda}_1(t)|e_1|\left(1 - \frac{2\delta_1}{|e_1|+\delta_1}\right)\\
&\le -\frac{1}{3}\hat{\lambda}_1(t)|e_1|\\
&\le -\frac{2}{3}\delta_1\hat{\lambda}_1(0),
\end{aligned} \tag{19}$$

where $\hat{\lambda}_1(t)\ge\hat{\lambda}_1(0)>0$ is employed in the last step. Thus, $V_1(t)$ decreases monotonically from $V_1(0)$ at a speed faster than $\frac{2}{3}\delta_1\hat{\lambda}_1(0)$ if $|e_1(t)|>2\delta_1$. So, it takes finite time that $V_1(t)$ decreases to any possibly nonnegative value if the presupposition $|e_1(t)|>2\delta_1$ holds. Further, by noticing the fact $V_1(t)\ge|e_1(t)|^2$, from (19), it can be seen that the condition $|e_1(t)|>2\delta_1$ cannot be satisfied forever as $V_1(t)$ decreases monotonically at a speed faster than $\frac{2}{3}\delta_1\hat{\lambda}_1(0)$. Therefore, the condition $|e_1(t)|\le 2\delta_1$ can be satisfied in finite time. Then, there exists $t_1>0$ such that $|e_1(t)|\le 2\delta_1$ as $t>t_1$, and $V_1(t)$ is uniformly bounded for $0\le t\le t_1$ (which also means that $|e_1(t)|$ and $\hat{\lambda}_1(t)$ are also uniformly bounded for $0\le t\le t_1$). By (11), it can be seen that $\hat{\lambda}_1(t)=\hat{\lambda}_1(t_1)$ for $t>t_1$. Therefore, $\hat{\lambda}_1(t)$ and $|e_1(t)|$ are uniformly bounded for all $t\ge 0$.

Similarly, it can be proved that $|e_2(t)|$ and $\hat{\lambda}_2(t)$ are uniformly bounded, and there exists $t_2>0$ such that $|e_2(t)|\le 2\delta_2$ for all $t>t_2$.

Step 2 For $i=1,2$, $|\dot{e}_i(t)|$ are uniformly bounded, and there exist $T_i>t_i$ such that $|\dot{e}_i(t)|\le\varepsilon_i(\delta_i)$ as $t>T_i$, where $\varepsilon_i(\delta_i)\to 0$ as $\delta_i\to 0$.

For $t>t_1$, differentiating the first equation in (14) yields

$$\ddot{e}_1(t) = \frac{d}{dt}\left((b_1 - b_3 y_1)e_3\right) - \hat{\lambda}_1(t)\frac{\dot{e}_1\delta_1}{\left(|e_1|+\delta_1\right)^2}, \tag{20}$$

where $\dot{\hat{\lambda}}_1(t)=0$ (as $t>t_1$) is employed. Differentiating $(\dot{e}_1(t))^2$ yields

$$\begin{aligned}
\frac{d}{dt}(\dot{e}_1(t))^2 &= 2\dot{e}_1(t)\cdot\frac{d}{dt}\left((b_1 - b_3 y_1)e_3\right) - 2\hat{\lambda}_1(t)\frac{\dot{e}_1^2\delta_1}{\left(|e_1|+\delta_1\right)^2}\\
&\le 2\dot{e}_1(t)\cdot\frac{d}{dt}\left((b_1 - b_3 y_1)e_3\right) - 2\hat{\lambda}_1(t_1)\frac{\dot{e}_1^2}{9\delta_1},
\end{aligned} \tag{21}$$

where the facts $|e_1(t)|\le 2\delta_1$ and $\hat{\lambda}_1(t)\ge\hat{\lambda}_1(t_1)$ are employed.

By calculating $\frac{d}{dt}\left((b_1 - b_3 y_1)e_3\right)$, it can be easily known that $\frac{d}{dt}\left((b_1 - b_3 y_1)e_3\right)$ is a bounded signal by observing Remarks 2-3, the assumptions (A1)-(A3), and the result in Step 1.

