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ROBUST H_∞ FILTERING FOR UNCERTAIN SYSTEM WITH TIME-DELAYED MEASUREMENT

Yunze Cai, Xing He, Xiaoming Xu , Weidong Zhang

Dept. of Automation Shanghai Jiaotong University
Shanghai, 200030 P. R. China
Tel/Fax: +86.21.62933329; Email : yzcai@263.net

Abstract: A robust H_∞ approach to filtering for continuous-time linear uncertain systems with time-delayed measurement is investigated. The filters such that the filtering process remains robustly stable and the transfer function from the disturbance inputs to error state outputs meets the prescribed H_∞-norm upper-bound constraints can be derived and obtained by solving two Riccati equations based on the lemma of robust stability for control system with delay, and the resulting filters are parameterized. Simulation examples prove the method is effective and feasible. *Copyright©2002 IFAC*

Keywords: Robust Stability, Robust Performance, Uncertainty, Time Delay, Riccati Equations

1 INTRODUCTION

All actuation and measurement devices are subject to time delay. Specifically, time delays arise in control actuation devices as well as computation delay in sensor measurement processing. Time delays often are the main cause of instability and poor performance of systems. In recent years a number of authors have devoted their attention to the stabilization of systems with state delays. There exist two main stabilization methods for the uncertain systems with time delay. One is Riccati equation (K.zhou and P.P.,1988; Shen et al.,1991;Lee et al, 1994;Ge et al,1996), and the other is Linear Matrix Inequality (LMI) (Lee et al,1994;Li et al,1996;Xu et al,2000). In K.zhou and P.P. (1988), a sufficient condition for memoryless stabilization of a class of uncertain linear systems with a variable-state delay and norm-bounded time-varying uncertainties is derived in term of an algebraic Riccati equation. Modified Riccati equation based design techniques were developed for feedback control of systems with delay in the system state dynamics in Lee et al (1994) and Ge et al (1996) and their references therein. LMI feasibility approaches for system stabilization with time delay were developed in Li et al (1996). Pila et al (1999) studied the problem of robust H_∞ filtering for linear certain systems with delay in measurement process using difference Riccati equation. However, the researches literature on the robust H_∞ filtering for systems with time-delayed measurement is few.

In this paper the problem of robust H_∞ filtering for linear uncertain systems with time-delayed measurement process is considered. The purpose is to realize a simple filter that assures a prescribed estimation performance for any disturbance in L_2. Based on the lemma of robust stability for control system with delay, the sufficient conditions of the existence of the robust filters are obtained. Moreover, the filter is characterized in the terms of positive solutions of two algebraic Riccati-like equations. It should be pointed out that, the filters are independent on the time-delay length for both time-delayed measurement and uncertain time-delayed measurement.

Notations: $\Re^n$ denotes the n dimensional Euclidean space, $\Re^{n\times m}$ is the set of all $n\times m$ real matrices. For any two symmetric matrices $\mathbf{X},\mathbf{Y}$ in $\Re^{n\times n}$, $\mathbf{X}>\mathbf{Y}(\mathbf{X}\geq\mathbf{Y})$ means that $\mathbf{X}-\mathbf{Y}$ is a positive-definite (semi-positive-definite) matrix. The H_∞ norm of a relational transfer function $H(s)$ is $\|H(s)\|_\infty$. $H_{zw}(s)$ is the transfer function from w to z and $\|\cdot\|$ means induced matrix 2-norm, and L_2 stands for the space of square integral vector functions. $\mathbf{I}$ is the identity matrix with appropriate dimensions and the superscript "T" stands for matrix transposition.

2 PROBLEM FORMULATIONS

Consider the uncertain time-delayed system.

