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# ON NONLINEAR DISTURBANCE ATTENUATION FOR HYDRAULIC DIFFERENTIAL CYLINDERS VIA TRACKING CONTROL

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Abstract: Hydraulic actuators are well described by nonlinear system modeling. Nonlinear control for differential cylinders may be realized in order to obtain disturbance decoupling. However, such a system cannot be decoupled from the disturbance and therefore it is necessary to compute a nonlinear disturbance attenuation controller that reduces the influence of the disturbance with respect to the system output. The controller synthesis of nonlinear disturbance attenuation is based on linear error dynamics. Measurements on a testbed represent the basis of this contribution and show the mode of operation for the disturbance attenuation controller under real conditions. The measurements yield very good tracking performance for the cylinder piston position up to a particular disturbance force margin.

Keywords: Disturbance attenuation, hydraulic actuators, nonlinear/tracking control

## 1. INTRODUCTION

Hydraulic drives are often applied in practical systems e.g. in concrete pumping manipulators that operate in wide ranges or carry heavy loads. As the main components for such a drive in this paper a differential cylinder and a proportional valve are investigated. A nonlinear mathematical modeling of the hydraulic system proves to be very useful for a controller determination.

An important objective of control engineering is to decouple disturbances such that they no longer affect the system outputs. The decision if a system is disturbance decouplable may be achieved by the nonlinear approaches *differential geometry* (Nijmeijer and van der Schaft, 1990; Isidori, 1995; Sastry, 1999; Schwarz, 2000; Tsirikos and Arvanitis, 2000) or *differential algebra* (Ritt, 1950; Fliess, 1987; Fliess and Glad, 1993; Bröcker, 2000). If the disturbance decoupling problem is not solvable, the question still remains whether

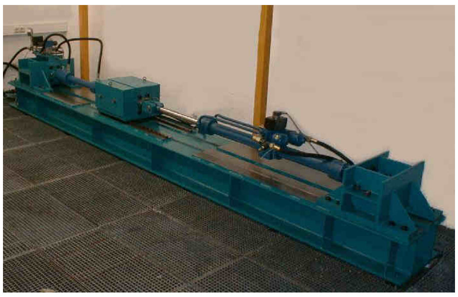

Fig. 1. Differential cylinder testbed

there exists at least a disturbance attenuation controller. Such a controller may not decouple the system outputs and disturbances but may reduce the influence of the disturbances.

After substantial results for disturbance attenuation in linear control theory, see e.g. (Knobloch and Kwakernaak, 1985; Del Muro-Cuellar and Martínez-García, 2000), there have been worked out recently interesting research works in Lyapunov stability based methods, see e.g. the $H_\infty$ approach (Jiang and Jiang, 1997; Ding *et al.*, 2000) respectively the $L_2$-gain analysis (van der Schaft, 1992; Isidori, 1996; Qian *et al.*, 2001) and the robust adaptive control (Ding, 1999). In robotics the tracking control approach, see e.g. (Spong, 1986; Studenny *et al.*, 1991; Tafazoli *et al.*, 1996; Liu and Peng, 2000), is a traditional method to warrant trajectory tracking regarding sensitivity and robustness. By the extension to nonlinear systems considering the relative degree as the highest order of the linear differential equations for the tracking error, (Slotine and Li, 1991) and (Isidori, 1995) adapt the tracking control approach to differential geometry. To the author's knowledge disturbances are not considered by this approach. Only (Khalil, 1996) mentions the possibility to apply tracking control for disturbance attenuation.

This paper presents a nonlinear disturbance attenuation contoller for quadratic (same number of in- and outputs) analytical input-affine multiple input – multiple output systems that is based on fundamental linear differential equations for the tracking error. The controller is derived and implemented on a differential cylinder testbed (see figure 1). Additionally, the testbed consists of a force control unit in order to guarantee a tunable disturbance force trajectory via a PI-controller. The experimental results of disturbance attenuation control demonstrate good tracking performance. Nevertheless, the applied controller is also limited to a particular disturbance force margin which is typical for the disturbance attenuation.

