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CONTROL OF THE SIMPLEST WALKING MODEL WITH LAMBDA MODEL

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Abstract: The analysis of the simplest biped walking machine can provide much insight into dynamics and control of human gait. The problem, in the sagittal plane, consists to control the impulse force under the swing foot and the torque between the legs in order to reach a stable limit cycle. The main contribution of this paper concerns the hierarchical control based on the physiological Lambda model and a pattern generator working with intermittent data. The feedback is tuned in order to obtain oscillations at the walking frequency. Simulation results present stable limit cycle. *Copyright © 2002 IFAC*

Keywords: walking, nonlinear, torque control, limit cycles, human.

1. INTRODUCTION

The analysis of biped gait motion provide much insight into the dynamics and control of human gait. The simplest model of bipedal locomotion is the ‘compass-gait’. It could walk in a dynamically stable manner (Miura and Shimoyarna, 1984). McGeer’s (1990) described one of them with semicircular feet and a point mass at the hip walking down an inclined plane The walking pattern is generated by passive interaction of gravity and inertia. Garcia et al. (1998) investigated a similar machine walking down an inclined plane, again under gravity. They demonstrated how the variation of one single parameter drives the system into bifurcation and chaotic step motions. Walking robots with more degrees of freedom have been studied. For example, Ching-Long S. et al. (1996) realized a 7 dof biped robot which is able to walk with a speed of 20cm/s. But the simplest model can give greater insight into human motion than more complex models. In this paper, the simplest walking model is considered without inclined plane. In order to recover energy lost, it must generated impulsive force under the feet, and it must applied torque between the legs The motion is confined to the sagittal plane, i.e. the system is stable in the lateral plan. The section 1 presents the mechanical model.

The control of a biped walking machine needs a trajectory generation pattern and a motor control. Qiang H. et al. (2001) experimented a method for planning walking patterns for a complicated biped robot. Tzafesta et al.(1998) presented the comparison between the control of a 5-link biped robot with computer torque control and sliding mode robust control. These papers are inspired by theories of automatic control, rather than physiological data.

An alternative approach is the Lambda model which is consistent with neurophysiological experimental data (Feldman, 1986). This model has successfully been used to simulate voluntary movements of the arm (Levin and Dimov, 1997) and jaw motion (Laboissière et al., 1996). The main advantage of the Lambda model is that complex control signals and system models are not required for movement (Gribble et al., 1998). Recently, the authors have demonstrated that Lambda-model is quite appropriate for postural control simulation (Micheau et al., 2001). On the other hand, recent papers focus on the possibility that the cerebellum contains an internal model of the motor system (Wolpert et al., 1998). These schemes imply complicated computations for the central nervous system to realise system inverse control. An alternative could be the ‘open-loop intermittent feedback optimal control’ (Ronco et al, 1998). In this scheme, system model and past information are used to compute open-loop actions.

The main contribution of this paper is to present a control based on the Lambda model as a torque control and a pattern generator working with intermittent data. The feedback law is tuned in order to obtain oscillation at the walking frequency. The section 2 presents the law used to maintain stable limit cycle. The section 3 presents simulations.

1. MODEL

1.1 The mechanical system

The model, shown on figure 1, has a left leg rigid body and right leg of equal length *L*, of mass *m*, and central inertia *I*, with the leg centre of mass at mid-length, *L*/2. The motion is confined in the sagittal plane. The index 1 refers to the swing leg. The index 2 refers to the stance leg. A concentrated mass, m_e, represents the upper body mass. At this articulation centre, an internal torque *T*, defined as acting from swing leg to stance leg, is inserted.

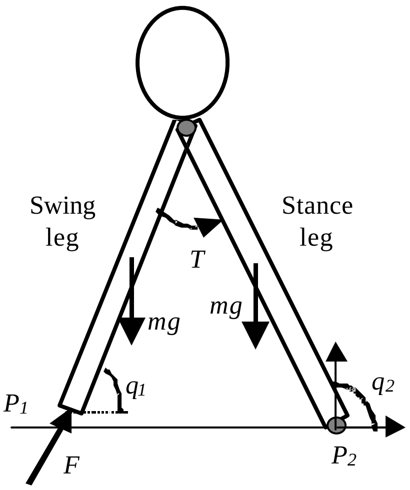


Fig. 1. Model for simulation under external forces and gravity.