$$\begin{aligned}
\dot{\mathbf{x}}(t) &= (\mathbf{A}+\Delta\mathbf{A}(t))\mathbf{x}(t)+\mathbf{B}\mathbf{w}(t)\\
\mathbf{y}(t) &= (\mathbf{C}+\Delta\mathbf{C}(t))\mathbf{x}(t)+(\mathbf{C}_d+\Delta\mathbf{C}_d(t))\\
&\quad\times\mathbf{x}(t-\tau)+\mathbf{D}\mathbf{w}(t)\\
\mathbf{x}(t) &= f(t),\qquad t=[-\tau,0]
\end{aligned}\tag{1}$$

where $\mathbf{x}(t)\in\Re^n$ is the system state, $\mathbf{y}(t)\in\Re^p$ is the controlled output, $\mathbf{w}(t)\in\Re^q$ is the disturbance input which is assumed to be an arbitrary signal in L_2. $\mathbf{A},\mathbf{B},\mathbf{C},\mathbf{C}_d$ and $\mathbf{D}$ are known constant real matrices of appropriate dimensions that describe the nominal system, $\Delta\mathbf{A}(t),\Delta\mathbf{C}(t)$ and $\Delta\mathbf{C}_d(t)$ are real time-varying matrix functions representing norm-bounded parameter uncertainties. τ denotes state delay, and $f(t)$ is a continuous vector-valued initial function.

The admissible parameter uncertainties in this paper is assumed to be modeled as

$$[\Delta\mathbf{A}(t)\quad\Delta\mathbf{C}(t)\quad\Delta\mathbf{C}_d(t)]=[\mathbf{H}_1\quad\mathbf{H}_2\quad\mathbf{H}_3]\mathbf{F}(t)\mathbf{E}$$

$\mathbf{H}_1,\mathbf{H}_2,\mathbf{H}_3,\mathbf{E}$ are known constant matrices with appropriate dimensions, $\mathbf{F}(t)$ is unknown matrix but satisfies the following:

$$\mathbf{F}^T(t)\mathbf{F}(t)\leq\mathbf{I}\tag{2}$$

We will design the filer such as

$$\dot{\mathbf{x}}_e(t)=\mathbf{A}_e\mathbf{x}_e(t)+\mathbf{K}_e\mathbf{y}(t),\mathbf{x}_e(0)=0\tag{3}$$

where $\mathbf{x}_e(t)$ denotes the state estimation, and $\mathbf{A}_e$ and $\mathbf{K}_e$ are the filter parameters to be estimated.

The state estimation error is

$$\mathbf{e}(t)=\mathbf{x}(t)-\mathbf{x}_e(t)$$

According to(1)and(3), we get the error dynamics

$$\begin{aligned}
\dot{\mathbf{e}}(t) &= \mathbf{A}_e\mathbf{e}(t)+(\mathbf{A}-\mathbf{A}_e-\mathbf{K}_e\mathbf{C}+\Delta\mathbf{A}(t)-\\
&\quad\mathbf{K}_e\Delta\mathbf{C}(t))\mathbf{x}(t)-\mathbf{K}_e(\mathbf{C}_d+\Delta\mathbf{C}_d(t)\\
&\quad)\mathbf{x}(t-\tau)+(\mathbf{B}-\mathbf{K}_e\mathbf{D})\mathbf{w}(t)\\
\mathbf{z}_e &= \mathbf{L}\mathbf{e}(t)
\end{aligned}\tag{4}$$

where $\mathbf{z}_e$ is the output of error, $\mathbf{L}$ is a known constant matrix. In the following sections, the variable t will be omitted if there is no risk of misunderstanding.

Therefore, by (1) and (4), it is easy to obtain the augmented system

$$\begin{aligned}
\dot{x}(t) &= \hat{\mathbf{A}}_c x+\hat{\mathbf{A}}_d x(t-\tau)+\mathbf{B}_c\mathbf{w}\\
\mathbf{z} &= \hat{\mathbf{L}}x
\end{aligned}\tag{5}$$

where $x=[\mathbf{x}^T\quad\mathbf{e}^T]^T$,

$$\hat{A}_c=\mathbf{A}_c+\Delta\mathbf{A}_c=\mathbf{A}_c+\mathbf{H}_c\mathbf{F}\mathbf{E}_c,$$

$$\hat{\mathbf{A}}_d=\overline{\mathbf{A}}_d+\Delta\overline{\mathbf{A}}_d=\overline{\mathbf{A}}_d+\mathbf{K}_e\overline{\mathbf{H}}_3\mathbf{F}\mathbf{E}_c,$$

$$\mathbf{A}_c=\begin{bmatrix}\mathbf{A} & 0\\ \mathbf{A}-\mathbf{A}_e-\mathbf{K}_e\mathbf{C} & \mathbf{A}_e\end{bmatrix},\mathbf{B}_c=\begin{bmatrix}\mathbf{B}\\ \mathbf{B}-\mathbf{K}_e\mathbf{D}\end{bmatrix}$$