## 2. FUNDAMENTALS OF THE NONLINEAR DISTURBANCE ATTENUATION

If a nonlinear system is not disturbance decouplable, the possibility remains to determine a nonlinear control law that reduces the influence of disturbances w.r.t. the system outputs. Some definitions of differential geometry are needed for the computation of the control law for the disturbance attenuation. For an *analytical input-affine system (ALS)* of the form

$$\Sigma_{\mathrm{ALS}} \begin{array}{ll} \dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{G}(\boldsymbol{x})\boldsymbol{u} + \boldsymbol{p}(\boldsymbol{x})d, & \boldsymbol{x} \in \mathbb{R}^n, \\ \boldsymbol{y} = \boldsymbol{h}(\boldsymbol{x}), & d \in \mathbb{R}, \quad \boldsymbol{u}, \boldsymbol{y} \in \mathbb{R}^m, \end{array} \tag{1}$$

with $\boldsymbol{G}(\boldsymbol{x}) := [\boldsymbol{g}_1(\boldsymbol{x}), \ldots, \boldsymbol{g}_m(\boldsymbol{x})]$ two characteristic values treated below are significant either for disturbance decoupling or for disturbance attenuation. The first characteristic value is the relative degree $\boldsymbol{r} = [r_1, \ldots, r_m]$ of the undisturbed system $\Sigma_{\mathrm{ALS}}^{d:=0}$ and is defined as (Isidori, 1995):

*Definition 1.*
A multivariable nonlinear system of the form in eq. (1) has a *relative degree* $\boldsymbol{r}$ at a point $\boldsymbol{x}_0$ if $\mathrm{L}_{\boldsymbol{g}_j}\mathrm{L}_{\boldsymbol{f}}^k h_i(\boldsymbol{x}) = 0$ for all $1 \leq j \leq m$, for all $0 < k < r_i - 1$, for all $1 \leq i \leq m$, and for all $\boldsymbol{x}$ in a neighborhood of $\boldsymbol{x}_0$, with $r_i = \min\{l | \mathrm{L}_{\boldsymbol{g}_j}\mathrm{L}_{\boldsymbol{f}}^{l-1} h_i(\boldsymbol{x}) \neq 0\}$, and where the decoupling matrix

$$\boldsymbol{A}(\boldsymbol{x}) := \begin{bmatrix} \mathrm{L}_{\boldsymbol{g}_1}\mathrm{L}_{\boldsymbol{f}}^{r_1-1}h_1(\boldsymbol{x}) & \cdots & \mathrm{L}_{\boldsymbol{g}_m}\mathrm{L}_{\boldsymbol{f}}^{r_1-1}h_1(\boldsymbol{x}) \\ \mathrm{L}_{\boldsymbol{g}_1}\mathrm{L}_{\boldsymbol{f}}^{r_2-1}h_2(\boldsymbol{x}) & \cdots & \mathrm{L}_{\boldsymbol{g}_m}\mathrm{L}_{\boldsymbol{f}}^{r_2-1}h_2(\boldsymbol{x}) \\ \vdots & \ddots & \vdots \\ \mathrm{L}_{\boldsymbol{g}_1}\mathrm{L}_{\boldsymbol{f}}^{r_m-1}h_m(\boldsymbol{x}) & \cdots & \mathrm{L}_{\boldsymbol{g}_m}\mathrm{L}_{\boldsymbol{f}}^{r_m-1}h_m(\boldsymbol{x}) \end{bmatrix}$$

is nonsingular at $\boldsymbol{x} = \boldsymbol{x}_0$.

The second characteristic value is the disturbance relative degree $\boldsymbol{r}_d = [r_{d,1}, \ldots, r_{d,m}]$ (Isidori, 1995):

*Definition 2.*
A multivariable nonlinear system of the form in eq. (1) has a *disturbance relative degree* $\boldsymbol{r}_d$ at a point $\boldsymbol{x}_0$ if $\mathrm{L}_{\boldsymbol{p}}\mathrm{L}_{\boldsymbol{f}}^k h_i(\boldsymbol{x}) = 0$ for all $0 < k < r_{d,i} - 1$, for all $1 \leq i \leq m$, and for all $\boldsymbol{x}$ in a neighborhood of $\boldsymbol{x}_0$, with $r_{d,i} = l | \mathrm{L}_{\boldsymbol{p}}\mathrm{L}_{\boldsymbol{f}}^{l-1} h_i(\boldsymbol{x}) \neq 0$.

