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15th Triennial World Congress, Barcelona, Spain

TRANSITION RATE BOUNDS FOR THE STABILITY ROBUSTNESS OF CONTINUOUS-TIME MARKOVIAN JUMP LINEAR SYSTEMS

Mehmet Karan * **Peng Shi** ** **C. Yalçın Kaya** ***

* *Boeing Australia Limited*
Current address: The Boeing Company, P.O. Box 3707,
Mail Stop: 8A-48, Seattle WA 98124-2207 USA
Email: Mehmet.Karan@ieee.org
** *Land Operations Division, Defence Science and Technology Organisation, PO Box 1500, Edinburgh, SA 5111, Australia.*
Email: peng.shi@dsto.defence.gov.au
*** *School of Mathematics, University of South Australia, Mawson Lakes, SA 5095, Australia.*
Email: yalcin.kaya@unisa.edu.au

Abstract: This paper considers the stability robustness of Markovian jump linear systems in continuous-time with respect to their transition rates. The system under study is a linear continuous-time one with Markovian jump parameters where the mode transition rate is perturbed. By using stochastic Lyapunov function approach and Kronecker product transformation techniques, we develop a sufficient condition for the robust stochastic stability of the underlying system, which is in terms of an upper bound of the perturbed transition rate. A numerical example is presented to illustrate the potential use of the proposed technique.

Keywords: Robust, hybrid, Markovian jump linear system, stability, perturbation

1. INTRODUCTION

As is well known, many physical systems have variable structures subject to random changes, which may result from the abrupt phenomena such as component and interconnection failures, parameters shifting, tracking, and the time required to measure some of the variables at different stages. Systems with this character may be modelled as hybrid ones, that is, the state space of the systems contains both discrete and continuous states. Among this kind of systems, jumping linear systems have been a subject of great practical importance which has attracted a lot of interest for the last three decades. In jumping linear systems, the dynamics of the discrete and continuous states are modelled, respectively, by a finite state Markov chain and linear differential equations subject to the discrete process. There has been a dramatic progress in jumping linear quadratic (JLQ) control theory since the pioneering work on JLQ control by Krasovskii and Lidskii (1961). The JLQ control problem was solved by Sworder (1969) using stochastic maximum principle for state feedback in finite horizon case. Wonham (1971) also obtained the same results using dynamic programming for both finite and infinite horizon cases. Mariton and Bertrand (1985) provided an approach to output feedback JLQ control problem. The continuous-time partially observable situation was studied by Fragoso (1988). An analysis of the discrete-time version of JLQ control problem was given in (Chizeck *et al.*, 1986) for the case of without driving noise, and (Fragoso, 1989) for the case of with driving noise, respectively. Recently, the problems of controllability, stabilizability, and continuous-time JLQ control have been theoretically addressed by Ji and Chizeck (1990) and the references therein. Feng and Loparo (1990) has studied the problem of almost sure instability of the random harmonic oscillator. The stochastic stability properties of jumping linear systems has been systematically investigated in (Feng *et al.*, 1992) and shown that several stability concepts are equivalent. Kalmanovich and Haddad (1994) tackled the problem of JLQ when the discrete Markov process in systems is not directly observable and obtained necessary conditions for optimality. The counterpart of H_∞ control of jump linear systems was investigated by Pan and Basar (1994) via zero-sum differential games. Also, Srichander and Walker (1989) made the stochastic stability analysis for continuous-time fault tolerant control systems, in which the system has two random processes with Markovian transition characteristics (one representing the random failures of the system and the other representing the failure detection and identification (FDI) decision behaviour).

On the other hand, design of control systems that can handle model uncertainties has been one of the most challenging problems and received considerable attention from control engineers and scientists in the past decades. There are two major issues in robust controller design. The first is concerned with the robust stability of the uncertain closed-loop system (see for example, Khargonekar *et al.* (1990) and the references therein), and the other is robust performance (see for example, (Karan *et al.*, 1994) and (Xie *et al.*, 1992). The problems of robust stochastic stability, stabilization, and control have been extensively studied in the past ten years. For some representative prior work on this general topic, we refer the reader to (Boukas, 1993; Boukas, 1995; Boukas and Shi, 1998; Shi and Boukas, 1997; Shi *et al.*, 1998; Shi *et al.*, 1999; Shi *et al.*, 2000). and the references therein. However, to the best of our knowledge, the study of robust stochastic stability for jumping linear continuous-time systems with perturbed transition rates has not yet been fully conducted. This problem is quite important in real physical systems, simply because the system mode jumping from one state to another is not always certain. The transition rate almost always has some kind of perturbation with a known upper bound.