A typical walking step is composed of :

i) for $t \in [0;\Delta t]$ an impulsive force, *F*, is applied at P_1;

ii) for $t \in [\Delta t; t_f]$ the torque, *T*, control the swing leg;

iii) at $t = t_f / 2$, the swing leg overtakes the stance leg;

iv) at $t = t_f$, the swing leg hits the ground and becomes the stance leg, and vice-versa (1↔2); a new step starts. To simplify the notation, the next step starts at the time *t*=0 (instead of t_f^+, $2t_f^+$, $3t_f^+$, ...).

1.2 The dynamic model

The equation of motion is written in the form:

$$\mathbf{M}(q)\ddot{q} + \mathbf{F}_i(q,\dot{q}) + \mathbf{F}_g(q) = \mathbf{Bu} \quad (1)$$

where $q = [q_1 \quad q_2]^T$ is the vector of angular coordinates.

The mass matrix $\mathbf{M} = [m_{ij}]$ for *i*=1,2 and *j*=1,2 is given by $m_{21} = m_{12} = -\dfrac{\cos(q_1 - q_2)}{2}$, $m_{11} = \dfrac{I}{mL^2} + \dfrac{1}{4}$, and

$$m_{22} = \frac{I}{mL^2} + \frac{5}{4} + \frac{m_E}{m}.$$

The vector of nonlinear inertial effects (Coriolis force and centrifugal force) is

$$\mathbf{F}_i(q,\dot{q}) = \begin{bmatrix} -\frac{1}{2}\sin(q_1 - q_2)\,\dot{q}_2^2 \\ +\frac{1}{2}\sin(q_1 - q_2)\,\dot{q}_1^2 \end{bmatrix} \quad (2)$$

The vector of gravitational external force is

$$\mathbf{F}_g(q) = \begin{bmatrix} -\frac{1}{2}\frac{g}{L}\cos(q_1) \\ +\left(\frac{m_E}{m} + \frac{3}{2}\right)\frac{g}{L}\cos(q_2) \end{bmatrix} \quad (3)$$

The control inputs, the torque *T* and the impulse force *F*, are written in the input vector $\mathbf{u} = [T \quad F]^T$. The matrix of inputs is

$$\mathbf{B} = [\mathbf{B}_T \quad \mathbf{B}_F] \quad (4)$$

with $\mathbf{B}_T = \begin{bmatrix} \frac{1}{mL^2} & -\frac{1}{mL^2} \end{bmatrix}^T$ and $\mathbf{B}_T = \begin{bmatrix} 0 & \frac{\sin(q_1 - q_2)}{mL} \end{bmatrix}^T$

1.3 Modelling of the ground impact

The collision between the swing foot and the ground occurs at the time t_f defined by the following geometric condition :

$$q_1(t_f) + q_2(t_f) = p \quad (5)$$

When the swing foot hits the ground, the vertical and horizontal speeds of the swing foot is forced to zero, $\dot{p}_1(t_f^+) = \mathbf{0}$, and the names of the two legs are permuted with the matrix $\mathbf{P} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$:

$$q(t_f^+) = \mathbf{P}q(t_f^-) \quad (6)$$

The angular speeds just after the collision are computed with the transition rule (Paulin, 2001):

$$\dot{q}(t_f^+) = \mathbf{P}\left[\dot{q}(t_f^-) - \mathbf{M}^{-1}\mathbf{J}^T\left(\mathbf{J}\mathbf{M}^{-1}\mathbf{J}^T\right)^{-1}\dot{p}_1(t_f^-)\right] \quad (7)$$

where $\dot{p}_1 = \begin{bmatrix} -L\sin(q_2)\dot{q}_2 + L\sin(q_1)\dot{q}_1 \\ L\cos(q_2)\dot{q}_2 + L\cos(q_1)\dot{q}_1 \end{bmatrix}$ is the speed of the swing foot. The matrix **J** is the Jacobian defined by

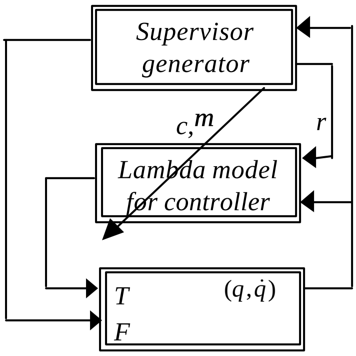
$$\mathbf{J} = \frac{\partial \dot{p}_1}{\partial \dot{q}} = \begin{bmatrix} L\sin(q_1) & -L\sin(q_2) \\ L\cos(q_1) & L\cos(q_2) \end{bmatrix} \quad (8)$$

2. CONTROL

2.1 The objectives

The first objective is to reach and maintain a stable limit cycle characterised by a step length of X meters and a step duration of t_f seconds.