$$\overline{\mathbf{A}}_d=\begin{bmatrix}0 & 0\\ -\mathbf{K}_e\mathbf{C}_d & 0\end{bmatrix},\mathbf{H}_c=\begin{bmatrix}\mathbf{H}_1 & 0\\ \mathbf{H}_1-\mathbf{K}_e\mathbf{H}_2 & 0\end{bmatrix},$$

$$\overline{\mathbf{H}}_3=\begin{bmatrix}0 & 0\\ -\mathbf{H}_3 & 0\end{bmatrix},\mathbf{E}_c=\begin{bmatrix}\mathbf{E} & 0\\ 0 & 0\end{bmatrix},\hat{\mathbf{L}}=[0\quad\mathbf{L}].$$

The objective of this paper is to design the filer (3), such that the filtering system (5) is asymptotically stable and its H_∞ norm less than a prescribed lever g, that is $\|H_{zw}\|_\infty \le g$, for all admissible uncertainties satisfying (2).

When $\Delta\mathbf{C}_d = 0$, system (1) changes into

$$\Sigma: \begin{array}{l} \dot{\mathbf{x}}(t) = (\mathbf{A} + \Delta\mathbf{A}(t))\mathbf{x}(t) + \mathbf{Bw}(t) \\ \mathbf{y}(t) = (\mathbf{C} + \Delta\mathbf{C}(t))\mathbf{x}(t) + \mathbf{C}_d\mathbf{x}(t-t) + \mathbf{D}w(t) \\ \mathbf{x}(t) = \mathbf{f}\,(t), \qquad t = [-t, 0] \end{array}$$

The admissible uncertainties are norm-bounded form

$$[\Delta\mathbf{A}(t) \quad \Delta\mathbf{C}(t)] = [\mathbf{H}_1 \quad \mathbf{H}_2]\mathbf{F}(t)\mathbf{E}$$

$\mathbf{F}(t)$ is an unknown time-varying matrix function satisfying (2). We will design the filer such as (3).

Therefore, it is easy to obtain the augmented system

$$\begin{array}{l} \dot{x}(t) = \hat{\mathbf{A}}_c x + \hat{\mathbf{A}}_d x(t-t) + \mathbf{B}_c\mathbf{w} \\ \mathbf{z} = \hat{\mathbf{L}}x \end{array} \qquad (6)$$

where $\hat{\mathbf{A}}_c = \mathbf{A}_c + \Delta\mathbf{A}_c = \mathbf{A}_c + \mathbf{H}_c\mathbf{F}\mathbf{E}_c$, $\hat{\mathbf{A}}_d = \begin{bmatrix} 0 & 0 \\ -\mathbf{K}_e\mathbf{C}_d & 0 \end{bmatrix}$, $x, \mathbf{A}_c, \mathbf{B}_c, \mathbf{H}_c, \mathbf{E}_c$ and $\hat{\mathbf{L}}_c$ are the same to the system (5).

3 MAIN RESULTS FOR $\Delta\mathbf{C}_d = 0$

Lemma 1 (Wang, et al, 1994): Given matrices $\mathbf{H}, \mathbf{F}(t)$ and $\mathbf{E}$ of appropriate dimensions, then for any $\mathbf{F}(t)$ satisfying $\|\mathbf{F}(t)\| \le 1$ and $e_1 > 0$, we have the following inequality

$$\mathbf{HFE} + \mathbf{E}^T\mathbf{F}^T\mathbf{H}^T < e\mathbf{HH}^T + e^{-1}\mathbf{E}^T\mathbf{E}$$

Lemma 2 (Pila, et al, 1999): If there are positive-definite matrices ***P*** and ***Q*** such that

$$\begin{array}{l} \hat{\mathbf{A}}_c^T\mathbf{P} + \mathbf{P}\hat{\mathbf{A}}_c + \mathbf{P}\hat{\mathbf{A}}_d\mathbf{Q}^{-1}\hat{\mathbf{A}}_d^T\mathbf{P} + \mathbf{Q} + \hat{\mathbf{L}}^T\hat{\mathbf{L}} + \\ g^{-2}\mathbf{PB}_c\mathbf{B}_c^T\mathbf{P} < 0 \end{array} \qquad (7)$$

holds, then system (5) is stable and $\|H_{zw}\|_\infty \le g$.