In this paper, we consider the stability robustness of continuous-time Markovian jump linear systems (MJLS) with respect to transition rates. Note that stability of MJLS has been investigated via the notion of stochastic stability introduced in (Ji and Chizeck, 1990), while necessary and sufficient conditions for the stochastic stability of MJLS have been obtained using stochastic Lyapunov functions in the work by Feng and Loparo (1990) and Feng *et al.* (1992). By using stochastic Lyapunov function approach, together with the help of Kronecker product transformation techniques, we develop a sufficient condition for the robust stochastic stability of the underlying system, which is in terms of an upper bound of the perturbed transition rate. A numerical example is presented to show the potential of the proposed techniques.

Notation. The notations in this paper are quite standard. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n dimensional Euclidean space and the set of all $n \times m$ real matrices. The superscript "T" denotes the transpose and the notation $X \geq Y$ (respectively, $X > Y$) where X and Y are symmetric matrices, means that $X - Y$ is positive semi-definite (respectively, positive definite). I is the identity matrix with compatible dimension. $\mathsf{E}\{\cdot\}$ is the expectation operator with respective to some probability measure P. The trace and i-th eigenvalue of a matrix M are denoted by $tr(M)$ and $\lambda_i(M)$ respectively. Minimum and maximum singular values of a matrix M are denoted by $\sigma_{\min}(M)$ and $\sigma_{\max}(M)$ respectively where $\sigma_{\max}(M) = \lambda_{\max}(M^T M)$.

2. MAIN RESULT

Consider the following autonomous Markovian jump linear system (MJLS) with the state vector $x(t) \in \mathbb{R}^N$

$$\overline{\mathcal{S}} \;:\; \dot{x}(t) \;=\; A(\eta_t)\, x(t) \tag{1}$$

The system mode $\{\eta_t,\ t \geq 0\}$ is a time homogeneous Markov process with right continuous trajectories and taking values in a finite set $\mathbb{M} = \{1, 2, \cdots, M\}$ with stationary transition probabilities

$$\begin{aligned} &Prob\,(\eta_{t\,+\,h} = j|\eta_t = i) \\ &\quad = \begin{cases} \overline{\pi}_{ij}h \,+\, o(h), & i \neq j \\ 1 \,+\, \overline{\pi}_{ii}h \,+\, o(h), & i = j \end{cases} \end{aligned}$$

where $h > 0$, $\lim_{h\to 0} \frac{o(h)}{h} = 0$ and $\overline{\pi}_{ij} \geq 0$ is the transition rate from mode i at time t to mode j at time $t + h$, and

$$\overline{\pi}_{ii} \;=\; -\sum_{\substack{j=1\\ j\neq i}}^{m} \overline{\pi}_{ij}.$$

Let $x(t, x_0, \eta_0)$ denote the trajectory of the state $x(t)$, of the system $\overline{\mathcal{S}}$ in (1), from the initial state x_0 with an initial system mode η_0. As introduced in (Ji and Chizeck, 1990), *stochastic stability* of a system can be defined as follows.

Definition 1. The system $\overline{\mathcal{S}}$ in (1) is said to be stochastically stable about the equilibrium point 0, if for any initial state $x_0 \in \mathbb{R}^n$ and for any initial mode $\eta_0 = i$ where $i \in \mathbb{M}$, the following holds

$$\int_0^\infty \mathsf{E}\left\{\, \|x(t, x_0, \eta_0)\|^2 \,\right\}\, dt \;<\; \infty.$$

Stochastic stability of Markovian jump linear systems can be checked using the following result given in (Feng *et al.*, 1992).

Lemma 1. ((Feng *et al.*, 1992)) The system $\overline{\mathcal{S}}$ in (1) is stochastically stable if and only if there exists a set of symmetric positive definite matrices $\overline{P}_i$ satisfying

$$A_i^T\,\overline{P}_i \;+\; \overline{P}_i\,A_i \;+\; \sum_{j=1}^{M} \pi_{ij}\,\overline{P}_j \;+\; \overline{Q}_i \;=\; 0 \quad (2)$$

for any given set of symmetric positive definite matrices $\overline{Q}_i$ where $i \in \mathbb{M}$.

