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GENERAL ADAPTIVE CONTROLS FOR NONLINEARLY PARAMETERIZED SYSTEMS UNDER GENERALIZED MATCHING CONDITION

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Abstract: This paper is devoted to the adaptive control design for a class of nonlinearly parameterized systems assuming the so-called generalized matching condition. A simple adaptive controller with a linear-in-parameter-like structure is designed to account for general parameter-dependent plant nonlinearities. An important feature of our approach is that compactness of parameter sets is not required. Global boundedness of the overall adaptive system and convergence to zero equilibrium state with any prescribed accuracy are established. Our construction technique takes advantage of Lipschitzian properties with respect to the parameter of the plant nonlinearity. *Copyright ©2002 IFAC*

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1. INTRODUCTION

In this paper, we consider the adaptive control problem for a class of nonlinearly parameterized (NP) systems satisfying the so-called generalized matching condition. Without loss of generality, we focus on the second-order case, i.e.

$$\begin{aligned} \dot{x}_1 &= x_2 + \varphi(x_1, \theta), \\ \dot{x}_2 &= u, \end{aligned} \tag{1}$$

where $u \in R$ is the control input, $x = [x_1, x_2]^T$ is the system state. Function $\varphi(x_1, \theta)$ is nonlinear in both the variable x_1 and the unknown parameter $\theta \in R^p$. The problem is to design a stabilizing state-feedback control u such that the state $x_1(t)$ converges to 0. As clarified later in the paper, our approach can be extended to systems of any order in a streamlined manner.

Classically, a useful methodology for designing controllers of this class is the adaptive backstepping method (Krstic *et al.*, 1995), under the assumption of a linear parameterization (LP) in the unknown parameter θ, i.e. the function $\varphi(x_1, \theta)$ in (1) is assumed linear in θ. The basic idea is to design a "stabilizing function", which prescribes a desired behavior for x_2 so that $x_1(t)$ is stabilized. Then, an effective control $u(t)$ is synthesized to regulate x_2 to track this stabilizing function. Very few results, however, are available in the literature that address adaptive backstepping for NP systems of the general form (1) (Kojic *et al.*, 1999). The difficulty here is attributed to two main factors inherent in the adaptive backstepping. The first one is how to construct the stabilizing function for x_1 in the presence of non-

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linear parameterizations. The second one arises from the fact that as the actual control $u(t)$ involves derivatives of the stabilizing function, the later must be constructed in such a way that it does not lead to multiple parameter estimates (or overparameterization) (Krstic *et al.*, 1995). The idea of convexity/concavity-based nonlinear adaptive control (A.M Annaswamy, 1998) has been exploited in (Kojic *et al.*, 1999) when $\varphi(x_1, \theta)$ is additive and moreover either convex or concave in θ. Because of the complexity of the proposed min-max adaptive controller, its stabilizing function is restricted to depend only on state x_1. Otherwise, it will lead to a controller whose structure is significantly more complex with multiple parameter estimates. Also, such stabilizing function can be found only under the assumption of compactness of the parametric set (i.e. the unknown parameter θ is known belonging to a prescribed compact set). Additionally, the projection strategy is employed to ensure that the estimate $\hat{\theta}(t)$ of θ lies in the same definition set as θ. The resulting control may be sensitive to the size of the parametric set and thus may be unnecessarily high gain. Other difficulties in implementation of the min-max approach is due to the non-trivial step of identifying convex or concave structures for not only the nonlinear function $\varphi(x_1, \theta)$ but also for its multiplication by the derivative of the designed stabilizing function. Moreover, solving the min-max problem is a very costly operation. Finally, form candidates for the stabilizing function seem to remain an immature matter and very few simulation results are explicitely discussed in the literature (Kojic *et al.*, 1999).

In this paper, we utilize the idea of monotonicity-based approach (Tuan *et al.*, 2001) to address the adaptive backstepping for the system (1). Our approach enables the design of the stabilizing function containing estimates of the unknown parameter θ without overparameterization. The compactness of parametric sets is not required. The proposed approach is naturally applicable not only to smooth, convex, concave nonlinearities but also to the broader class of Lipschitzian functions.