The disturbance attenuation controller requires additionally a difference relative degree $\boldsymbol{\varrho}$ and a maximum difference relative degree $\tilde{\varrho}$. Both values result from *Definition 1/2* and represent a new point of interest.

*Definition 3.*
If a multivariable nonlinear system of the form in eq. (1) is not disturbance decouplable, there exists a *difference relative degree* $\boldsymbol{\varrho} = \boldsymbol{r} - \boldsymbol{r}_d$, with $\varrho_i = r_i - r_{d,i}$, and a maximum difference relative degree $\tilde{\varrho} = \max(\{\varrho_1 \ \ldots \ \varrho_m\}), \ \forall \ i = 1, 2, \ldots, m$.

The terms $\mathrm{L}_{\boldsymbol{f}}^k h_i(\boldsymbol{x})$ are defined as the so called Lie derivatives of the order $k$ of any real-valued function $h_i$ along a vector field $\boldsymbol{f}$:

$$\mathrm{L}_{\boldsymbol{f}}^k h_i(\boldsymbol{x}) := \frac{\partial \mathrm{L}_{\boldsymbol{f}}^{k-1} h_i(\boldsymbol{x})}{\partial \boldsymbol{x}} \cdot \boldsymbol{f}(\boldsymbol{x}).$$

On the basis of these definitions the disturbance attenuation (note that we face $r_{d,i} < r_i$, with $i = 1, \ldots, m$ for systems that cannot be decoupled from the disturbance) may be described by the fundamental linear differential equations for the tracking error:

$$e_i^{(r_i)} + \sum_{j=1}^{r_i} c_{r_i-j,i} \, e_i^{(r_i-j)} = 0, \tag{2}$$

$\forall \ i = 1, \ldots, m$, with

$$e_i = y_{\mathrm{rt},i} - y_i \quad , \forall \ i = 1, \ldots, m \tag{3}$$

and its time derivatives

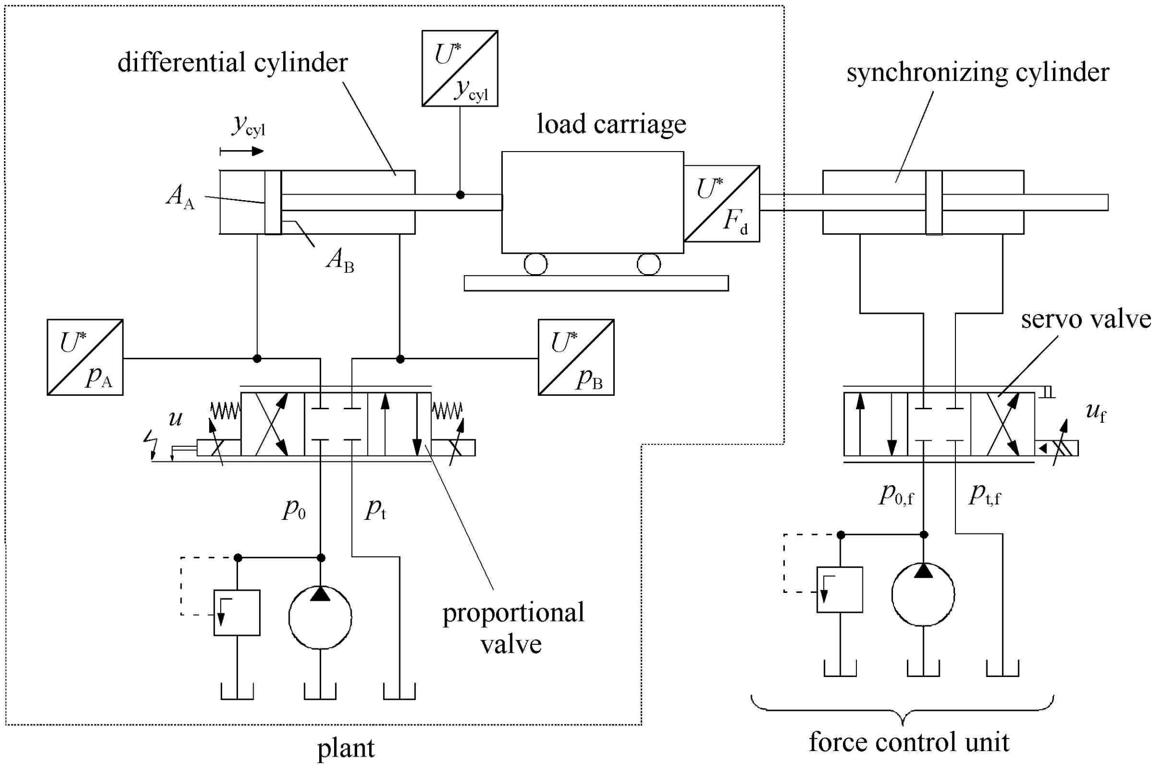


Fig. 2. Testbed: plant (differential cylinder with proportional valve) and force control unit

$$e_i^{(j)} = y_{\mathrm{rt},i}^{(j)} - y_i^{(j)}, \tag{4}$$

$\forall\ (i = 1, \dots, m) \wedge (j = 1, \dots, r_i)$. Differentiating the system outputs $y_i$

$$\dot{y}_i, \dots, y_i^{(r_i)}, \tag{5}$$

and then substituting the system outputs $y_i$ and its time derivatives (eq. (5)) in eq. (3) and eq. (4) and finally into eq. (2) leads to the nonlinear disturbance attenuation control law

$$\boldsymbol{u}_{\mathrm{att}} = \tilde{\boldsymbol{f}}(y_{\mathrm{rt},i}, \dot{y}_{\mathrm{rt},i}, \dots, y_{\mathrm{rt},i}^{(r_i)}, \boldsymbol{x}, \tilde{\boldsymbol{d}}, c_{r_i - j,i}), \tag{6}$$

with $\tilde{\boldsymbol{d}} = [d, \dot{d}, \ddot{d}, \dots, d^{(\tilde{\varrho})}]^{\mathrm{T}}$. The control law consists mainly of the reference trajectory $y_{\mathrm{rt},i}$ and its time derivatives as the controller inputs. The feedback also comprises the states, the disturbance and its time derivatives. The control parameters $c_{r_i - j,i}$ have to be optimized.

## 3. TESTBED MODELING

The nonlinear disturbance attenuation is proved on a testbed as shown in figure 2. The testbed consists of two independently driven subsystems: the plant and a force control unit. For hydraulic drives in practice the important components differential cylinder and proportional valve are chosen as the plant. To simulate diverse disturbance force profiles that act on the load carriage a force control unit is applied. This unit comprises a synchronizing cylinder and a servo valve and guarantees a tunable disturbance force trajectory via a PI-controller. The quantities used to compute the control law are measured at the testbed by piezo-resistive sensors for the pressures, an incremental position sensor and a force sensor with a sampling time $T = 0.004$ s. Since the proportional valve is characterized by an overlap a digitally implemented valve compensation ensures zero-lap behavior (adapted valve voltage $\hat{u}$).

The plant *differential cylinder/servo valve* is represented by the state-space model (this model can be rewritten into an ALS of the form in eq. (1)) (Bröcker, 2001)

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \frac{1}{m_{\mathrm{ta}}}\left(\left(x_3 - \frac{x_4}{\varphi}\right) A_{\mathrm{A}} - F_{\mathrm{f}}(x_2) - F_{\mathrm{d}}\right) \\
\dot{x}_3 &= \frac{E_{\mathrm{oil}}}{V_{\mathrm{A}}(x_1)}(-A_{\mathrm{A}} x_2 + B_{\mathrm{v}} K_{\mathrm{v}} a_1(x_3) U) \\
\dot{x}_4 &= \frac{E_{\mathrm{oil}}}{V_{\mathrm{B}}(x_1)}\left(\frac{A_{\mathrm{A}}}{\varphi} x_2 - B_{\mathrm{v}} K_{\mathrm{v}} a_2(x_4) U\right),
\end{aligned} \tag{7}$$

with the states

$x_1 = y_{\mathrm{cyl}}$ cylinder piston position [m],
$x_2 = \dot{y}_{\mathrm{cyl}}$ cylinder piston velocity [$\mathrm{ms}^{-1}$],
$x_3 = p_{\mathrm{A}}$ oil pressure in chamber A [Pa],
$x_4 = p_{\mathrm{B}}$ oil pressure in chamber B [Pa].