The above equation can be rewritten to obtain the following result.

Lemma 2. The system $\overline{\mathcal{S}}$ in (1) is stochastically stable if and only if there exists a block diagonal symmetric positive definite matrix $\overline{P}$ satisfying the following equation for any given symmetric block diagonal positive definite matrix $\overline{Q}$.

$$A^T\,\overline{P} \;+\; \overline{P}\,A \;+\; \Gamma\,[I \,\otimes\, (\overline{P}\,E)] \;+\; \overline{Q} \;=\; 0 \quad (3)$$

where $\otimes$ denotes the Kronecker product as defined in (Graham, 1981) and

$$A = \begin{bmatrix} A_1 & \mathbf{0} & & \mathbf{0} \\ \mathbf{0} & A_2 & & \mathbf{0} \\ \vdots & & \ddots & \vdots \\ \mathbf{0} & \dots & \mathbf{0} & A_M \end{bmatrix} \in \mathbb{R}^{NM\times NM},$$

$$\overline{P} = \begin{bmatrix} \overline{P}_1 & \mathbf{0} & & \mathbf{0} \\ \mathbf{0} & \overline{P}_2 & & \mathbf{0} \\ \vdots & & \ddots & \vdots \\ \mathbf{0} & \dots & \mathbf{0} & \overline{P}_M \end{bmatrix} \in \mathbb{R}^{NM\times NM},$$

$$\Gamma = \begin{bmatrix} v_1 & \mathbf{0} & \dots\dots & \mathbf{0} \\ \mathbf{0} & v_2 & 0 \;\dots & \mathbf{0} \\ \vdots & & \ddots & \vdots \\ \vdots & & \ddots & 0 \\ \mathbf{0} & \dots\dots & \mathbf{0} & v_M \end{bmatrix} \in \mathbb{R}^{NM\times NM^2}$$

where v_i is defined by $v_i \;=\; \begin{bmatrix}\pi_{i1}I \;\; \pi_{i2}I \;\; \dots \;\; \pi_{iM}I\end{bmatrix}$ and

$$\overline{Q} = \begin{bmatrix} \overline{Q}_1 & \mathbf{0} & \dots\dots & \mathbf{0} \\ \mathbf{0} & \overline{Q}_2 & \mathbf{0} \;\dots & \mathbf{0} \\ \vdots & & \ddots & \vdots \\ \vdots & & \ddots & \vdots \\ \mathbf{0} & \dots & \mathbf{0} & \overline{Q}_M \end{bmatrix} \in \mathbb{R}^{NM\times NM},$$

$$E = \begin{bmatrix} I \; \dots \; I \end{bmatrix}^T \;\in\; \mathbb{R}^{NM\times N}.$$

From the above lemma, we obtain the following theorem which gives bounds on the transition rates for the stochastic stability of Markovian jump linear systems.

Theorem 1. Consider a MJLS $\mathcal{S}$ identical to $\overline{\mathcal{S}}$ in (1), but with jumping rates π_{ij} such that

$$\pi_{ij} \;=\; \overline{\pi}_{ij} + \Delta\pi_{ij}$$

Then this system is stable if

$$\max_i \left\{ \left(\sum_{j=1}^{M} \Delta\pi_{ij}^2 \right)^{1/2} \right\} \;\leq$$

$$\min_i \left\{ \sigma_{\min}(\overline{Q}_i) \right\} \quad \lambda_{\max}^{-1} \left(\sum_{i=1}^{M} \overline{P}_i^2 \right) \qquad (4)$$

and

$$\sum_{j=1}^{M} \Delta\pi_{ij} \;=\; 0,$$

for all $i = 1, \dots, M$ where

$$\Delta\pi_{ij} \;>\; -\pi_{ij} \qquad (5)$$

for $i \neq j$ where $i, j = 1, \dots, M$. In (4), the matrices $\overline{P}_i$ and $\overline{Q}_i$ are positive definite matrices satisfying (2) for $i = 1, \dots, M$.

