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RATE AND MAGNITUDE-BOUNDED ACTUATORS: SCHEDULED STATE FEEDBACK DESIGN

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Abstract: This paper studies the problem of controller synthesis for systems subject to amplitude and rate saturation constraints in the actuators. The main approach assumes that each actuator has a dynamic model of order at least one. The controllers presented here are only state feedback, though extension to the output feedback is also possible. The solvability conditions for the problem are expressed as finite-dimensional linear matrix inequalities. In addition to stabilization, disturbance attenuation in terms of the L_2 gain from the disturbance to output is considered.

Keywords: Saturation, rate bounds, disturbance attenuation.

1. INTRODUCTION

Actuator saturation has been an important topic of research for decades. Recent progress in several areas (e.g., numerical linear algebra, robust and LPV control) however, has led to a great deal of interest in a variety of issues related to systems with bounded actuators. A comprehensive review is not feasible here and interested readers can consult Bernstein and Michel (1995) or Stoorvogel and Saberi (1999) for recent surveys. For a representative sample of work in anti-windup area, one can consult Campo et al (1990), De Dona and Goodwin (2000), Kappor et al (1998), Kothare et al (1994), Teel (1999), Barbu et al (2000), Mulder et al 2001, while a variety of results based on taking into account the saturation bounds explicitly can be found in Lin and Saberi (1995), Lin (1998a), Kapila et al (1999) among of the available references).

The bounds on the *rate* of the actuator force (torque, thrust, etc.) have also been identified as a source of severe performance degradation or instability in aerospace applications - as well as many other instances where power limitations pose a critical limitation (see Hess and Snell (1997) or Snell and Hess (1998) for example). This has led to much interest in actuator rate bounds. In some cases, the actuator rate is modeled as an abstract operation (Stoorvogel and Saberi (1999)). In other instances it is modeled through use of a first order model representing the actuator dynamics, which was suggested in Berg et al (1996) (see e.g., Lin (1998a), Chellaboina, et al (2000), Nguyen and Jabbari (2000), Tyan and Bernstein (1995)).

In this paper, we present synthesis results for systems with bounded actuator amplitudes and rates. The actuator models used are generalizations of the typical first order models; i.e., the rate is incorporated through actuator dynamics of order *at least* one and can have a general LTI form (indeed, extension to an LPV model for the actuator dynamics to account for common nonlinearities is relatively straightforward). Our results concern disturbance attenuation (in the

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sense of a small L_2 gain from disturbance to the controlled output), though other performance measures (e.g., peak-to-peak gain estimates or energy to peak gains) are also possible. For disturbance attenuation, the L_2-gain from the disturbance to the controlled output is minimized, while an estimate of the reachable set is established simultaneously. The resulting problem relies on an LMI-based multi-objective control approach, which searches for a common Lyapunov matrix for both problems (L_2 gain and reachable set).

In Kose and Jabbari (2001), the potential conservatism associated with a single LTI controller was discussed. In such techniques, the controllers are often designed for the worst case disturbance. This leads to conservatism for controllers that are designed to avoid actuator limits (i.e., low gain controllers). At times, through use of oversaturation and similar techniques, the gain of the controller is increased - see Lin and Saberi (1995), Nguyen and Jabbari (1999) and De Dona et al (2000) as examples). These high gain controllers however do not provide better guaranteed performance generally (see e.g., Kiyama and Iwasaki (2001)), though they show better performance in simulations. Here, to provide better guaranteed performance, we present a scheduling approach. The concept of scheduling controllers to avoid actuator saturation goes back at least to the early work of Gutman and Hadanger (1985) or Megretski (1996) and has been receiving increased attention recently (e.g., Lin (1998b), Teel (1995), Wu, et al (2000), Henrion et al (1998), Shewchun and Feron (1998) among others).

The technique used here is similar to that in Srivastava and Jabbari (2000), where the controller is dependent on the system response (e.g., the state), by using state dependent ellipsoids. These in turn are used to obtain a scheduling parameter for adjusting (scheduling) the controller. In smaller ellipsoids, larger gains are possible, which can result in better performance (and lead to parameter dependent performance measures). If the state vector moves further from the origin (e.g., due to disturbance), smaller gains are used. As in Srivastava and Jabbari (2000), the controller is thus a function of the system response and not any a priori estimate of the worst cases disturbance.

While the results here are state feedback, generalization to output feedback is possible (though not trivial). Also, for brevity, only the single input case is presented. As explained in the results section, the extension for the multi-input case is immediate. The main results are stated first in a from that enforces the magnitude bounds directly. As explained below, this is not particularly useful for scheduling of the controller and an alternative approach, that exploits the structure (and dynamics) of the actuator will be used.