The area ratio is defined as $\varphi = A_{\mathrm{A}}/A_{\mathrm{B}}$ and the oil volumes of each pipe and chamber are calculated as $V_{\mathrm{A}}(x_1) = V_{\mathrm{pipe,A}} + x_1 A_{\mathrm{A}}$ and $V_{\mathrm{B}}(x_1) =$

$V_{\text{pipe,B}} + (H - x_1)\frac{A_\text{A}}{\varphi}$ (where $H$ is the cylinder stroke). The quantity $B_\text{v}$ denotes the flow resistance value and $K_\text{v}$ the valve amplification. The friction force $F_\text{f}$ can be approximated by the "Stribeck–curve" as a combination of viscious friction $f_\text{vi}$, static friction $F_\text{s}$ and coulomb friction $F_\text{c}$ with the distinction of cases for the cylinder piston velocity

$$F_\text{f}(x_2) = \begin{cases} f_\text{vi}x_2 + F_\text{c} + F_\text{s}\text{e}^{-\frac{x_2}{c_\text{s}}}, \forall\ x_2 \geq 0 \\ f_\text{vi}x_2 - F_\text{c} - F_\text{s}\text{e}^{\frac{x_2}{c_\text{s}}}, \quad \forall\ x_2 < 0. \end{cases} \tag{8}$$

The functions $a_1(x_3)$ and $a_2(x_4)$ are defined as

$$\begin{aligned} a_1(x_3) &= \begin{cases} \text{sgn}(p_0 - x_3)\sqrt{|p_0 - x_3|}\ , \forall\ U \geq 0 \\ \text{sgn}(x_3 - p_\text{t})\sqrt{|x_3 - p_\text{t}|}\ , \forall\ U < 0, \end{cases} \\ a_2(x_4) &= \begin{cases} \text{sgn}(x_4 - p_\text{t})\sqrt{|x_4 - p_\text{t}|}\ , \forall\ U \geq 0 \\ \text{sgn}(p_0 - x_4)\sqrt{|p_0 - x_4|}\ , \forall\ U < 0. \end{cases} \end{aligned} \tag{9}$$

The system input is defined as the normalized valve voltage $U = u/u_\text{max}$. The system output is the cylinder piston position $y_\text{cyl}$ and the disturbance is the disturbance force $F_\text{d}$. The analytical model is valid assuming the following simplifications:

- no gravitational effects,
- no leakage,
- constant total accelerated mass $m_\text{ta}$, oil elasticity $E_\text{oil}$, pump and tank pressure $p_0, p_\text{t}$,
- zero-lap and proportional behavior of the valve.

## 4. CONTROLLER FOR THE DISTURBANCE ATTENUATION

The relative degree $r$ and the disturbance relative degree $r_d$ for the system differential cylinder/proportional valve amount to $r = 3$ and $r_d = 2$. Consequently, the condition $r_d < r$ holds and the system is not disturbance decouplable. Therefore, the disturbance attenuation is applied as follows. The control law for disturbance attenuation can be derived from eq. (2)–(5). The first three derivatives of the cylinder piston position are computed as:

$$\dot{y}_\text{cyl} = x_2, \tag{10}$$

$$\ddot{y}_\text{cyl} = \frac{1}{m_\text{ta}}\left(\left(x_3 - \frac{x_4}{\varphi}\right)A_\text{A} - F_\text{f}(x_2) - F_\text{d}\right), \tag{11}$$