Let

$$K=\max_{t\ge t_1}\left(\left|\frac{d}{dt}\left((b_1-b_3y_1)e_3\right)\right|\right). \qquad (22)$$

If $|\dot{e}_1(t)|>\frac{10\delta_1 K}{\hat{\lambda}_1(t_1)}$, then, from (21), it yields

$$\frac{d}{dt}(\dot{e}_1(t))^2\le 2|\dot{e}_1(t)|\left(K-\hat{\lambda}_1(t_1)\frac{|\dot{e}_1(t)|}{9\delta_1}\right)$$

$$\le 2|\dot{e}_1(t)|\left(K-\frac{10}{9}K\right)<-\frac{20\delta_1K^2}{9\hat{\lambda}_1(t_1)}, \qquad (23)$$

i.e. $|\dot{e}_1(t)|$ decreases monotonically at a speed faster than $\frac{20\delta_1K^2}{9\hat{\lambda}_1(t_1)}$. Thus, the presupposition $|\dot{e}_1(t)|>\frac{10\delta_1 K}{\hat{\lambda}_1(t_1)}$ cannot be satisfied forever. Then, there exists an instant $T_1\ge t_1$ such that

$$|\dot{e}_1(t)|\le\frac{10\delta_1 K}{\hat{\lambda}_1(t_1)} \qquad (24)$$

for all $t>T_1$.

Therefore, $|\dot{e}_1(t)|$ is very small as $t>T_1$ by choosing very small δ_1. Further, from (23), it can be seen that the decreasing speed of $|\dot{e}_1(t)|$ can be increased by choosing a very small δ_1.

Similarly, it can be proved that there exists an instant $T_2\ge t_2$ such that $|\dot{e}_2(t)|$ is bounded and very small as $t>T_2$ by choosing very small δ_2. Thus, the lemma is proved.

Remark 5: In the constructed system (6), $\hat{y}_1(t)$ and $\hat{y}_2(t)$ are the auxiliary outputs. The dynamics of $\hat{y}_1(t)$ and $\hat{y}_2(t)$ are introduced in order to estimate the unknown signals $(b_1-b_3y_1)e_3$ and $(b_2-b_3y_2)e_3$. Lemma 1 tells us that their corresponding estimates are $\hat{\lambda}_1(t)\frac{e_1}{|e_1|+\delta_1}$ and $\hat{\lambda}_2(t)\frac{e_2}{|e_2|+\delta_2}$ (see the first two equations in (14)) (see also Remarks 3-4). These estimates are employed in the third equation in (6) to force $\hat{y}_3(t)$ to be very close to $y_3(t)$ as t is very large.

The next theorem gives the condition to guarantee the generated signal $\hat{y}_3(t)$ in (6) to be very close to $y_3(t)$ as t is very large.

Theorem 1. Suppose there exist a positive constant β and a very small positive constant ρ such that

$$\int_t^{t+\rho}\phi^T(\tau)\phi(\tau)d\tau = \int_t^{t+\rho}\left((b_1-b_3y_1(\tau))^2+(b_2-b_3y_2(\tau))^2\right)d\tau\ge\beta \qquad (25)$$

for all $t\ge 0$. If the parameter α is chosen large enough, then $e_3(t)$ is uniformly bounded and decreases exponentially. Further, there exist $T_0\ge 0$ and $\varepsilon(\delta_1,\delta_2)>0$ such that

$$|e_3(t)|\le\varepsilon(\delta_1,\delta_2) \qquad (26)$$

as $t\ge T_0$, where $\varepsilon(\delta_1,\delta_2)\to 0$ as $\sum_{i=1}^{2}\delta_i\to 0$. Thus, $\hat{y}_3(t)$ generated in (6) is the approximate estimate of $y_3(t)$ as t is very large by choosing very small parameters δ_1 and δ_2.

Proof: By the assumptions (A1)-(A3) and Remarks 2-3, it can be easily seen that $|(a_{31}y_1+a_{32}y_2+a_{33})+b_3(y_3+\hat{y}_3)|$ is a bounded signal, i.e. there exists $\gamma>0$ such that

$$\gamma=\sup_{t>0}\left\{|(a_{31}y_1+a_{32}y_2+a_{33})+b_3(y_3+\hat{y}_3)|\right\}. \qquad (27)$$

Let

$$\kappa(t)=\left|2\alpha e_3(t)\phi^T(t)\begin{bmatrix}\dot{e}_1(t)\\ \dot{e}_2(t)\end{bmatrix}\right|. \qquad (28)$$

From Lemma 1, it can be seen that there exists $d(\delta_1,\delta_2)>0$ such that

$$\kappa(t)\le d(\delta_1,\delta_2) \qquad (29)$$

as $t>\max(T_1,T_2)$, where $d(\delta_1,\delta_2)\to 0$ as $\sum_{i=1}^{2}\delta_i\to 0$.