We use the following notation: For a vector $x \in \mathbb{R}^n$, the Euclidean norm is defined as $\|x\|_E \triangleq \sqrt{x^T x}$. We denote by $\mathcal{E}_P$ the ellipsoid $\{x \in \mathbb{R}^n : x^T P x \leq 1\}$ for a given $P = P^T > 0$ in $\mathbb{R}^{n \times n}$. For a signal $x \in \mathcal{L}_2^n[0, \infty)$, the $\mathcal{L}_2$-norm of x is $\|x\|_2 \triangleq \left(\int_0^\infty x(t)^T x(t)\, dt\right)^{1/2}$. In matrices defined through subblocks, the symbol "$\star$" at the (i,j) block stands for the transpose of block (j,i).

2. PRELIMINARIES

In this paper, we consider systems of the form

$$\dot{x} = Ax + B_1 w + B_2 u \tag{1a}$$
$$z = C_1 x + D_{11} w + D_{12} u, \tag{1b}$$

where $w(t) \in \mathbb{R}^{m_1}$ is the external disturbance on the system, $u(t) \in \mathbb{R}$ is the single control input to the system and $z(t) \in \mathbb{R}^{p_1}$ denotes the controlled output of the system. The actuator dynamics are given by

$$\dot{x}_v = A_v x_v + B_v v \tag{2a}$$
$$u = C_v x_v + D_v v \tag{2b}$$

where $v(t) \in \mathbb{R}$ denotes the control command, $x_v(t) \in \mathbb{R}^{n_v}$, $n_v \geq 1$. Throughout the paper, we assume that the actuator is strictly proper, i.e., $D_v = 0$. Note that this description is a simple generalization of the typical first order models used in many papers and can be used to bound other actuator-related entities.

Combining plant and actuator dynamics, we obtain

$$\dot{\tilde{x}} = \underbrace{\begin{bmatrix} A & B_2 C_v \\ 0 & A_v \end{bmatrix}}_{\tilde{A}} \tilde{x} + \underbrace{\begin{bmatrix} B_1 \\ 0 \end{bmatrix}}_{\tilde{B}_1} w + \underbrace{\begin{bmatrix} 0 \\ B_v \end{bmatrix}}_{\tilde{B}_2} v \tag{3a}$$
$$z = \underbrace{\begin{bmatrix} C_1 & D_{12} C_v \end{bmatrix}}_{\tilde{C}_1} \tilde{x} + D_{11} w \tag{3b}$$

We also assume that the magnitude and rate saturation bounds are given for u,

$$|u(t)| \leq r \quad \text{and} \quad |\dot{u}(t)| \leq d \quad \forall t \geq 0. \tag{4}$$

Our overall goal is to design controllers such that the conditions above are not violated while maintaining a good disturbance attenuation from w to z.

Finally, we assume that the peak value of the disturbance can be estimated (possibly conservatively) through

$$\max_t \|w(t)\|_E \leq w_{max}.$$

As shown below, conservatism associated with this bound is not critical since the scheduled controllers are not designed for the worst case, but are chosen based on the actual response of the system. Throughout this paper we refer to ellipsoids with the following form

$$\mathcal{E}(P,c) \triangleq \{x \in \mathbb{R}^n : x^T P x \le c\} .$$

3. MAIN RESULT

We start with the following theorem that establishes the structure and the main properties of the proposed controllers.

Theorem 1. Suppose there exist a function $\rho : \mathbb{R}^+ \to [\rho_{min}, \rho_{max}]$ with $\rho_{min} = 1/w_{max}^2$, a scalar $\alpha > 0$, parameter-dependent matrices $X(\rho)$, $F(\rho)$ and a function $\gamma(\rho)$ (with appropriate levels of continuity) such that $dX/d\rho \le 0$ and

$$\begin{bmatrix} M(\rho_{min}) + \alpha X(\rho_{min}) & \star \\ \tilde{B}_1^T & -\alpha I \end{bmatrix} < 0 \qquad (5)$$

and for all $\rho \in [\rho_{min}, \rho_{max}]$,

$$\begin{bmatrix} M(\rho) - \dot{X}(\rho) & \star & \star \\ \tilde{B}_1^T & -\gamma(\rho) I & \star \\ \tilde{C}_1 X(\rho) & \tilde{D}_{11} & -\gamma(\rho) I \end{bmatrix} < 0 \qquad (6)$$