The organization of the paper is as follows. Section 2 addresses the adaptive backstepping problem for the system (1) in the case of Lipschitzian parameterizations. Then, the case of smooth nonlinearity $\varphi(x_1, \theta)$ is more specialized in Section 3. Numerical simulations are discussed in Section 4 to verify the validity of the proposed approach. Finally, some concluding remarks are given in Section 5.

Throughout the paper, the following saturation function is used

$$\mathrm{sat}(e/\epsilon) = \begin{cases} e/\epsilon & \text{when } -\epsilon \le e \le \epsilon \\ 1 & \text{when } e > \epsilon \\ -1 & \text{when } e < -\epsilon \end{cases} \qquad (2)$$

Then given $\epsilon > 0$ and $e_\epsilon = e - \epsilon \mathrm{sat}(e/\epsilon)$, the following relations obviously hold true whenever $|e| > \epsilon$,

$$e_\epsilon^2 \le e_\epsilon e, \ \mathrm{sat}(e/\epsilon) = \mathrm{sgn}(e_\epsilon). \qquad (3)$$

We shall use the absolute value of a vector, which is defined as

$$|\theta| = [\, |\theta_1| \quad |\theta_2| \quad \dots \quad |\theta_p| \,]^T, \quad \forall\, \theta \in R^p.$$

Also, by $\theta \ge \bar{\theta}$, we mean that $\theta_j \ge \bar{\theta}_j, j = 1, 2, ..., p$.

In order to simplify the derivations throughout the paper, it is assumed that [2]

$$\theta \in R_+^p, \quad \text{i.e. } \theta_j \ge 0, \ j = 1, 2, 3, ..., p. \qquad (4)$$

2. ADAPTIVE BACKSTEPPING FOR LIPSCHITZIAN PARAMETERIZATIONS

Assumption 1. Function $\varphi(x_1, \theta)$ is Lipschitzian in θ, i.e. there are continuous functions $0 \le L_j(x_1) < +\infty, j = 1, 2, ..., p$, such that $\forall \bar{\theta}, \theta \in R^p$,

$$|\varphi(x_1, \bar{\theta}) - \varphi(x_1, \theta)| \le L(x_1)|\bar{\theta} - \theta|, \qquad (5)$$

with $L(x_1) = [\, L_1(x_1) \quad L_2(x_1) \quad \dots \quad L_p(x_1) \,]$. Additionally, it is assumed that

- $L(x_1)$ is differentiable,
- $\varphi(x_1, 0)$ is smooth in x_1.

It is worth noting that Lipschitzian parameterizations includes convex, concave or smooth parameterizations as special cases.

The next lemma will be used frequently in subsequent developments.

Lemma 1. (Tuan *et al.*, 2001) The following inequality holds true for any $e(t)$,

$$e(t)\varphi(x_1, \theta) \le e(t)(\varphi(x_1, 0) + \mathrm{sgn}(e(t))L(x_1)\theta).$$

We start our design procedure by rewriting system (1) in the presence of a stabilizing function $\alpha(x_1, \hat{\theta})$ for $x_1(t)$,

$$\begin{aligned} \dot{x}_1 &= z + \alpha(x_1, \hat{\theta}) + \varphi(x_1, \theta), \\ \dot{x}_2 &= u, \end{aligned} \qquad (6)$$

where

$$z = x_2 - \alpha(x_1, \hat{\theta}) \qquad (7)$$

[2] In Remark 2 of Section 2, we will show how the general case $\theta \in R^p$ can be easily retrieved from our results.

is the error between the stabilizing function $\alpha(x_1, \hat{\theta})$ and the state $x_2(t)$ of the system.
Given an arbitrary $\epsilon > 0$, define

$$V_1(t) := \frac{1}{2}x_{1\epsilon}^2(t) + \frac{1}{2}z^2(t),$$

with

$$x_{1\epsilon} = x_1 - \epsilon \text{sat}(x_1/\epsilon). \tag{8}$$

Whenever $|x_1| \leq \epsilon$, one has $x_{1\epsilon} = 0$ and hence,

$$\dot{V}_1 = z(u - \frac{d}{dt}\alpha(x_1, \hat{\theta})). \tag{9}$$

On the other hand, by Lemma 1, for $|x_1| > \epsilon$,

$$\begin{aligned}\dot{V}_1 &= x_{1\epsilon}(z + \alpha(x_1, \hat{\theta}) + \varphi(x_1, \theta)) \\ &\quad + z(u - \frac{d}{dt}\alpha(x_1, \hat{\theta})) \\ &\leq x_{1\epsilon}(z + \alpha(x_1, \hat{\theta}) + \varphi(x_1, 0) + \text{sgn}(x_{1\epsilon})L(x_1)\theta) \\ &\quad + z(u - \frac{d}{dt}\alpha(x_1, \hat{\theta})). \end{aligned} \tag{10}$$