$$\begin{aligned} y_\text{cyl}^{(3)} = {} & \frac{A_\text{A}E_\text{oil}(-A_\text{A}x_2 + B_\text{v}K_\text{v}a_1(x_3)U)}{V_\text{A}(x_1)m_\text{ta}} - \frac{\dot{F}_\text{d}}{m_\text{ta}} \\ & - \frac{\frac{\text{d}F_\text{f}(x_2)}{\text{d}x_2}\left(\left(x_3 - \frac{x_4}{\varphi}\right)A_\text{A} - F_\text{f}(x_2) - F_\text{d}\right)}{m_\text{ta}^2} \\ & - \frac{A_\text{A}E_\text{oil}\left(\frac{A_\text{A}x_2}{\varphi} - B_\text{v}K_\text{v}a_2(x_4)U\right)}{V_\text{B}(x_1)\varphi m_\text{ta}}. \end{aligned} \tag{12}$$

The tracking error of the cylinder piston position and its time derivatives are given by

$$\begin{aligned} e &= y_\text{rt} - y_\text{cyl}, \\ \dot{e} &= \dot{y}_\text{rt} - \dot{y}_\text{cyl}, \\ \ddot{e} &= \ddot{y}_\text{rt} - \ddot{y}_\text{cyl}, \\ e^{(3)} &= y_\text{rt}^{(3)} - y_\text{cyl}^{(3)}. \end{aligned} \tag{13}$$

The linear differential equation for the tracking error is then specified as

$$e^{(3)} + c_2\ \ddot{e} + c_1\ \dot{e} + c_0\ e = 0. \tag{14}$$

Substituting eq. (10)–(12) into eq. (13) yields

$$e = y_\text{rt} - x_1, \tag{15}$$

$$\dot{e} = \dot{y}_\text{rt} - x_2, \tag{16}$$

$$\ddot{e} = \ddot{y}_\text{rt} - \frac{\left(x_3 - \frac{x_4}{\varphi}\right)A_\text{A} - F_\text{f}(x_2) - F_\text{d}}{m_\text{ta}}, \tag{17}$$

$$\begin{aligned} e^{(3)} = {} & y_\text{rt}^{(3)} - \frac{A_\text{A}E_\text{oil}(-A_\text{A}x_2 + B_\text{v}K_\text{v}a_1(x_3)U)}{V_\text{A}(x_1)m_\text{ta}} \\ & + \frac{\frac{\text{d}F_\text{f}(x_2)}{\text{d}x_2}\left(\left(x_3 - \frac{x_4}{\varphi}\right)A_\text{A} - F_\text{f}(x_2) - F_\text{d}\right)}{m_\text{ta}^2} \\ & + \frac{A_\text{A}E_\text{oil}\left(\frac{A_\text{A}x_2}{\varphi} - B_\text{v}K_\text{v}a_2(x_4)U\right)}{V_\text{B}(x_1)m_\text{ta}\varphi} \\ & + \frac{\dot{F}_\text{d}}{m_\text{ta}}. \end{aligned} \tag{18}$$

Afterwards, substituting eq. (15)–(18) into eq. (14) and solving the equation with respect to the normalized valve voltage $U$ leads to the control law