where

$$M(\rho) \triangleq \tilde{A}X(\rho) + X(\rho)\tilde{A}^T + \tilde{B}_2 F(\rho) + F(\rho)^T \tilde{B}_2^T,$$

$$-\rho\, u_{sat}^2 I + \begin{bmatrix} 0 & C_v \end{bmatrix} X(\rho) \begin{bmatrix} 0 & C_v \end{bmatrix}^T < 0 \qquad (7)$$

and

$$\begin{bmatrix} -X(\rho) & \star \\ \begin{bmatrix} 0 & C_v A_v \end{bmatrix} X(\rho) + C_v B_v F(\rho) & -\rho\, d_{sat}^2 I \end{bmatrix} \le 0. \qquad (8)$$

Then, if $\rho(t)$ is chosen such that

$$x(t)^T X(\rho)^{-1} x(t) \le \frac{1}{\rho} \qquad (9)$$

the control law

$$u = F(\rho) X(\rho)^{-1} x \qquad (10)$$

satisfies the following:

(i) For the closed-loop state vector, the set $\mathcal{E}\big(X(\rho_{min})^{-1}, 1/\rho_{min}\big)$ is invariant. That is, for a disturbance with $w(t)^T w(t) \le w_{max}^2$ and any $x(0) \in \mathcal{E}\big(X(\rho_{min})^{-1}, 1/\rho_{min}\big)$, we have $x(t) \in \mathcal{E}\big(X(\rho_{min})^{-1}, 1/\rho_{min}\big)$ for all $t \ge 0$.

(ii) The closed-loop system is internally stable with

$$\int_0^\infty \gamma\big(\rho(t)\big)^{-1} z(t)^T z(t)\, dt < \int_0^\infty \gamma\big(\rho(t)\big) w(t)^T w(t)\, dt. \qquad (11)$$

(iii) The control input satisfies (4). ■

Remarks:

(i) In the theorem above, condition (5) leads to (i) and (6) to (ii). Statement (iii) follows from conditions (5), (7) and (8) jointly.

(ii) We stress that the plant itself is LTI, whereas the control law (10) is parameter varying. So far, the only rule for choosing the parameter ρ is that it satisfy (9). It will soon become clear that ρ is in fact a measure of the proximity of x to the origin and the controller is scheduled accordingly.

(iii) The theorem above establishes the controller as the solution of a parameter-varying problem. As in Srivastava and Jabbari (2000), we use spline functions as approximations for $\gamma(\rho)$ and $F(\rho)$ and a smooth version of a spline function for $X(\rho)$, as the following. Consider a collection of points $0 < \eta_1 < \cdots < \eta_{n_\eta}$, and a corresponding collections of matrices M_k. Then, a linear spline function based on η_k's and M_k's is defined by

$$M_S(\rho) \triangleq M_k + \frac{\rho - \eta_k}{\eta_{k+1} - \eta_k}(M_{k+1} - M_k) \quad (12)$$

for $\rho \in [\eta_k, \eta_{k+1}]$.

As the theorem below indicates, this permits finding the appropriate variables through a finite number of linear matrix inequalities and an appropriately defined parameter $\rho(t)$.

Lemma 2. Let $1/w_{max}^2 = \eta_1 < \cdots < \eta_{n_\eta}$. Suppose there exist matrices X_k and F_k and scalars α and γ_k such that

$$\begin{bmatrix} M_1 + \alpha X_1 & \star \\ \tilde{B}_1^T & -\alpha I \end{bmatrix} < 0 \qquad (13)$$

and for all $k = 1 : n_\eta$, and $m = k-1, k$

$$\begin{bmatrix} M_k + d_{max}\Delta X_m & \star & \star \\ \tilde{B}_1^T & -\gamma_k I & \star \\ \tilde{C}_1 X_k & \tilde{D}_{11} & -\gamma_k I \end{bmatrix} < 0, \qquad (14)$$

where $M_k \triangleq \tilde{A}X_k + X_k\tilde{A}^T + \tilde{B}_2 F_k + F_k^T \tilde{B}_2^T$,

$$X_{k+1} \le X_k, \qquad (15)$$

$$-\eta_k\, u_{sat}^2 I + \begin{bmatrix} 0 & C_v \end{bmatrix} X_k \begin{bmatrix} 0 & C_v \end{bmatrix}^T < 0 \qquad (16)$$

and

$$\begin{bmatrix} -X_k & \star \\ \begin{bmatrix} 0 & C_v A_v \end{bmatrix} X_k + C_v B_v F_k & -\eta_k\, d_{sat}^2 I \end{bmatrix} \le 0. \qquad (17)$$