In order to make the first term in the RHS of inequality (10) nonpositive, i.e. to stabilize the state $x_1(t)$, we choose the stabilizing function $\alpha(x_1, \hat{\theta})$ in the form

$$\begin{aligned}\alpha(x_1, \hat{\theta}) &= -k_1 x_1 - \varphi(x_1, 0) \\ &\quad -h(x_1)L(x_1)\hat{\theta},\end{aligned} \tag{11}$$

with an estimate $\hat{\theta}$ of the unknown parameter θ and

$$h(x_1) = \begin{cases} \sin(\frac{\pi}{2\epsilon}x_1) & \text{when } |x_1| \leq \epsilon \\ \text{sgn}(x_1) & \text{when } |x_1| > \epsilon\,. \end{cases} \tag{12}$$

Note that when $|x_1| \leq \epsilon$, instead of $\sin(\frac{\pi}{2\epsilon}x_1)$, $h(x_1)$ can be any function such that $h(x_1)$ is smooth on R. In case of (12), $h(x_1)$ is indeed smooth with its derivative given by

$$h'(x_1) = \begin{cases} \frac{\pi}{2\epsilon}\cos(\frac{\pi}{2\epsilon}x_1) & \text{when } |x_1| \leq \epsilon \\ 0 & \text{when } |x_1| > \epsilon\,. \end{cases} \tag{13}$$

With

$$\begin{aligned}\frac{d}{dt}\alpha(x_1, \hat{\theta}) &= \frac{\partial \alpha}{\partial x_1}(x_2 + \varphi(x_1, \theta)) \\ &\quad - h(x_1)L(x_1)\dot{\hat{\theta}},\end{aligned}$$

in view of (3),(10),(12) and Lemma 1, it follows that whenever $|x_1| > \epsilon$,

$$\begin{aligned}\dot{V}_1 &\leq (-k_1 x_{1\epsilon}^2 + x_{1\epsilon}z + |x_{1\epsilon}|L(x)\tilde{\theta}) \\ &\quad + z(u - \frac{\partial \alpha}{\partial x_1}(x_2 + \varphi(x_1, \theta)) + h(x_1)L(x_1)\dot{\hat{\theta}}) \\ &\leq (-k_1 x_{1\epsilon}^2 + x_{1\epsilon}z + |x_{1\epsilon}|L(x)\tilde{\theta}) \\ &\quad + z(u - \frac{\partial \alpha}{\partial x_1}x_2 + h(x_1)L(x_1)\dot{\hat{\theta}} - \frac{\partial \alpha}{\partial x_1}\varphi(x_1, 0) \\ &\quad + \text{sgn}(z)|\frac{\partial \alpha}{\partial x_1}|L(x_1)\theta), \end{aligned} \tag{14}$$

where $\tilde{\theta} = \theta - \hat{\theta}$ is parameter error.

In the view of (9) and (14), the following designed control input $u(t)$

$$\begin{aligned}u &= -x_{1\epsilon} - k_2 z + \frac{\partial \alpha}{\partial x_1}x_2 - h(x_1)L(x_1)\dot{\hat{\theta}} \\ &\quad + \frac{\partial \alpha}{\partial x_1}\varphi(x_1, 0) - \text{sgn}(z)|\frac{\partial \alpha}{\partial x_1}|L(x_1)\hat{\theta},\end{aligned} \tag{15}$$

together with the following Lyapunov function for the system (1)

$$V(t) = V_1(t) + \frac{1}{2}||\tilde{\theta}||^2, \tag{16}$$

result in

$$\dot{V} \leq \begin{cases} -k_2 z^2 + |z\frac{\partial \alpha}{\partial x_1}|L(x_1)\tilde{\theta} \\ -\dot{\hat{\theta}}^T\tilde{\theta}, \quad \text{when } |x_1| \leq \epsilon \\ \\ -k_1 x_{1\epsilon}^2 + |x_{1\epsilon}|L(x_1)\tilde{\theta} \\ -k_2 z^2 + |z\frac{\partial \alpha}{\partial x_1}|L(x_1)\tilde{\theta} \\ -\dot{\hat{\theta}}^T\tilde{\theta}, \quad \text{when } |x_1| > \epsilon \end{cases} \tag{17}$$