Proof : Define an energy function $\mathcal{E}(X)$ for the system in (1) as

$$\mathcal{E}(x(t), \eta_t) \;=\; x^T(t)\,\overline{P}_{\eta_t}\,x(t)$$

Then the system is stable if

$$\begin{aligned} &\mathcal{L}(\mathcal{E})(x(t), \eta_t) \;= \\ &\quad x^T(t) \left(A(\eta_t)^T\,\overline{P}_{\eta_t} \;+\; \overline{P}_{\eta_t}\,A(\eta_t) \right. \\ &\qquad \left. +\; \sum_{j=1}^{M} \pi_{\eta_t, j}\,\overline{P}_j \right) x(t) \\ &< 0 \end{aligned}$$

where $\mathcal{L}$ is the infinitesimal generator acting on $\mathcal{E}(x, \eta)$. Therefore

$$\begin{aligned} &A^T\,\overline{P} \;+\; \overline{P}\,A \\ &\quad +\; \left(\overline{\Gamma} + \Delta\Gamma\right)\,[I \,\otimes\, (\overline{P}\,E)] \;\leq\; 0 \end{aligned}$$

$$\begin{aligned} &A^T\,\overline{P} \;+\; \overline{P}\,A \;+\; \overline{\Gamma}\,[I \,\otimes\, (\overline{P}\,E)] \\ &\quad +(\Delta\Gamma)\,[I \,\otimes\, (\overline{P}\,E)] \;\leq\; 0 \end{aligned}$$

$$-\overline{Q} + (\Delta\Gamma)\,[I \,\otimes\, (\overline{P}\,E)] \;\leq\; 0$$

Thus, a sufficient condition can be given as

$$\begin{aligned} \sigma_{\max}\left\{(\Delta\Gamma)\,[I \,\otimes\, (\overline{P}\,E)]\right\} \;&\leq\; \sigma_{\min}(\overline{Q}) \\ &=\; \min_i \left\{\sigma_{\min}(\overline{Q}_i)\right\} \qquad (6) \end{aligned}$$

where $\sigma_{\max}(\cdot)$ and $\sigma_{\min}(\cdot)$ denote the maximum and minimum singular values of $(\cdot)$ respectively. Note that

$$\begin{aligned} &\sigma_{\max}\left\{(\Delta\Gamma)\,[I \,\otimes\, (\overline{P}\,E)]\right\} \;\leq \\ &\quad \sigma_{\max}\{(\Delta\Gamma)\}\,\sigma_{\max}[I \,\otimes\, (\overline{P}\,E)] \end{aligned}$$

Now, we can choose $\Delta\Gamma$ such that

$$\sigma_{\max}(\Delta\Gamma)\,\sigma_{\max}[I\otimes(\overline{P}E)] \;\leq\; \min_i\left\{\sigma_{\min}(\overline{Q}_i\right\} \quad (7)$$

We should note that the above inequality provides a more conservative bound on the permissible perturbation $\Delta\Gamma$ since any value of $\Delta\Gamma$ which satisfies the bound in (7) will also satisfy the inequality in (6). On the other hand,

$$\begin{aligned}\sigma_{\max}[I\;\otimes\;(\overline{P}\,E)] &= \rho_{\max}^{1/2}\left(\overline{P}\,E\right)\\ &= \rho_{\max}^{1/2}\left(\sum_{i=1}^{M}\overline{P}_i^2\right)\\ &= \lambda_{\max}\left(\sum_{i=1}^{M}\overline{P}_i^2\right)\end{aligned}$$

Moreover,

$$\begin{aligned}\sigma_{\max}\left\{\Delta\Gamma\right\} &= \pi_{\max}^{1/2}(\Delta\Gamma^T\,\Delta\Gamma)\\ &= \pi_{\max}^{1/2}(\Delta\Gamma\,\Delta\Gamma^T)\\ &= \max_{i\in\mathbb{M}}\left\{\Delta v_i\,\Delta v_i^T\right\}^{1/2}\\ &= \max_{i\in\mathbb{M}}\left\{\left(\sum_{j=1}^{M}\Delta\pi_{ij}^2\right)^{1/2}\right\}\end{aligned}$$

Consequently, the perturbed system is stable if the bound in (4) is satisfied.

Next, we give an example to illustrate the above result for the case of scalar Markovian jump linear systems.