Then, the parameter $\rho(t)$ and functions $X\big(\rho(t)\big)$, $F\big(\rho(t)\big)$, and $\gamma\big(\rho(t)\big)$ defined as below satisfy the conditions (5)-(8) in Theorem 1:

$\underline{\rho(t)}$: Given $x(t)$, determine $k \triangleq \max j$ such that $x(t)^T X_j^{-1} x(t) \le 1/\eta_j$, and let

$$\rho'(t) \triangleq \begin{cases} \dfrac{1}{x(t)^T X_S(r)^{-1} x(t)} & \text{if } k < n_\eta \\ \eta_{n_\eta} & \text{if } k = n_\eta \end{cases}$$

with X_S defined as in (12). Then, for a $T > 0$ small enough,

$$\rho(t) \triangleq \frac{1}{T} \int_{t-T}^{t} \rho'(s)\, ds. \tag{18}$$

$\underline{X(\rho)}$: Given ρ, for a $L > 0$ small enough,

$$X(\rho) \triangleq \frac{1}{L} \int_{\rho-L/2}^{\rho+L/2} X'(s)\, ds \tag{19}$$

where

$$X'(s) \triangleq \begin{cases} X_1 & \text{if } \eta_1 - L/2 \leq s \leq \eta_1 \\ X_S(s) & \text{if } \eta_k \leq s \leq \eta_{k+1},\ 1 \leq k < n_\eta \\ X_{n_\eta} & \text{if } \eta_{n_\eta} \leq s \leq \eta_{n_\eta} + L/2 \end{cases} \tag{20}$$

and $F(\rho)$ and $\gamma(\rho)$ are defined similar to $X'(\rho)$. ■

Remarks:

(i) In the formulation above, the values of η_k's, d_{max} and β_k's are design variables. Although we skip much detail due to space limitations, it is worth noting that a major role is played by the largest value of η_k's. Beyond a certain value, increasing η_{n_η} has only little effect. The number of η_k's determines how sharply γ_k's drop with η_k's.

(ii) The variables T and L that are used in the construction of $\rho(t)$ and $X(\rho)$ do not affect the solvability conditions, but their existence is guaranteed. In numerical simulations, we simply use $\rho'(t)$ and $X'(\rho)$.

(iii) Inequality (16) does not involve F. As such, it does not benefit fully from scheduling. To maximize the effects of scheduling, we can modify this inequality and place a bound not on u, but on the command v. To do this, we use the peak-to-peak gain (Abedor, *et al.*, 1996) of the actuator dynamics. Suppose this gain is found to be δ. Then, the requirement $|u(t)| \leq u_{sat}$ is satisfied if $|v(t)| \leq \delta^{-1} u_{sat}$. In this case, (16) can be replaced by

$$\begin{bmatrix} -X_k & F_k^T \\ F_k & -\eta_k \dfrac{usat^2}{\delta^2} \end{bmatrix} < 0. \tag{21}$$

Due to explicit appearance of F_k in the inequality above, better use can be made of parametric dependence. This is what we do in the following example.

4. NUMERICAL EXAMPLE

Consider the linearized model of an inverted pendulum with damping:

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 10 \end{bmatrix} w + \begin{bmatrix} 0 \\ 5 \end{bmatrix} u$$
$$z = \begin{bmatrix} 1 & 0 \end{bmatrix} x.$$

Suppose the actuator dynamics is given by the transfer function

$$U(s) = \frac{10}{s+10} V(s).$$

We assume saturation bounds for the magnitude and the rate of the control input are given by $u_{sat} = 10$ and $d_{sat} = 100$, respectively.

We apply Lemma 2 to the system and set $n_\eta = 20$ and let η_k's be 20 linearly spaced points between 1 and 1000. We solve inequalities (13)-(17) (with (21) instead of (16)) while minimizing $\sum_{k=1}^{n_\eta} \gamma_k$. The resulting values for γ_k and the norms of controller gains at these nodes, $\|K_k\|$ are given in figures Figure 1 and Figure 2. Clearly, much higher