The second objective is to control the system with the Lambda model. The figure 2 presents the structure of the control system. The physiological Lambda model, presented as an adaptive controller, realises a feedback loop based on position (q), speed ($\dot{q}$), reference (r) and adjustable parameters (c,m). The section 2.5 explains that this function is performed by motoneurones. The supervisor which could represent the central nervous system generates the command of open-loop impulse force F, the reference (r) and parameters (c,m). These signals are computed by the generator with intermittent information obtained after the ground collision.



Mechanical system

Fig. 2. Hierarchical structure of the control system

2.2 Simplified model

In order to design the control strategy a simplified linearized model is written. The simplifications are based on the analysis by simulation of a typical human gait (Bourassa et al., 2000).

For the swing phase, the stance leg is uncontrollable, $\ddot{q}_2 = 0$, and its trajectory only depends from the initial conditions:

$$q_2(t) = \dot{q}_2(\Delta t)(t - \Delta t) + q_2(\Delta t) \text{ for } t \in \left[\Delta t; t_f\right] \tag{9}$$

For the swing phase, the swing leg is controllable and can be written as

$$J_m \ddot{q}_1 = T \tag{10}$$

with $J_m = \dfrac{mL^2\left[m_{11}m_{22} - \frac{1}{4}\right]}{m_{22} - \frac{1}{2}}$ the equivalent inertia because $m_{21} = m_{12} \approx -1/2$ with an error of 4%.

2.3 The initial conditions

In the case of a one periodic limit cycle, the angular initial conditions after the ground collision and for $\Delta t \to 0$ must be :

$$q_1(0) = \operatorname{acos}\left(\frac{X}{2L}\right) \tag{11}$$

$$q_2(0) = \boldsymbol{p} - q_1(0) \tag{12}$$

The objective is to reach the collision condition (5) at the desired time t_f. This leads to

$$\dot{q}_2(\Delta t) = \frac{\boldsymbol{p} - q_1(0)}{t_f - \Delta t} \tag{13}$$

According to the equation (7) $\dot{q}_1(0) \approx \dot{q}_2(t_f^-)$ if the velocity increment are neglected. The equation (9) clearly shows that $\dot{q}_2(t_f) = \dot{q}_2(\Delta t)$. Moreover, if the initial impulse force is neglected and $\Delta t \to 0$, the velocity initial conditions can be approximated by

$$\dot{q}_1(0) \approx \dot{q}_2(0) \approx \frac{\boldsymbol{p} - q_1(0)}{t_f} \tag{14}$$

Because the simplified linearized model of the mechanical systems was used in this section, the equations (11), (12) and (14) only give an approximation of the initial conditions.

2.4 Control of the impulse force

The impulse force F is used to reach the desired angular velocity (13). In other words, the impulse force is used to adjust the trajectory of the stance leg by controlling the initial condition, $\dot{q}_2(\Delta t)$.

According to equation (1), after a constant impulse force F for $t \in [0;\Delta t]$ the velocity increments are given by :

$$\dot{q}(\Delta t) = \dot{q}(0^+) + \mathbf{M}^{-1}\left(q\left(0^+\right)\right)\mathbf{B}_F F + \boldsymbol{e} \tag{15}$$

where $\boldsymbol{e} \to 0$ for $\Delta t \to 0$. In order to reach this initial angular velocity condition (13), the equation (15) is used to give the expression of the positive impulse force for $t \in [0;\Delta t]$:

$$F = \frac{m_{11}m_{22} - m_{12}^2}{m_{11}} \frac{ML}{\Delta t \sin(q_1 - q_2)} [\Delta \dot{q}_2]^+ \tag{16}$$

where $[X]^+ = 0$ when $X < 0$.