3. EXAMPLE

In this section, we consider the robust stability of a scalar Markovian jump linear system given by

$$\mathcal{S}\;:\;\begin{cases}\dot{x}(t) &= a(\eta_t)\,x(t), \qquad t\leq 0\\ x(0) &= x_0\in\mathbb{R}\end{cases} \qquad (8)$$

where the transition rate matrix $\overline{\Pi}$, associated with the system mode η_t, is given by

$$\overline{\Pi} \;=\; \begin{bmatrix}-\overline{\pi}_1 & \overline{\pi}_1\\ \overline{\pi}_2 & -\overline{\pi}_2\end{bmatrix} \qquad (9)$$

where $\overline{\pi}_i > 0$. Our aim is to investigate the stability of the system $\mathcal{S}$ which is identical to the nominal system $\overline{\mathcal{S}}$ except that the transition rates are perturbed as

$$\pi_i \;=\; \overline{\pi}_i + \Delta\pi_i \qquad (10)$$

Note from (Feng *et al.*, 1992) that the perturbed system $\mathcal{S}$ is stable if and only if

$$2a_1p_1 + \pi_1(p_2-p_1)+1 = 0, \qquad (11)$$
$$2a_2p_2 + \pi_2(p_1-p_2)+1 = 0 \qquad (12)$$

where $a_i = a(i)$ and $\overline{q}_1 = \overline{q}_2 = 1$. The above equations can be solved as

$$p_1 = \frac{n_1}{d}, \qquad p_2 = \frac{n_2}{d} \qquad (13)$$

$$n_1 = \pi_1+\pi_2-2a_2, \qquad (14)$$
$$n_2 = \pi_1+\pi_2-2a_1, \qquad (15)$$
$$d = 4a_1a_2-2a_2\pi_1-2a_1\pi_2. \qquad (16)$$

Therefore the conditions $p_1 > 0$ and $p_2 > 0$ are satisfied if and only if

$$n_1>0,\; n_2>0,\; d>0$$

or

$$n_1<0,\; n_2<0,\; d<0 \qquad .$$

The first set of constraints can be rewritten with the additional constraints on π_1, π_2

$$\pi_1+\pi_2 > 2\max\{a_1,a_2\}, \qquad \pi_1>0, \qquad (17)$$
$$2a_1a_2 > a_2\pi_1+a_1\pi_2, \qquad \pi_2>0 \qquad (18)$$

and the second set of constraints becomes

$$\pi_1+\pi_2 < 2\min\{a_1,a_2\}, \qquad \pi_1>0, \qquad (19)$$
$$a_2\pi_1+a_1\pi_2 < 2a_1a_2, \qquad \pi_2>0. \qquad (20)$$

However, (19) cannot be satisfied if either a_1 or a_2 is negative. Thus, the constraints in (17) and (18) are necessary and sufficient conditions for the stability of the system in terms of π_1 and π_2 when either of the subsystems is stable. However, the above necessary and sufficient conditions (17)-(20) are, in general, very difficult to calculate for higher dimensional systems.

We can also use (4) to obtain a bound on $\Delta\pi_i$ in (10) for $i=1,2$ for the stochastic stability of the perturbed system $\mathcal{S}$ as

$$\max_{i=1,2}\{\mid\Delta\pi_i\mid\} \;<\; \frac{1}{\sqrt{2}}\;\frac{\min_{i=1,2}(\sqrt{\overline{q}_i}\,)}{\sqrt{\overline{p}_1^2+\overline{p}_2^2}} \qquad (21)$$

where $\overline{p}_1$ and $\overline{p}_2$ can be calculated for the system $\overline{\mathcal{S}}$ from (11) with the transition rates $\overline{\pi}_1$ and $\overline{\pi}_2$.

Now, let us consider the example given in (Ji and Chizeck, 1990) where the system is a scalar MJLS as given in (8) where

$$a_1 \;=\; 1/3, \qquad a_2 \;=\; -4/3 \qquad (22)$$

and the probability transition rate matrix $\overline{\pi}$ in (9) is given by

$$\overline{\pi}_1 \;=\; 1, \qquad \overline{\pi}_2 \;=\; 1.$$

It can be found that this system is stable since from (13) and (14), $\overline{p}_1$ and $\overline{p}_2$ can be calculated with the above values of the transition rates $\overline{\pi}_i$ as

$$\overline{p}_1 \;=\; 21, \qquad \overline{p}_2 \;=\; 6\,.$$

The admissible values of (π_1,π_2) for the stochastic stability of the perturbed system $\mathcal{S}$ with the parameter values in (22) can be found from (17)-(20) as

$$\mathcal{R} = \left\{ (\pi_1, \pi_2) \middle| \pi_1 + \pi_2 > \frac{2}{3}, \right.$$
$$\left. 4\pi_1 - \pi_2 > \frac{8}{3}, \pi_1 > 0,\ \pi_2 > 0 \right\}$$

Note that the above bounds are necessary and sufficient for the stochastic stability of the perturbed system $\mathcal{S}$, however these bounds are in general very difficult to compute for higher dimensional systems.