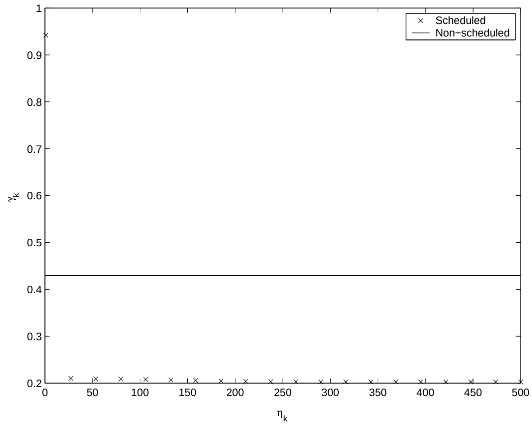

Fig. 1. γ_k versus η_k

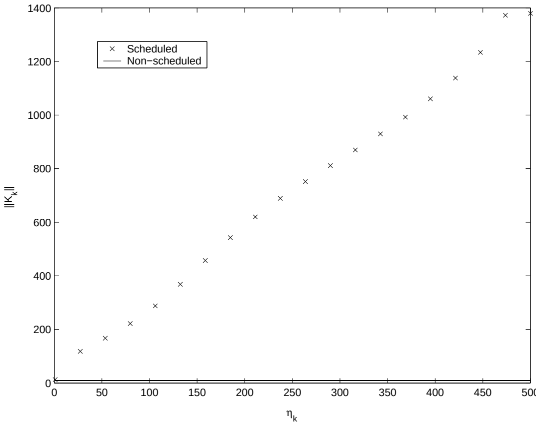

Fig. 2. Norm of controller gains versus η_k

controller gains are allowed when the state vector moves closer to the origin, yielding much better disturbance attenuation. In implementation, we use the disturbance signal given in Figure 3. Implementation of the scheduled controller to the system results in Figure 4 for the control input, Figure 5 for the control input rate and Figure 6 for the controlled output.

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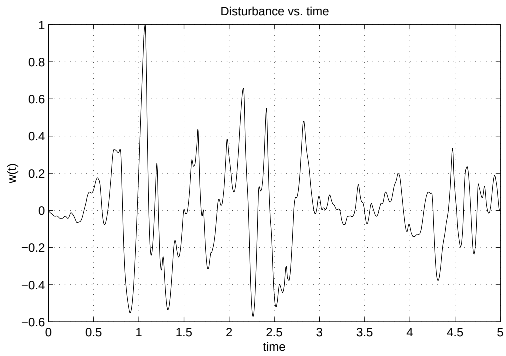

Fig. 3. Disturbance signal.

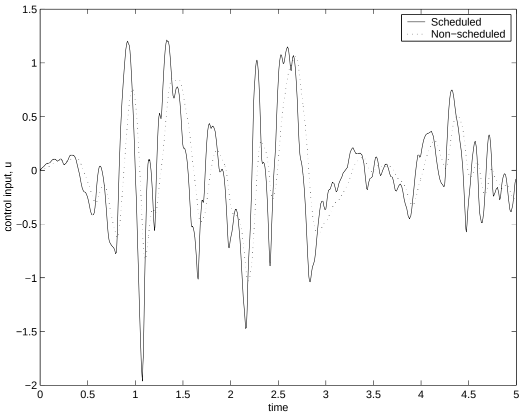

Fig. 4. Control input.

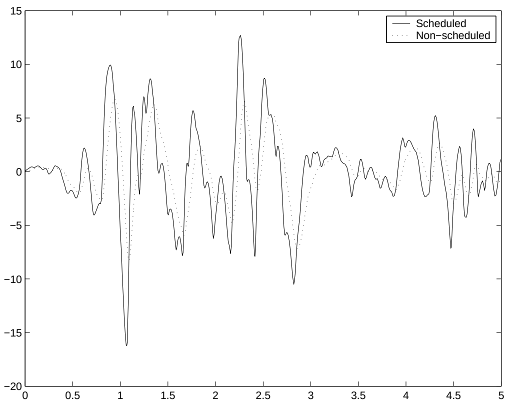

Fig. 5. Control input rate.

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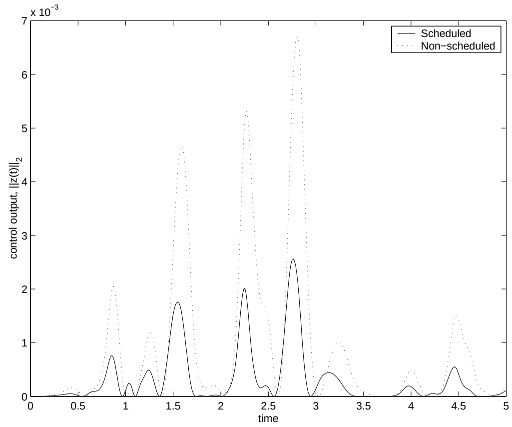

Fig. 6. Controlled output, $\|z(t)\|_2$.

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