2.5 Lambda Model

The fig. 3 presents the lambda model of feedback law implemented. The muscles are controlled by two variables, l and m, specified by the central nervous system (Feldman, 1986). The variable l is the static threshold of motoneuronal recruitment. The variable m is a time–dimensional coefficient of the rate of change in the joint angle. These variables determine the dynamic threshold of motoneuronal recruitment : $l^* = l - m\dot{q}$. When the angle q exceeds the dynamic threshold, $q > l^*$, there is muscle activation. Both the level of muscle activation and its active torque are increasing positive function of $\left[l^* - q\right]^+$. Only positive torque can be generated. Thus, an extensor muscle and an flexor muscle are used to generate a signed torque T at the joint. The total torque is

$$T = T_e - T_f \tag{14}$$

where the torque of the extensor muscle is

$$T_e = a\left(\exp\left(a[l_e - q_1 - m\dot{q}_1]^+\right) - 1\right) \tag{15}$$

and the torque of the flexor muscle is

$$T_f = a\left(\exp\left(a\left[-l_f + q_1 + m\dot{q}_1\right]^+\right) - 1\right) \tag{16}$$

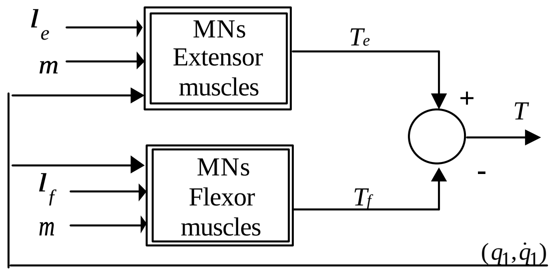


Fig. 3. Feedback law based on Lambda Model.

The equations (14), (15) and (16) give the following equivalent form of the lambda model

$$T = 2a\mathrm{e}^{ac}\sinh[a\,(r - q_1 - m\dot{q}_1)] \tag{17}$$

where $r = \left(\dfrac{l_f + l_e}{2}\right)$ is the reference signal and $c = \left(\dfrac{l_f - l_e}{2}\right)$ the coactivation. The physiological parameter a modulates the nonlinear effect of the hyperbolic sinus.

2.6 Tuning the lambda model

The reference signal with lambda model is usually chosen as ramp shaped. Thus, the reference signal *r(t)* is written as a function of initial conditions :

$$r(t) = q_1(0) + \left(p - 2q_1(0)\right)\frac{t}{t_f} \tag{18}$$

The linearized form of (17) is a state feedback $\mathbf{K} = [K_p \quad K_d]$ on the state variables $\mathbf{x}^T = [q_1 \quad \dot{q}_1]$:

$$T(q_1) \cong K_p r - \mathbf{Kx} \tag{19}$$

where $K_p = 2aa\exp(+ac)$ and $K_d = mKp$.

In order to avoid ground collision before the final time and a perfect symmetric pattern, the feedback law is used to obtain a oscillatory system. The period of oscillations are tuned to step duration. Thus, for $\Delta t \to 0$, the feedback law (19) and the reference (18), the gains must be

$$K_p = J_m\left(\frac{2p}{t_f}\right)^2 \quad \text{and} \quad K_d = 0 \tag{20}$$

which implies the Lambda parameters $c = \dfrac{1}{a}\ln\left(\dfrac{K_p}{2aa}\right)$ and $m = 0$.

2.6 Analysis in Poincaré Section

The Poincaré section is at the start of a step, just after the ground impact and the permutation. The definition of the section is given by equation (5). It reduces the problem in 4D state space to a 3D maps $\mathbf{f}$.

$$x[k+1] = \mathbf{f}\left(x[k]\right) \tag{19}$$

Where $x[k] = \left[q_1(kt_f^+) \quad \dot{q}_1(kt_f^+) \quad q_2(kt_f^+)\right]^T$ is the state just after the ground-collision at the $k^{\underline{\text{th}}}$ step.

A period-one gait cycle is a fixed-point, x^* , of the map defined by $x^* = \mathbf{f}\left(x^*\right)$. A limit cycle is stable if any perturbations to the fixed-point, $dx[k] = x[k] - x^*$, will decay. For sufficiently small perturbations, the perturbations will decrease according to :

$$dx[k+1] = \mathbf{J}dx[k] \tag{20}$$

if the Jacobian $\mathbf{J} = \dfrac{\partial \mathbf{f}}{\partial x}(x^*)$ of the map $\mathbf{f}$ has all of its eigenvalues inside the unit circle.