It follows that the following update law for the estimate $\hat{\theta}$

$$\dot{\hat{\theta}} = [|x_{1\epsilon}| + |z\frac{\partial \alpha}{\partial x_1}|]L(x_1)^T, \tag{18}$$

leads to the inequalities:

$$\dot{V}(t) \leq \begin{cases} -k_2 z^2 & \text{when } |x_1| \leq \epsilon \\ -k_1 x_{1\epsilon}^2 - k_2 z^2 & \text{when } |x_1| > \epsilon \end{cases} \tag{19}$$

The last inequalities imply that $V(t)$ is decreasing, and thus is bounded by $V(0)$. Consequently, $x_{1\epsilon}(t), z(t)$ and $\tilde{\theta}(t)$ must be bounded quantities by virtue of definition (16). Also, relation (19) gives $\int_0^T x_{1\epsilon}^2(t)dt \leq V(0), \int_0^T z^2(t)dt \leq V(0), \forall T > 0$, i.e. $x(t)_{1\epsilon}, z(t) \in L_2$. Applying Barbalat's Lemma (K.J. Astrom, 1995, p. 205) yields $\lim_{t\to\infty} x_{1\epsilon}(t) = 0$, $\lim_{t\to\infty} z(t) = 0$. Finally, let us mention that the update law (18) guarantees $\hat{\theta}(t) \in R_+^p, \forall t > 0$ provided that $\hat{\theta}(0) \in R_+^p$. We are now in a position to formulate the following result.

Theorem 1. Under assumption 1, the adaptive controller defined by equations (11),(15), and (18) stabilizes system (1) in the sense that all signals in the closed-loop system are globally bounded and the system state $x_1(t)$ asymptotically tracks 0 within a precision of ϵ.

The control determined by (11),(15) and (18) is discontinuous at $z(t) = 0$. However, we can modify it to get a continuous version with the following modified stabilizing function $\alpha(x_1, \hat{\theta})$

$$\begin{aligned} \alpha(x_1, \hat{\theta}) &= -k_1 x_1 - \varphi(x_1, 0) \\ &\quad -h(x_1)(\epsilon_z + L(x_1)\hat{\theta}), \end{aligned} \tag{20}$$

and its associated continuous control input $u(t)$

$$\begin{aligned} u &= -x_{1\epsilon} - k_2 z + \frac{\partial \alpha}{\partial x_1} x_2 \\ &\quad -h(x_1)L(x_1)\dot{\hat{\theta}} + \frac{\partial \alpha}{\partial x_1}\varphi(x_1, 0) \\ &\quad -\mathrm{sat}(z/\epsilon_z)|\frac{\partial \alpha}{\partial x_1}|L(x_1)\hat{\theta}, \\ \dot{\hat{\theta}} &= (|x_{1\epsilon}| + |z_\epsilon \frac{\partial \alpha}{\partial x_1}|)L(x_1)^T. \end{aligned} \tag{21}$$

The error $z(t)$ of the system converges to 0 within a precision of ϵ_z. As before, the system state $x_1(t)$ asymptotically tracks 0 within precision of ϵ.

Remark 1 It is also possible to design an adaptive controller for system (1) with a new one-dimensional observer $\hat{\theta}$ independent of the dimension of the unknown parameter θ. For that purpose, define $L_{max}(x_1) := \max_{j=1,2,\ldots,p} L_j(x_1)$, with $L(x_1)$ in (5). By taking a Lyapunov function in the form

$$V(t) := V_1(t) + \frac{1}{2}(\sum_{j=1}^{p} \theta_j - \hat{\theta})^2,$$

it can be readily shown that Theorem 1 is still satisfied when the one-dimensional observer

$$\dot{\hat{\theta}} = [|x_{1\epsilon}| + |z\frac{\partial \alpha}{\partial x_1}|]L_{max}(x_1)$$

is used in the adaptive controller (11),(15) with $L(x_1)$ replaced by $L_{max}(x_1)$.