Now, from (14), differentiating $(e_3(t))^2$ yields

$$\frac{d}{dt}(e_3(t))^2=2e_3(t)\left(-(a_{31}y_1+a_{32}y_2+a_{33})e_3-b_3(y_3+\hat{y}_3)e_3\right) -2\alpha e_3(t)\phi^T(t)w(t)$$

$$\le 2\gamma(e_3(t))^2-2\alpha e_3(t)\phi^T(t)\left(\phi(t)e_3(t)-\begin{bmatrix}\dot{e}_1(t)\\ \dot{e}_2(t)\end{bmatrix}\right)$$

$$\le -2\left(\alpha\phi^T(t)\phi(t)-\gamma\right)(e_3(t))^2+\kappa(t). \qquad (30)$$

Let

$$\varsigma(t)=2\left(\alpha\phi^T(t)\phi(t)-\gamma\right). \qquad (31)$$

It can be seen that there exist a positive constant μ such that

$$0<\mu\le\int_t^{t+\rho}\varsigma(\tau)d\tau \qquad (32)$$

for all $t\ge 0$, if α is chosen as $\alpha>\frac{0.5\mu+\gamma\rho}{\beta}$.

From the assumption (25), it can be easily seen that there exist a constant θ such that

$$\theta\le\int_t^{t+\vartheta}\varsigma(\tau)d\tau \qquad (33)$$

for all $t\ge 0$ and any ϑ satisfying $0\le\vartheta\le\rho$. Let

$$s(t)=\frac{d}{dt}(e_3(t))^2+\varsigma(t)(e_3(t))^2-\kappa(t). \qquad (34)$$

Then, $s(t) \le 0$. Solving the differential equation (34) yields

$$(e_3(t))^2 = e^{-\int_T^t \varsigma(\tau)d\tau}(e_3(T))^2 + \int_T^t e^{-\int_\iota^t \varsigma(\tau)d\tau}(s(\iota)+\kappa(\iota))d\iota$$

$$\le e^{-\int_T^t \varsigma(\tau)d\tau}(e_3(T))^2 + d(\delta_1, \delta_2)\int_T^t e^{-\int_\iota^t \varsigma(\tau)d\tau} d\iota \, , \quad (35)$$

where $T > \max(T_1, T_2)$.

Now, express $t-T$ as $t-T = k\rho + \sigma$, where $0 \le \sigma < \rho$. Then, we have

$$\int_T^t \varsigma(\tau)d\tau = \sum_{l=1}^{k}\int_{T+(l-1)\rho}^{T+l\rho}\varsigma(\tau)d\tau + \int_{T+k\rho}^{T+k\rho+\sigma}\varsigma(\tau)d\tau \ge k\mu + \theta \, . \quad (36)$$

For $T+(l-1)\rho \le \iota < T+l\rho$, by expressing $\iota = T+(l-1)\rho+\omega$ with $0 \le \omega < \rho$, we have

$$\int_\iota^t \varsigma(\tau)d\tau = \int_{T+(l-1)\rho+\omega}^{T+k\rho+\sigma}\varsigma(\tau)d\tau$$

$$= \begin{cases} \int_{T+(l-1)\rho+\omega}^{T+k\rho+\omega}\varsigma(\tau)d\tau + \int_{T+k\rho+\omega}^{T+k\rho+\sigma}\varsigma(\tau)d\tau & if \ \ \sigma \ge \omega \\ \int_{T+(l-1)\rho+\omega}^{T+(k-1)\rho+\omega}\varsigma(\tau)d\tau + \int_{T+(k-1)\rho+\omega}^{T+k\rho+\sigma}\varsigma(\tau)d\tau & if \ \ \sigma < \omega \end{cases}$$

$$\ge \begin{cases} (k-l+1)\mu + \theta & if \ \ \sigma \ge \omega \\ (k-l)\mu + \theta & if \ \ \sigma \ge \omega \end{cases}$$