It is easy to see that Lemma 2 is also fit for the system (6).

For sake of simplicity, we give the following definitions:

$$\mathbf{M} = g^{-2}\mathbf{BB}^T + e\mathbf{H}_1\mathbf{H}_1^T, \quad \mathbf{N} = g^{-2}\mathbf{DB}^T + e\mathbf{H}_2\mathbf{H}_1^T,$$

$$\mathbf{R} = g^{-2}\mathbf{DD}^T + e\mathbf{H}_2\mathbf{H}_2^T + \mathbf{C}_d\mathbf{Q}_{11}^{-1}\mathbf{C}_d^T,$$

$$\mathbf{A}_e = \mathbf{A} + \mathbf{MP}_1, \quad \mathbf{C}_e = \mathbf{C} + \mathbf{NP}_1,$$

$$\hat{\mathbf{A}} = \mathbf{A}_e - \mathbf{N}^T\mathbf{R}^{-1}\mathbf{C}_e$$

Theorem 1: Let d_1, d_2 be sufficiently small positive constants. If there exists positive scalar e and a positive-definite block-diagonal matrix $\mathbf{Q} = \begin{bmatrix} \mathbf{Q}_{11} & \\ & \mathbf{Q}_{22} \end{bmatrix}$ such that Riccati equations

$$\begin{array}{l} \mathbf{A}^T\mathbf{P}_1 + \mathbf{P}_1\mathbf{A} + \mathbf{P}_1\mathbf{MP}_1 + e^{-1}\mathbf{E}^T\mathbf{E} + \mathbf{Q}_{11} \\ + d_1\mathbf{I} = 0 \end{array} \qquad (8)$$

$$\begin{array}{l} \hat{\mathbf{A}}^T\mathbf{P}_2 + \mathbf{P}_2\hat{\mathbf{A}} + \mathbf{P}_2(\mathbf{M} - \mathbf{N}^T\mathbf{R}^{-1}\mathbf{N})\mathbf{P}_2 + \mathbf{L}^T\mathbf{L} \\ + \mathbf{Q}_{22} + d_2\mathbf{I} = 0 \end{array} \qquad (9)$$

have symmetric positive-definite solutions $\mathbf{P}_1 > 0$ and $\mathbf{P}_2 > 0$ respectively, then the filter with parameters

$$\mathbf{K}_e = \mathbf{N}^T\mathbf{R}^{-1}, \qquad (10)$$

$$\mathbf{A}_e = \mathbf{A}_e - \mathbf{K}_e\mathbf{C}_e \qquad (11)$$

will be such that, for all admissible parameter uncertainties, the system (6) is asymptotically stable and H_∞ norm satisfying $\|H_{zw}\|_\infty \le g$.

Proof: It does not matter that let $\mathbf{P} = \begin{bmatrix} \mathbf{P}_1 & \\ & \mathbf{P}_2 \end{bmatrix}$.

For $e > 0, \mathbf{P} > 0$, from Lemma1, it is easy to know

$$\begin{array}{l} (\mathbf{A}_c + \Delta\mathbf{A}_c)^T\mathbf{P} + \mathbf{P}(\mathbf{A}_c + \Delta\mathbf{A}_c) + \hat{\mathbf{L}}^T\hat{\mathbf{L}} + \\ \mathbf{P}\hat{A}_d\mathbf{Q}^{-1}\hat{\mathbf{A}}_d^T\mathbf{P} + g^{-2}\mathbf{PB}_c\mathbf{B}_c^T\mathbf{P} + \mathbf{Q} \le A_c^T\mathbf{P} + \\ \mathbf{PA}_c + \mathbf{P}\hat{\mathbf{A}}_d\mathbf{Q}\hat{\mathbf{A}}_d^T\mathbf{P} + e\mathbf{PH}_c\mathbf{H}_c^T\mathbf{P} + \\ e^{-1}\mathbf{E}_c^T\mathbf{E}_c + \hat{\mathbf{L}}^T\hat{\mathbf{L}} + g^{-2}\mathbf{P}\hat{\mathbf{B}}\hat{\mathbf{B}}^T\mathbf{P} \end{array} \qquad (12)$$