$$U_\text{att} = \frac{U_\text{n}}{U_\text{d}}, \text{ with} \tag{19}$$

$$\begin{aligned} U_\text{n} = {} & y_\text{rt}^{(3)}V_\text{A}(x_1)V_\text{B}(x_1)\varphi^2 m_\text{ta}^2 + E_\text{oil}A_\text{A}^2 m_\text{ta}V_\text{B}(x_1)\varphi^2 x_2 \\ & + E_\text{oil}A_\text{A}^2 m_\text{ta}V_\text{A}(x_1)x_2 + \dot{F}_\text{d}V_\text{A}(x_1)V_\text{B}(x_1)\varphi^2 m_\text{ta} \\ & + \frac{\text{d}F_\text{f}(x_2)}{\text{d}x_2}V_\text{A}(x_1)V_\text{B}(x_1)\varphi\left(\varphi A_\text{A}x_3 - A_\text{A}x_4\right) \\ & - \frac{\text{d}F_\text{f}(x_2)}{\text{d}x_2}V_\text{A}(x_1)V_\text{B}(x_1)\varphi^2\left(F_\text{f}(x_2) + F_\text{d}\right) \\ & + c_2V_\text{A}(x_1)V_\text{B}(x_1)\varphi m_\text{ta}\left(\varphi m_\text{ta}\ddot{y}_\text{rt} - \varphi A_\text{A}x_3 + A_\text{A}x_4\right) \\ & + c_2V_\text{A}(x_1)V_\text{B}(x_1)\varphi^2 m_\text{ta}\left(F_\text{f}(x_2) + F_\text{d}\right) \\ & + c_1V_\text{A}(x_1)V_\text{B}(x_1)\varphi^2 m_\text{ta}^2\left(\dot{y}_\text{rt} - x_2\right) \\ & + c_0V_\text{A}(x_1)V_\text{B}(x_1)\varphi^2 m_\text{ta}^2\left(y_\text{rt} - x_1\right) \end{aligned}$$

$$U_\text{d} = E_\text{oil}A_\text{A}m_\text{ta}\varphi B_\text{v}K_\text{v}\left(V_\text{B}(x_1)\varphi a_1(x_3) + V_\text{A}(x_1)a_2(x_4)\right)$$

that depends on various quantities – not only the measured quantities $y_\text{cyl}, p_\text{A}, p_\text{B}, F_\text{d}$ – but also the time derivative of the disturbance force $\dot{F}_\text{d}$.

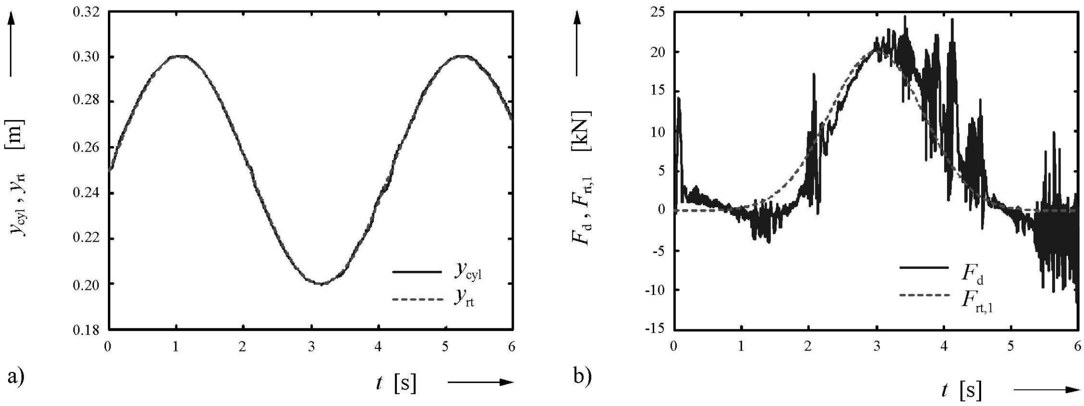


Fig. 3. Experimental results for a bell-shaped disturbance force profile $F_{\mathrm{rt},1}$: (a) cylinder piston position (b) disturbance force

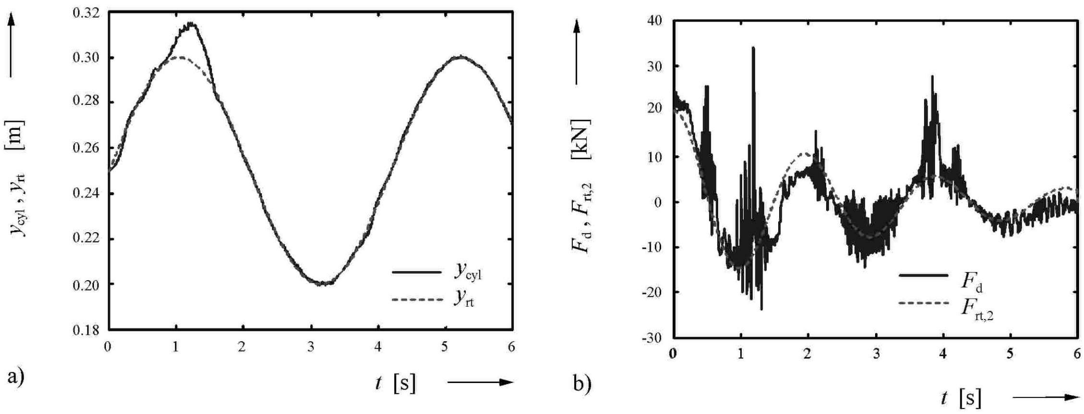


Fig. 4. Experimental results for disturbance force profile in the style of a damped vibration $F_{\mathrm{rt},2}$: (a) cylinder piston position (b) disturbance force