Remark 2 For the general case $\theta \in R^p$, it follows in a straightforward manner from relation (5) and Lemma 1 that (Tuan *et al.*, 2001) for all $e(t)$

$$e\varphi(x_1, \theta) \le e(\varphi(x_1, 0) + \mathrm{sgn}(e)L(x_1)|\theta|).$$

Therefore, using a Lyapunov function defined as

$$V(t) = V_1(t) + \frac{1}{2}|||\theta| - \hat{\theta}||^2,$$

Theorem 1 remains valid for $\theta \in R^p$. We refer interested readers to reference (Tuan *et al.*, 2001) for more details on this technique.

Remark 3 The results of this section can be directly applied to the design of adaptive controller for the following class of systems considered in (Kojic *et al.*, 1999)

$$\begin{aligned} \dot{x}_1 &= x_2 + \sum_{i=1}^{n} \sigma_i f_i(x_1, \theta), \\ \dot{x}_2 &= u, \end{aligned} \tag{22}$$

where parameter θ is assumed to be in a compact set Θ, $\sigma_i \in R$, functions $f_i(x_1, \theta)$ are nonlinear in both variable x_1 and unknown parameter θ. In this case, $\varphi(x_1, \theta, \sigma) = \sum_{i=1}^{n} \sigma_i f_i(x_1, \theta)$ can be considered as a Lipschitzian function in σ and satisfies assumption 1 for unknown parameter σ, where

$$L(x_1) \ge [\sup_{\theta \in \Theta} |f_1(x_1, \theta)| \quad \ldots \quad \sup_{\theta \in \Theta} |f_n(x_1, \theta)|].$$

Such term $L(x_1)$ can always be found, since the parameter θ is assumed to lie in a compact set. The resulting controller is simpler than the proposed adaptive controller in (Kojic *et al.*, 1999).

3. ADAPTIVE BACKSTEPPING FOR A CLASS OF SMOOTH NONLINEARITY

In this section, we show that when the nonlinear function $\varphi(x_1, \theta)$ in system (1) is continuously differentiable (or smooth), our proposed adaptive control for this case will be better structured by exploiting the smoothness of the nonlinear function $\varphi(x_1, \theta)$. The smooth function $\varphi(x_1, \theta)$ can be decomposed as follows

$$\begin{aligned} \varphi(x_1, \theta) &= \varphi(0, \theta) + A(x_1, \theta)x_1, \\ A(x_1, \theta) &= \int_0^1 \frac{\partial \varphi}{\partial x_1}|_{\rho x_1} d\rho. \end{aligned} \tag{23}$$

Assumption 2. $A(x_1, \theta)$ is Lipschitzian in θ, i.e. there are continuous functions $0 \le L_j(x_1) < +\infty, j = 1, 2, \ldots p$ such that for all $\bar{\theta}$, $\theta \in R^p$

$$|A(x_1, \bar{\theta}) - A(x_1, \theta)| \le L(x_1)|\bar{\theta} - \theta|, \tag{24}$$

with $L(x_1) = [\, L_1(x_1) \quad L_2(x_1) \quad \ldots \quad L_p(x_1)\,]$.

Under this assumption, the following result is immediate

$$x_1^2 A(x_1, \theta) \le x_1^2 A(x_1, 0) + x_1^2 L(x_1)\theta. \tag{25}$$

Furthermore, with the representation (23), the process model (6) is rewritten as

$$\begin{aligned} \dot{x}_1 &= z + \alpha(x_1, \hat{\theta}) + \varphi(0, \theta) + A(x_1, \theta)x_1, \\ \dot{x}_2 &= u. \end{aligned} \tag{26}$$