Let the right side of (12) $= y := \begin{bmatrix} y_{11} & y_{12} \\ y_{12}^T & y_{22} \end{bmatrix}$, where

$$\begin{array}{l} y_{11} = \mathbf{A}^T\mathbf{P}_1 + \mathbf{P}_1\mathbf{A} + \mathbf{P}_1(g^{-2}\mathbf{BB}^T + \\ e\mathbf{H}_1\mathbf{H}_1^T)\mathbf{P}_1 + e^{-1}\mathbf{E}^T\mathbf{E} + \mathbf{Q}_{11} \end{array} \qquad (13)$$

$$\begin{array}{l} y_{12} = (\mathbf{A} - \mathbf{A}_e - \mathbf{K}_e\mathbf{C})^T\mathbf{P}_2 + \mathbf{P}_1(g^{-2}\mathbf{B}(\mathbf{B} - \\ \mathbf{K}_e\mathbf{D})^T + e\mathbf{H}_1(\mathbf{H}_1 - \mathbf{K}_e\mathbf{H}_2)^T)\mathbf{P}_2 \end{array} \qquad (14)$$

$$\begin{array}{l} y_{22} = \mathbf{A}_e^T\mathbf{P}_2 + \mathbf{P}_2\mathbf{A}_e + \mathbf{P}_2(\mathbf{K}_e\mathbf{C}_d\mathbf{Q}_{11}^{-1}\mathbf{C}_d^T\mathbf{K}_e^T \\ + g^{-2}(\mathbf{B} - \mathbf{K}_e\mathbf{D})(\mathbf{B} - \mathbf{K}_e\mathbf{D})^T + e(\mathbf{H}_1 - \\ \mathbf{K}_e\mathbf{H}_2)(\mathbf{H}_1 - \mathbf{K}_e\mathbf{H}_2)^T)\mathbf{P}_2 + \mathbf{L}^T\mathbf{L} + \mathbf{Q}_{22} \end{array} \qquad (15)$$

Equation (8) means that $y_{11} = -d_1\mathbf{I} < 0$, and the expression of $\mathbf{A}_e$ in (11) and equation (14) imply $y_{12} = 0$. According to the expression (10) and (11), it is not difficult to obtain $y_{22} = -d_2\mathbf{I} < 0$.

Now we have the conclusion that $y < 0$, and thus from Lemma 2 that there exist positive-definite matrices ***P*** and ***Q***, such that the expression (7) holds and the system(6)is stable and $\|H_{zw}\|_\infty \le g$. This proves Theorem 1.

From the proof of Theorem 1, we can see that the robust filters design for time-delay systems with parameter uncertainty is mainly based on the positive-definite solutions for two algebraic Riccatic equation (8) and (9), and the parameters of the filter can guarantee the robust performance of systems independent of delay and uncertainties. Two positive constants d_1, d_2 are just to ensure that $y_{11} < 0, y_{22} < 0$ hold.

4 MAIN RESULTS FOR $\Delta\mathbf{C}_d \neq 0$

Lemma 3 (Mahmoud, et al, 2000): Given matrices $\overline{\mathbf{H}}_3, \mathbf{E}_c, \mathbf{F}(t)$ and ***Q*** of appropriate dimensions and with ***Q*** positive-definite, $\mathbf{F}^T(t)\mathbf{F}(t) \le \mathbf{I}$, then for any $e_2 > 0$ satisfying $\mathbf{Q} - e_2\mathbf{E}_c^T\mathbf{E}_c > 0$, the following inequality holds:

$$(\overline{\mathbf{A}}_d + \overline{\mathbf{H}}_3\mathbf{F}\mathbf{E}_c)\mathbf{Q}^{-1}(\overline{\mathbf{A}}_d + \overline{\mathbf{H}}_3\mathbf{F}\mathbf{E}_c)^T \le e_2^{-1}\overline{\mathbf{H}}_3\overline{\mathbf{H}}_3^T + \overline{\mathbf{A}}_d(\mathbf{Q} - e_2\mathbf{E}_c^T\mathbf{E}_c)^{-1}\overline{\mathbf{A}}_d^T$$