## 5. EXPERIMENTAL RESULTS

As the reference trajectory for the cylinder piston position $y_{\mathrm{rt}} = 0.05\sin(1.5t) + 0.25$ [m] is chosen. The pump and tank pressure of the plant are set to $p_0 = 80\cdot 10^5$ Pa and $p_{\mathrm{t}} = 1\cdot 10^5$ Pa, respectively. The force control unit is driven by the pressures $p_{0,\mathrm{f}} = 100\cdot 10^5$ Pa and $p_{\mathrm{t,f}} = 1\cdot 10^5$ Pa. As a first disturbance force profile $F_{\mathrm{rt},1}$ a bell-shaped curve is chosen and the second profile $F_{\mathrm{rt},2}$ is in the style of a damped vibration

$$F_{\mathrm{rt},1} = F_{\mathrm{a}} \mathrm{e}^{-\frac{(t-t_0)^2}{b_0}}, \text{ and} \tag{20}$$

$$F_{\mathrm{rt},2} = \sqrt{F_2^2 + \frac{(D\omega_2 F_2)^2}{\nu_2^2}}\, \mathrm{e}^{-D\omega_2 t} \cdot \cos(\nu_2 t - \varphi_2), \tag{21}$$

with $F_{\mathrm{a}} = 20$ kN, $t_0 = 3$ s, $b_0 = 1$ s$^2$, $F_2 = 20$ kN, $\omega_2 = 3.25$ s$^{-1}$, $D = 0.1$, $\nu_2 = 3.2337$ s$^{-1}$, $\varphi_2 = 0.1002$ rad. To place the poles $P_1 = -300$ s$^{-1}$, $P_{2,3} = -50 \pm 50\,j$ [s$^{-1}$] for the disturbance attenuation controller, the coefficients read $c_0 = 1500000$ s$^{-3}$, $c_1 = 35000$ s$^{-2}$, $c_2 = 400$ s$^{-1}$ and an optimal control behavior is obtained. Figure 3(a) demonstrates very good tracking performance for the cylinder piston position $y_{\mathrm{cyl}}$ with the disturbance attenuation controller when the bell-shaped profile $F_{\mathrm{rt},1}$ is tracked. The controlled disturbance force $F_{\mathrm{d}}$ in figure 3(b) oscillates around the trajectory marginally. If the disturbance force $F_{\mathrm{d}}$ (see figure 4(b)) is controlled w.r.t. $F_{\mathrm{rt},2}$, the tracking error for the cylinder piston position (see figure 4(a)) in the first reversion of the sinusoidal trajectory is relatively high. The reason for this can be traced in the high force interval of the disturbance force at that space of time. The disturbance attenuation controller reduces the tracking error only for a limited disturbance force to a tolerable limit. In this time interval the reserve of the adapted valve voltage $\hat{u}$ ($\pm 10$ V) is reached. This is the space of time where the cylinder piston position cannot exactly follow the reference trajectory.

## 6. CONCLUSIONS

A nonlinear disturbance attenuation controller has been derived and implemented on a testbed choosing a differential cylinder and a proportional valve as the plant. Additionally, the testbed consists of a force control unit in order to guarantee a tunable disturbance force trajectory via

PI-control. The nonlinear disturbance attenuation controller shows very good tracking performance for the cylinder piston position if a bell-shaped curve as the disturbance force profile is chosen. A second disturbance force profile in the style of a damped vibration demonstrates the limits of disturbance attenuation. The control is limited to a particular disturbance force margin which is typical for disturbance attenuation.

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