Next, the function $V_2(t) = \frac{1}{2}x_1^2 + \frac{1}{2}z^2$ by relation (25) satisfies

$$\begin{aligned} \dot{V}_2(t) &= x_1(z + \alpha(x_1, \hat{\theta}) + \varphi(0, \theta) + A(x_1, \theta)x_1) \\ &\quad +z(u - \frac{d}{dt}\alpha(x_1, \hat{\theta})) \\ &\le x_1(z + \alpha(x_1, \hat{\theta}) + \varphi(0, \theta) + x_1 A(x_1, 0) \\ &\quad +x_1 L(x_1)\theta) + z(u - \frac{d}{dt}\alpha(x_1, \hat{\theta})). \end{aligned}$$

Naturally, an adaptive controller for this case should consist of a traditional update law $\hat{\varphi}_0$ for adaptation to linear parameter $\varphi(0,\theta)$ and a newly designed update law $\hat{\theta}$ for adaptation to nonlinear parameter θ. For that purpose, the stabilizing function α is chosen as

$$\begin{aligned}\alpha(x_1,\hat{\varphi}_0,\hat{\theta}) = & -k_1x_1 - x_1A(x_1,0) \\ & -x_1L(x_1)\hat{\theta} - \hat{\varphi}_0,\end{aligned} \quad (27)$$

with its derivative calculated by

$$\begin{aligned}\frac{d}{dt}\alpha(x_1,\hat{\varphi}_0,\hat{\theta}) = & \frac{\partial\alpha}{\partial x_1}(x_2+\varphi(0,\theta) \\ & +A(x_1,\theta)x_1) - x_1L(x_1)\dot{\hat{\theta}} - \dot{\hat{\varphi}}_0.\end{aligned}$$

Hence,

$$\begin{aligned}\dot{V}_2(t) \le & (-k_1x_1^2 + x_1z + x_1\tilde{\varphi}_0 + x_1^2L(x_1)\tilde{\theta}) \\ & +z(u - \frac{\partial\alpha}{\partial x_1}(x_2+\varphi(0,\theta) \\ & +A(x_1,\theta)x_1) + x_1L(x_1)\dot{\hat{\theta}} + \dot{\hat{\varphi}}_0),\end{aligned} \quad (28)$$

where $\tilde{\theta} = \theta - \hat{\theta}$, $\tilde{\varphi}_0 = \varphi(0,\theta) - \hat{\varphi}_0$ are parameter errors. Applying Lemma 1 for the term $(-zx_1\frac{\partial\alpha}{\partial x_1})A(x_1,\theta)$ in the RHS of inequality (28), it follows that

$$\begin{aligned}\dot{V}_2(t) \le & (-k_1x_1^2 + x_1z + x_1\tilde{\varphi}_0 + x_1^2L(x_1)\tilde{\theta}) \\ & +z(u - \frac{\partial\alpha}{\partial x_1}x_2 + x_1L(x_1)\dot{\hat{\theta}} \\ & +\dot{\hat{\varphi}}_0 - \frac{\partial\alpha}{\partial x_1}\varphi(0,\theta) - x_1\frac{\partial\alpha}{\partial x_1}A(x_1,0) \\ & +\mathrm{sgn}(z)|\frac{\partial\alpha}{\partial x_1}x_1|L(x_1)\theta).\end{aligned} \quad (29)$$

In view of (29), the following Lyapunov candidate function

$$V(t) = V_2(t) + \frac{1}{2}\tilde{\varphi_0}^2 + \frac{1}{2}||\tilde{\theta}||^2,$$

together with the following design of control input

$$\begin{aligned}u(t) = & -x_1 - k_2z + \frac{\partial\alpha}{\partial x_1}x_2 - x_1L(x_1)\dot{\hat{\theta}} - \dot{\hat{\varphi}}_0 \\ & +x_1\frac{\partial\alpha}{\partial x_1}A(x_1,0) - \mathrm{sgn}(z)|\frac{\partial\alpha}{\partial x_1}x_1|L(x_1)\hat{\theta} \\ & +\frac{\partial\alpha}{\partial x_1}\hat{\varphi}_0,\end{aligned} \quad (30)$$

results in

$$\begin{aligned}\dot{V}(t) \le & -k_1x_1^2 - k_2z^2 \\ & +(x_1 - z\frac{\partial\alpha}{\partial x_1})\tilde{\varphi}_0 - \dot{\hat{\varphi}}_0\tilde{\varphi}_0 \\ & +(x_1^2L(x_1) + |z\frac{\partial\alpha}{\partial x_1}x_1|L(x_1))\tilde{\theta} - \dot{\hat{\theta}}^T\tilde{\theta}.\end{aligned}$$