For sake of simplicity, we also give the following definitions:

$$\mathbf{M} = e_1^{-1}\mathbf{H}_1\mathbf{H}_1^T + g^{-2}\mathbf{B}\mathbf{B}^T,$$
$$\mathbf{N} = e_1^{-1}\mathbf{H}_2\mathbf{H}_1^T + g^{-2}\mathbf{D}\mathbf{B}^T,$$
$$\mathbf{R} = g^{-2}\mathbf{D}\mathbf{D}^T + e_1^{-1}\mathbf{H}_2\mathbf{H}_2^T + e_2^{-1}\mathbf{H}_3\mathbf{H}_3^T + \mathbf{C}_d(\mathbf{Q}_{11} - e_2\mathbf{E}^T\mathbf{E})^{-1}\mathbf{C}_d^T$$
$$\mathbf{A}_e = \mathbf{A} + \mathbf{M}\mathbf{P}_1, \mathbf{C}_e = \mathbf{C} + \mathbf{N}\mathbf{P}_1,$$
$$\hat{\mathbf{A}} = \mathbf{A}_e - \mathbf{N}^T\mathbf{R}^{-1}\mathbf{C}_e.$$

Theorem 2: Let d_1, d_2 be sufficiently small positive constants. If there exist positive scalars e_1, e_2 and a positive-definite block-diagonal matrix $\mathbf{Q} = \begin{bmatrix} \mathbf{Q}_{11} & \\ & \mathbf{Q}_{22} \end{bmatrix}$ such that Riccati equations

$$\mathbf{A}^T\mathbf{P}_1 + \mathbf{P}_1\mathbf{A} + \mathbf{P}_1\mathbf{M}\mathbf{P}_1 + e_1\mathbf{E}^T\mathbf{E} + \mathbf{Q}_{11} + d_1\mathbf{I} = 0 \quad (16)$$

$$\hat{\mathbf{A}}^T\mathbf{P}_2 + \mathbf{P}_2\hat{\mathbf{A}} + \mathbf{P}_2(\mathbf{M} - \mathbf{N}^T\mathbf{R}^{-1}\mathbf{N})\mathbf{P}_2 + \mathbf{L}^T\mathbf{L} + \mathbf{Q}_{22} + d_2\mathbf{I} = 0 \quad (17)$$

have symmetric positive-definite solutions $\mathbf{P}_1 > 0$ and $\mathbf{P}_2 > 0$ respectively, then the filter's parameters

$$\mathbf{K}_e = \mathbf{N}^T\mathbf{R}^{-1}, \ \mathbf{A}_e = \mathbf{A}_e - \mathbf{K}_e\mathbf{C}_e$$

will be such that, for all admissible parameter uncertainties, the system(5)is asymptotically stable and H_∞ norm satisfying $\|H_{zw}\|_\infty \le g$.

Proof: It does not matter that let $\mathbf{P} = \begin{bmatrix} \mathbf{P}_1 & \\ & \mathbf{P}_2 \end{bmatrix}$.
For $e_1 > 0$, $\mathbf{P} > 0$, $e_2 > 0$ and positive-definite block-diagonal matrix $\mathbf{Q} = \mathbf{Q}^T > 0$, satisfying $\mathbf{Q} - e_2\mathbf{E}_c^T\mathbf{E}_c > 0$, from Lemma 1 and Lemma 3, it is easy to know that

$$(\mathbf{A}_c + \Delta\mathbf{A}_c)^T\mathbf{P} + \mathbf{P}(\mathbf{A}_c + \Delta\mathbf{A}_c) + \mathbf{P}(\overline{\mathbf{A}}_d + \Delta\overline{\mathbf{A}}_d)\mathbf{Q}^{-1}(\overline{\mathbf{A}}_d^T + \Delta\overline{\mathbf{A}}_d^T)\mathbf{P} + \hat{\mathbf{L}}^T\hat{\mathbf{L}} + g^{-2}\mathbf{P}\mathbf{B}_c \times \mathbf{B}_c^T\mathbf{P} + \mathbf{Q} \le \mathbf{A}_c^T\mathbf{P} + \mathbf{P}\mathbf{A}_c + \mathbf{P}\overline{\mathbf{A}}_d(\mathbf{Q} - e_2\mathbf{E}_c^T\mathbf{E}_c)^{-1}\overline{\mathbf{A}}_d^T\mathbf{P} + e_1^{-1}\mathbf{P}\mathbf{H}_c\mathbf{H}_c^T\mathbf{P} + e_1\mathbf{E}_c^T\mathbf{E}_c + \hat{\mathbf{L}}^T\hat{\mathbf{L}} + e_2^{-1}\mathbf{P}\overline{\mathbf{H}}_3\overline{\mathbf{H}}_3^T\mathbf{P} + g^{-2}\mathbf{P}\mathbf{B}_c\mathbf{B}_c^T\mathbf{P} + \mathbf{Q} \quad (18)$$