3. SIMULATIONS

To solve the equations of the kinematics and dynamics of the robot mechanisms, the equations was implemented on Matlab/Simulink with ODE5 and

T_e=0.0001s. The table 1 presents the parameters used for the simulation.

The simplest walking model is configured for a step length of 26cm and a step duration of 0.3s. The initial conditions computed with the equation (11), (12) and (14) are $x[0]=[1.440\ -0.79\ -0.79]^T$. The table 1 presents the parameters of the Lambda model. At each step, the force impulse is computed with equation (16) for $\Delta t = 0.02\text{s}$. Its value fluctuates near 300Nm. The generator computes the reference signal with the equation (18) and the initial angular value of the swing leg $q_1(0)$.

Table 1 Table of parameters

Var.	Values	Var.	Val.
L	1 m	a	10 rad $^{-1}$
m	10 kg	$\boldsymbol{a}$	0.0545 Nm
m_e	40 kg	c	$\pi/4$ rad
I	5 kgm^2		
g	10 m/s^2		

The figure 4 shows the trajectory of the swing foot for a one period limit cycle. The swing foot movement starts at –25cm, lifts high enough to 0.8cm, goes forward, overtakes the stance leg at (0,0) , and lifts high to 7mm before to takeoff at +25cm.

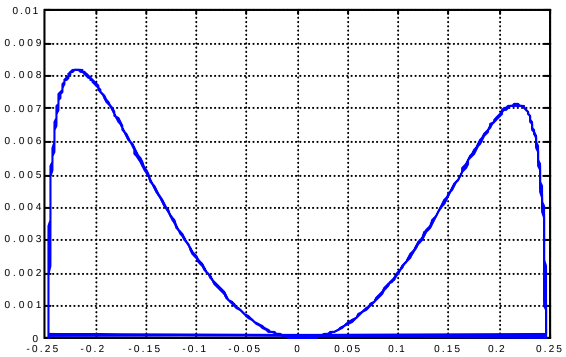


Fig. 4. The trajectory of the swing foot P_1 (x-y coordinates in meters).

The figure 5 is a typical phase diagram of one leg. The Poincaré's section is defined just after the ground-collision. The fixed point is $x^* = [1.447\ -0.774\ -0.781]^T$. The Jacobian matrix has been constructed with a perturbation of δx_j=0.005 for j=1,2,3. The eigenvalues are in the unit circle (-0.629, -0.166 and -0.030).

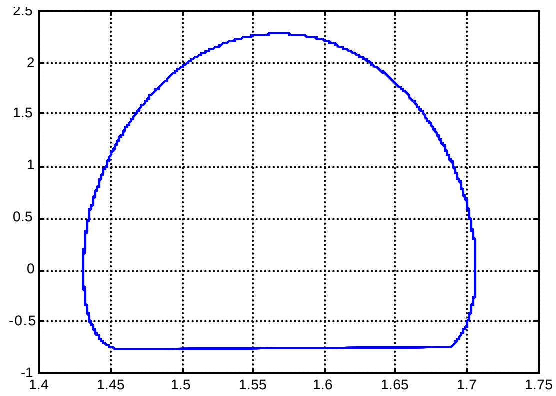


Fig. 5. Phase diagram of $(q_1,\dot{q}_1)$ presents the Limit cycle.

The figure 6 shows that the torque have a sinusoidal waveform of amplitude 240 Nm. From 0ms to 165ms, the torque is positive to accelerate the swing leg. From 165ms to 304ms, the torque is negative to decelerate the swing leg.

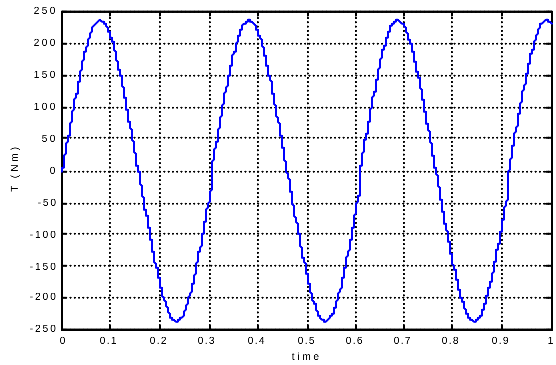


Fig. 6. The torque T (Nm) versus time.