Therefore, the following update laws

$$\begin{aligned}\dot{\hat{\varphi}}_0 &= x_1 - z\frac{\partial\alpha}{\partial x_1}, \\ \dot{\hat{\theta}} &= (x_1^2 + |z\frac{\partial\alpha}{\partial x_1}x_1|)L(x_1)^T,\end{aligned} \quad (31)$$

lead to $\dot{V}(t) \le -k_1x_1^2 - k_2z^2$, which like Theorem 1 guarantees that $\lim_{t\to\infty} x_1 = 0$, $\lim_{t\to\infty} z(t) = 0$. We summarize these results in the following theorem.

Theorem 2. Under assumption 2, the adaptive controller defined by equations (23),(27), (30), and (31) stabilizes system (1) in the sense that all signals in the closed-loop system are globally bounded and the system state $x_1(t)$ asymptotically tracks 0 as $t \to \infty$.

As before, the control law determined by (30) and (31) is discontinuous at $z(t) = 0$. In the same way as described in section 2, it can be modified into a continuous one whose the resulting error $z(t)$ and system state $x_1(t)$ converge to 0 within a precision of ϵ.

4. SIMULATION EXAMPLES

Consider system (1) with

$$\varphi(x_1,\theta) = \theta_1\mathrm{sgn}(x_1) + e^{-x_1^2\theta_2}. \quad (32)$$

In this case, $\theta = [\theta_1,\theta_2]^T \in R_+^2$ is the unknown parameter. The nonlinear function $\varphi(x_1,\theta)$ is *Lipschitzian* in θ with $L(x_1) = \begin{bmatrix} 1 & x_1^2 \end{bmatrix}$. Thus, the adaptive controller (20), (21) stabilizes the system (1),(32) by Theorem 1. In simulations, the values of the parameters and initial values of the system are chosen as $x_1(0) = 1(rad), \theta_1 = 0.3(rad), \theta_2 = 0.5(rad)$. Figure 1 shows performances of the above designed system whose feedback gains are set to $k_1 = 1, k_2 = 1$ and $\epsilon_z = 0.02$.

Next, consider system (1) where

$$\begin{aligned}\varphi(x_1,\theta) &= l\theta \\ & +\frac{1}{2}\ln(1 + (g(x_1) + h(x_1)\theta)^2)x_1\end{aligned} \quad (33)$$

and $\theta = [\theta_1,\theta_2]^T \in R_+^2$, $l = \begin{bmatrix} l_1 & l_2 \end{bmatrix}, g(x_1) = x_1, w(x_1) = \begin{bmatrix} x_1+1 & x_1^2 \end{bmatrix}$. Clearly, $\varphi(x_1,\theta)$ is a *smooth* function in θ. Thus, it can be decomposed as $\varphi(x_1,\theta) = \varphi(0,\theta) + A(x_1,\theta)x_1$, where

$$\begin{aligned}\varphi(0,\theta) &= l\theta, \\ A(x_1,\theta) &= \frac{1}{2}\ln(1 + (x_1 + w(x_1)\theta)^2).\end{aligned} \quad (34)$$

Noting that $A(x_1,\theta)$ is *Lipschitzian* in θ with $L(x_1) = \begin{bmatrix} x_1^2+2 & x_1^2 \end{bmatrix}$. Applying Theorem 2 to system (1) and (33), we have the system stabilized by adaptive controller (27),(30),(31).

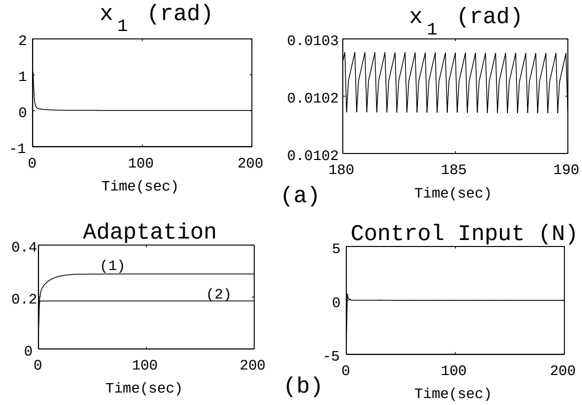

Fig. 1. Performances of system (1),(32) by controller (20), (21) with $\epsilon = .01$, $\epsilon_z = .02$. (a) $x_1(rad)$ at different scales, (b) Adaptation performance and control input(N): $(1) - \hat{\theta}_1, (2) - \hat{\theta}_2$.