Let the right side of (18) $= y := \begin{bmatrix} y_{11} & y_{12} \\ y_{12}^T & y_{22} \end{bmatrix}$, then we should prove $y < 0$. The proof is similar to Theorem 1, so we omit it here.

5 NUMERICAL EXAMPLES

In this section, two numerical examples are given to demonstrate the applicability of the proposed approach.

Example 1. Consider the linear uncertain time-delayed system Σ with parameters given by

$$\mathbf{A} = \begin{bmatrix} -2 & 1 \\ -5 & -10 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$
$$\mathbf{C} = [1 \quad 0], \mathbf{C}_d = [1 \quad 0.5], \mathbf{D} = [1 \quad 0], t = 0.5,$$
$$\mathbf{H}_1 = [0.5 \quad 1]^T, \mathbf{H}_2 = 0.1, \mathbf{F} = 1, \mathbf{E} = [-1 \quad -1],$$
$$\mathbf{L} = [0.1 \quad 1], \ \mathbf{f}(t) = 0.5, \quad t \in [-0.5, 0], g = 1.$$

According to the Theorem 1, let $e = 0.1$,

$$\mathbf{Q}_{11} = \begin{bmatrix} 0.3 & -0.5 \\ -0.5 & 1 \end{bmatrix}, \mathbf{Q}_{22} = \begin{bmatrix} 0.3 & -0.5 \\ -0.5 & 3 \end{bmatrix}$$

then the filter with parameters is

$$\mathbf{A}_e = \begin{bmatrix} -1.9357 & 1.0527 \\ -4.8791 & -9.9005 \end{bmatrix}, \mathbf{K}_e = \begin{bmatrix} 0.0032 \\ 0.0003 \end{bmatrix}$$

It is easily to verify that the specified robust stability as well as H_∞ disturbance rejection constraints is achieved.

Example 2: Consider the uncertain time-delayed measurement system (1) with

$$\mathbf{A} = \begin{bmatrix} -2 & 1 \\ -5 & -10 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$
$$\mathbf{C} = [1 \quad 0], \mathbf{C}_d = [1 \quad 0.5], \mathbf{D} = [1 \quad 0], t = 0.5,$$
$$\mathbf{H}_1 = [0.5 \quad 1]^T, \mathbf{H}_2 = 1, \mathbf{H}_3 = 0.1, \mathbf{F} = 1,$$
$$\mathbf{E} = [1 \quad -1], \mathbf{L} = [0.1 \quad 1],$$
$$\mathbf{f}(t) = 0.5, \qquad t \in [-0.5, 0]$$

Select the same $\mathbf{Q}_{11}$ and $\mathbf{Q}_{22}$ to Example 1, and let $e_1 = 10, e_2 = 0.1$, $g = 1$, then it is easily seen that $\mathbf{Q} - e_2 \mathbf{E}_c^T \mathbf{E}_c > 0$, then according to Theorem 2, the filer such as (3) is

$$\mathbf{A}_e = \begin{bmatrix} -2.0691 & 1.0011 \\ -5.0293 & -9.9965 \end{bmatrix}, \mathbf{K}_e = \begin{bmatrix} 0.0730 \\ 0.0352 \end{bmatrix}$$

It is also easily to verify that the specified robust stability as well as H_∞ disturbance rejection constraints is achieved.

6 CONCLUSIONS

In terms of the solution of Riccati –like equation, the thesis propose design methods of robust H_∞ filter for uncertain systems with time-delayed measurement. Two types of uncertainty have been considerer here. One is no uncertainty in time-delayed measurement, and another is the uncertainty in the time-delayed measurement. Sufficient conditions are derived which guarantee the stability of the uncertain time-delay system. Filters such that the underlying system is robust stable and has a robust H_∞ performance for all admissible unknown parameter uncertainty and time-delay are designed. The simulation results have shown the approached methods are effective and feasible.

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