On the other hand, we can also obtain bounds from (21) on $(\Delta\pi_1, \Delta\pi_2)$, or equivalently on (π_1, π_2) also using (10), as

$$\max_{i=1,2} \{ | \Delta\pi_i | \} < \frac{1}{\sqrt{2}} \frac{1}{\sqrt{21^2 + 6^2}} = 0.00324$$

Thus, the above result yields an admissible (π_1, π_2)-region $\mathcal{R}_1$ for the stochastic stability of the perturbed system $\mathcal{S}$ as

$$\mathcal{R}_1 = \left\{ (\pi_1, \pi_2) \middle| |\pi_1 - 1| < 0.00324, \right.$$
$$\left. |\pi_2 - 1| < 0.00324,\ \pi_1 > 0,\ \pi_2 > 0 \right\}$$

These two admissible regions $\mathcal{R}$ and $\mathcal{R}_1$ can be seen in Figure 1. It is not surprising that $\mathcal{R}_1$ allows relatively small perturbation bounds on the transition rates since the values of the nominal system transition rates are close to the boundary of the admissible transition rate region $\mathcal{R}$. On the other hand, if we choose the nominal system transition rates as

$$\overline{\pi}_1 = 2, \qquad \overline{\pi}_2 = 1$$

then from (13) and (14) we obtain

$$\overline{p}_1 = \frac{51}{26}, \qquad \overline{p}_2 = \frac{21}{26}.$$

Now, from (21) we obtain the following bound

$$\max_{i=1,2} \{ | \Delta\pi_i | \} < 0.3333.$$

The above bound defines an admissible transition rate region $\mathcal{R}_2$ as

$$\mathcal{R}_2 = \left\{ (\pi_1, \pi_2) \middle| |\pi_1 - 2| < 0.3333, \right.$$
$$\left. |\pi_2 - 1| < 0.3333,\ \pi_1 > 0,\ \pi_2 > 0 \right\}$$

This region is also shown in Figure 1 where it can be seen that $\mathcal{R}_2$ is much larger than $\mathcal{R}_1$ since it is obtained with nominal system transition rate values that are not close to the boundary of $\mathcal{R}$. We can conclude that the nominal system $\overline{\mathcal{S}}$, with $\overline{\pi}_1 = \overline{\pi}_2 = 1$, is not as robust against transition rate perturbations as the same system with $\overline{\pi}_1 = 2, \overline{\pi}_2 = 1$.

4. CONCLUSION

In this paper, we considered robust stability of continuous-time Markovian jump linear systems in terms of the perturbed transition rates. Using stochastic Lyapunov functions, a bound on the transition rates was given so that the system remains stochastically stable. An example was given to illustrate the result for scalar systems where there exists a necessary and sufficient condition on the robust stability of the system.

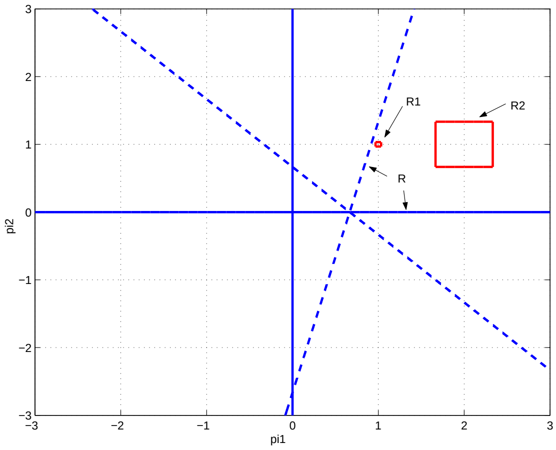

Fig. 1. Robust stability bounds on the transition rates for the scalar Markovian jump linear system with $a_1 = 1/3$, $a_2 = -4/3$. $\mathcal{R}$ is the necessary and sufficient admissible region (π_1, π_2). The nominal system transition rate values $(\overline{\pi}_1 = 1, \overline{\pi}_2 = 1)$ and $(\overline{\pi}_1 = 2, \overline{\pi}_2 = 1)$ are used to obtain $\mathcal{R}_1$ and $\mathcal{R}_2$, respectively.

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