$$\ge (k-l)\mu + \theta \, . \quad (37)$$

Thus,

$$\int_T^t e^{-\int_\iota^t \varsigma(\tau)d\tau} d\iota = \sum_{l=1}^{k}\int_{T+(l-1)\rho}^{T+l\rho} e^{-\int_\iota^t \varsigma(\tau)d\tau} d\iota + \int_{T+k\rho}^{t} e^{-\int_\iota^t \varsigma(\tau)d\tau} d\iota$$

$$\le \sum_{l=1}^{k}\int_{T+(l-1)\rho}^{T+l\rho} e^{-(k-l)\mu-\theta} d\iota + \int_{T+k\rho}^{t} e^{-\theta} d\iota$$

$$\le \rho\sum_{l=1}^{k} e^{-(k-l)\mu-\theta} + \rho e^{-\theta}$$

$$= \rho e^{-\theta}\frac{e^{-k\mu} - 2e^{\mu} + 1}{1-e^{\mu}} \, . \quad (38)$$

Then, substituting (36) and (38) into (35) yields

$$(e_3(t))^2 \le e^{-k\mu}e^{-\theta}(e_3(T))^2 +$$

$$e^{-k\mu}\frac{\rho e^{-\theta}}{1-e^{\mu}}d(\delta_1, \delta_2) + \rho e^{-\theta}\frac{1-2e^{\mu}}{1-e^{\mu}}d(\delta_1, \delta_2) \, , \quad (39)$$

i.e. $(e_3(t))^2$ decreases exponentially if it is not very small.

By the boundedness of the right hand side of (6), it follows that $t_i - t_{i-1}$ is greater than a positive constant, say χ (As ρ is very small, we can conclude that $\chi >> \rho$), for all *i*. On every such a time interval, the variable $(e_3(t))^2$ decreases exponentially (see (39)) and, at t_i, the estimation error $e_3(t)$ satisfies $|e_3(t_i+0)| < |e_3(t_i-0)|$. Thus, $|e_3(t)|$ decreases exponentially until it becomes very small. Therefore, $|e_3(t)|$ is bounded and (26) is proved.

Remark 6: The condition (25) can be thought of the observability condition for the perspective system. The condition (25) means that if $(b_1 - b_3y_1(t))^2 + (b_2 - b_3y_2(t))^2$ is very small or zero at some instant, it must increase fast enough thereafter. Further, it can be proved that if $(b_1 - b_3y_1(t))^2 + (b_2 - b_3y_2(t))^2$ is zero in some time interval, it must be identically zero. Moreover, the condition in [5] is much more strict than the condition (25) in this paper. Therefore, it can be seen that the assumption (25) is reasonable for a large class of perspective systems.

Remark 7: The design parameters $\alpha_i > 0$ and $\delta_i > 0$ ($i = 1, 2$) determine the estimating speed and the estimating precision. The parameters $\alpha_i > 0$ ($i = 1, 2$) should be chosen large enough to rapidly adjust $\hat{\lambda}_i(t)$.

4. EXAMPLES AND SIMULATIONS

In this section, we present the simulation results for two examples. The simulation is done by Matlab. The sampling period is chosen as 0.05. Suppose the measured image data is corrupted by 1% with random noise.

Example 1: Consider the movement of the object described by

$$\frac{d}{dt}\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} = \begin{bmatrix} -0.2 & 0.4 & -0.6 \\ 0.1 & -0.2 & 0.3 \\ 0.3 & -0.4 & 0.4 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 0.5 \\ 0.25 \\ 0.3 \end{bmatrix},$$

$$\begin{bmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 1.5 \\ 2.5 \end{bmatrix}. \quad (45)$$

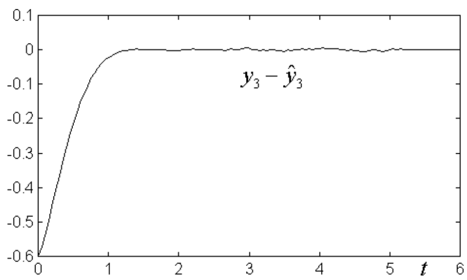

Fig. 1 The difference between y_3 and its estimate $\hat{y}_3$ by using the new observer for Example 1.