The figure 7 shows the reference signal generated with the inital conditions, the angles of the swing leg and of the stance leg. The swing leg overtakes the stance leg at time 165ms. At the end of the period gait cycle (t_f=0.304 s), the final values become the initial conditions, i.e. the variable names $q_2(t)$ and $q_1(t)$. are permuted. A new reference signal $r(t)$ is computed for the future with the new initial conditions.

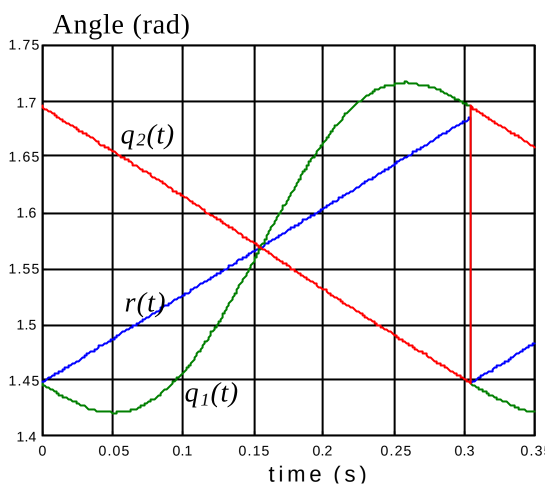


Fig. 7. Angles $q_1(t)$, $q_2(t)$, and $r(t)$ versus time over one step.

3. CONCLUSIONS

The proposed scheme generates gait pattern with intermittent data obtained after the ground collision of the swing leg. The generator computes the reference signal and the open-loop impulse force for the next step. The lambda control is capable to achieve the torque control of the swing leg. It was demonstrated that the linearized Lambda model is equivalent to an adaptable controller. The feedback loop is tuned in order to obtain oscillation at the walking frequency. The impulse force and the lambda controller are tuned in order to maintain the goal of desired step duration and step length. A symmetric gait pattern and a stable limit cycle were simulated. Future works will be to authenticate this model with experimental data.

REFERENCES

Bourassa P., Meier M.R., Micheau P., Buaka P. (2000), approximate solutions for gait simulation and control, *AMAM Symposium*, Montréal, Canada, august 7-12.

Ching-Long S. (1996), The dynamics and control of a biped walking robot with seven degrees of freedom, *J of Dynamic Systems, Measurement, and control*, **118**, 683-690.

Feldman A.G. (1986), Once more on the Equilibrium-point Hypothesis (λ Model) for Motor Control, *J Mot Behav*, **18**: 17-54.

Garcia M., Chatterjee A, Ruina A. Coleman M. (1998), The simplest walking model: stability, complexity and scaling, *J. Biomech Eng*, **120**:281-288.

Gribble PL, Ostry DJ, Sanguineti V, Laboissière R (1998), Are complex control signals required for human arm movement?, *The American Physiological Society*, 1409-1424.

Levin M F, Dimov M (1997), Spatial zones for muscle coactivation and the control of postural stability, *Brain Res*, **757**: 43-59

Micheau P. Kron A, Bourassa P. (2001), Analysis of human postural stability based on the inverted pendulum model with time-delay in feedback, *Proceeding of the ACC*, Arlington, 2297-2298.

Miura and Shimoyarna, (1984), Dynamic walk of biped locomotion, *Inter. Journal of Robotic Research*, **3**(2), 60-74.

McGeer T. (1990) Passive Dynamic walking, *Int. J Robot Res*, **9**(2):62-82.

Puaka P (2001), Modélisation, analyse dynamique et contrôle d'un robot marcheur bipède: cas d'un modèle à deux (quatre) degrés de liberté, *Master thesis*, Université de Sherbrooke, Canada.

Qiang H., Kazuhito Y., Shuuji K., Kenji K., Hirohiko A., Noriho K., Kazuo T. (2001), Planning walking patterns for a biped robot, *IEEE Transactions on robotics and automation*, **17**(3), 280-289.

Ronco E., Arsan T., Gawthrop P.J. (1999), Open-Loop intermittent feedback optimal control: a practical continuous-time GPC, *IEE Control theory and Applications*, **146**(5), 426-425.

Tzafesta S.G., Krikochoritis A.E., Tzafestas C.S. (1996), Robust sliding-mode control applied to 5-link biped robot, *Journal of Intelligent and Robotic Systems*, **15**, 68-133.

Wolpert DM, Miall R C, Kawato M (1998), Internal models in the cerebellum, *Trends in cognitive sciences* **2**:9 338-47.