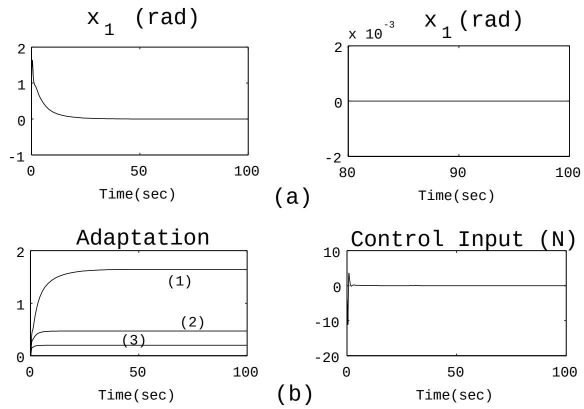

Fig. 2. Performances of system (1),(33) by controller (27),(30),(31). (a) $x_1(rad)$ at different scales, (b) Adaptation performance and control input(N): $(1) - \hat{\varphi}_0$, (2)-$\hat{\theta}_1, (3) - \hat{\theta}_2$.

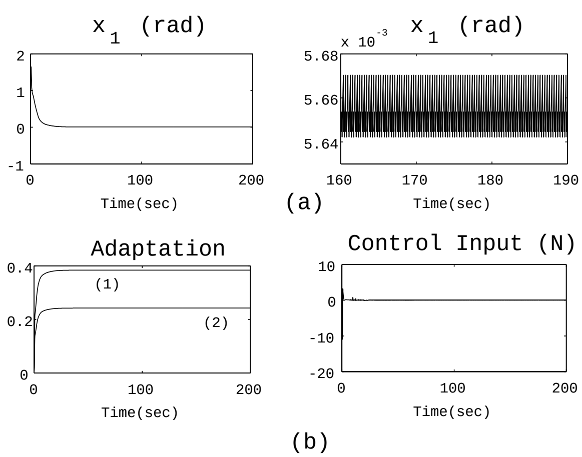

Fig. 3. Performances of system (1),(33) by controller (20), (21) with $\epsilon = .005$, $\epsilon_z = .05$. (a) $x_1(rad)$ at different scales, (b) Adaptation performance and control input(N): $(1) - \hat{\theta}_1, (2) - \hat{\theta}_2$.

For simulations, the values of the parameters and initial values of the system are chosen as $x_1(0) = 1(rad), \theta = [0.3 \quad 0.5](rad), l = [2 \quad 2]$. Figure 2 shows performances of the designed system.

On the other hand, it can be seen that $\varphi(x_1, \theta)$ in (33) is also a *Lipschitzian* function in θ with $L(x_1) = l + \frac{1}{2}(x_1^2+1)[x_1^2+2 \quad x_1^2]$. Thus, we can also have another stabilizing adaptive controller by applying Theorem 1 to system (1) and (33). Performances of such controller in Figure 3 shows how a better behaved controller is obtained by exploiting the smoothness of function $\varphi(x_1, \theta)$ to expand it into linear part $\varphi(0, \theta)$ and a nonlinear counterpart as in expression (23).

5. CONCLUSIONS

Thanks to simple structures of monotonic functions, adaptive backstepping can be designed for NP unknown parameter without conservatism attached to the size of the parameter set. Indeed, compactness of parameter sets is not required in our approach. A simple but effective adaptive controller is designed in the general situation where the nonlinearity of the system enjoys a general Lipschitzian structure. When nonlinear structures of the system is exploited more in depth as in the case of a smooth nonlinearity, we have also shown through our theory and simulations how a better behaved adaptive controller can be designed. The LP-like structure of the proposed adaptive control, whose unknown parameter estimator does not result in any overparameterization, is a key point to extend our approach to systems of arbitrary order in a natural and direct manner.

6. REFERENCES

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