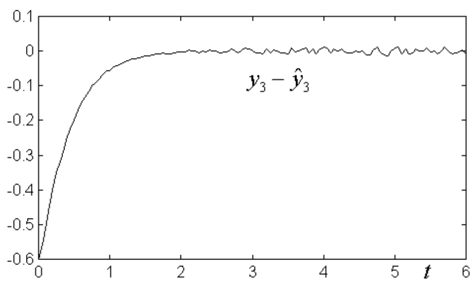

Fig. 2 The difference between y_3 and its estimate $\hat{y}_3$ by using the IBO for Example 1.

For this example, the condition (25) is satisfied for $\rho = 0.05$ and $\beta = 0.05$. In the observer design, $\hat{y}_3(0)$ is chosen as $\hat{y}_3(0) = 1$. The parameters are chosen as $\alpha = 20$, $\alpha_1 = \alpha_2 = 5$, $\delta_1 = \delta_2 = 0.3$, $M = 10$, $\hat{\lambda}_1(0) = \hat{\lambda}_2(0) = 0.2$.

Comparison between the proposed new observer and the IBO in Jankovic *et al.* (1995) is performed. Because of the trade-off of the estimation error and the converging speed, the estimation error is compared based on the same converging speed. Figure 1 shows the difference between y_3 and its estimate $\hat{y}_3$ by using the new observer. Figure 2 shows the difference between y_3 and its estimate $\hat{y}_3$ by using the IBO. It can be seen that the estimation error in Figure 1 is much smaller than that in Figure 2, i.e. the performance of the new observer is better than that of the IBO. Since the performance of the extended Kalman filter (EKF) is similar to that of IBO, it can be concluded that the performance of the new observer is also better than that of the EKF. Further, it can be seen that the formulation of the new observer is much simpler than that of the IBO, not to say EKF. And the *a priori* knowledge about the noise is not needed in the new method. Therefore, it can be concluded that the new observer is superior to the traditional IBO and EKF.

Example 2: Consider the periodic movement of the object described by

$$\frac{d}{dt}\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} = \begin{bmatrix} 0 & -2\pi & 0 \\ 2\pi & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 2\pi\cos(2\pi t) \end{bmatrix},$$

$$\begin{bmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}. \quad (46)$$

In the observer design, $\hat{y}_3(0)$ is chosen as $\hat{y}_3(0) = 1$. The parameters are chosen as $\alpha = 20$, $\alpha_1 = \alpha_2 = 5$, $\delta_1 = \delta_2 = 2$, $M = 10$, $\hat{\lambda}_1(0) = \hat{\lambda}_2(0) = 0.2$.

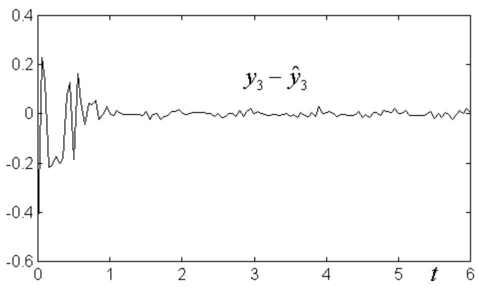

Fig. 3 The difference between y_3 and its estimate $\hat{y}_3$ by using the new observer for Example 2.

Figure 3 shows the difference between y_3 and its estimate $\hat{y}_3$ by using the new observer. It can be easily seen that $(b_1 - b_3 y_1(t))^2 + (b_2 - b_3 y_2(t))^2 = (2\pi\cos(2\pi t))^2\left((y_1(t))^2 + (y_2(t))^2\right)$ takes the value zero periodically. The method proposed by Jankovic *et al* (1995) cannot be applied to this movement.

5. CONCLUSIONS

In this paper, we consider the state identification problem of a class of perspective system, where the parameters are assumed known. The formulated problem can be converted into the observation of a dynamical system with nonlinearties. A new discontinuous observer, which is motivated by the sliding mode control method, is proposed to identify the state of the perspective systems. Minor *a priori* knowledge about the system is required. The attraction of the new method lies in that the algorithm is very simple and easy to be implemented, and can cope with a large class of perspective systems. Further, simulation results show the robustness to measurement noises and the superiority to the traditional ones.

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