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YOULA PARAMETRIZATION APPLIED TO A GPC CONTROLLED SISO SYSTEM

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Abstract: This paper presents a new approach based on Youla parametrization, to retune Generalised Predictive Control (GPC) law while preserving its two degrees of freedom structure. This results in a more robust controller with improved high frequency disturbance rejection properties. The proposed strategy expresses an initial controller in a powerful form via the Youla theory, each transfer function of the resulting closed loop system being independently modified. This structure is then particularised to GPC, to adjust the transfer between measurement noise and system output, without changes in the initial input/output closed loop, so that performance of the initial GPC controller is enhanced. *Copyright © 2002 IFAC*

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1. INTRODUCTION

A classical technique to enhance qualities of GPC controllers is to make use of a model parameter as an additional degree of freedom. In this way, defining the prediction model in the classical CARIMA form, the *C* polynomial modelling the noise influence may be used as a tuning parameter, since its identification is usually avoided. This approach is developed by Yoon and Clarke (1995). It has been shown that the *C* polynomial plays a crucial role in the robustness of the control law. More generally, this polynomial influences the robustness and disturbance rejection, unfortunately its choice remains complicated.

Another way to introduce extra parameters is given by the *Q*-parametrization. It was first used by Kouvaritakis, *et al.*, (1992) to enhance robustness of the control: a robust optimisation problem is defined and an optimal *Q* parameter is derived. This parameter is also considered by Ansay, *et al*., (1998) to adjust the compromise between disturbance rejection and robustness. This *Q*-parametrization can be referred to as a Youla type parametrization, see (Maciejowski, 1989) with additional implicit assumptions which restrict the result. It must be noted that links between the *C* polynomial and the *Q* parameter exist, see for example (Yoon and Clarke, 1995). A complementary interpretation of the parameter can then be obtained but the choice of extra parameters remains difficult.

This paper presents a "full" Youla parameter approach aiming at retuning GPC controllers, which in fact provides an original and more general structure compared to all strategies stated above. Moreover, it guaranties that all stabilizing controllers can be examined. Section 2 briefly reminds the different steps required for the GPC controller design. Then, as a starting point of the method, Section 3 considers a SISO system with an initial two-degrees of freedom controller transformed into a MIMO system with a one-degree of freedom controller.
From this model, Section 4 shows that the corresponding stabilizing controllers for the MIMO case can be parametrized by a 2×2 matrix of stable transfers using Youla theory. To maintain equivalence with the original SISO case, additional

constraints are then considered. As a result, a subset of the Youla parameter is obtained that characterizes all the two-degrees of freedom stabilizing controllers of the original SISO system. In connection with the proposed strategy, the characterization of stabilizing controllers which maintain particular closed loop transfer functions invariant is also studied. It is shown that this general approach, particularized to keep transfer between reference and output invariant for a two-degrees of freedom controller enables to recover classical results.
Finally, Section 5 provides simulation results obtained on a GPC controlled asynchronous motor. It is shown that this method can optimise noise rejection through an appropriate choice of the Q parameter.

2. GPC DESIGN

This part briefly reminds the basic steps of the GPC controller design, more details may be found in (Bitmead, *et al.*, 1990 or Dumur and Boucher, 1998). In the GPC theory, the plant is classically modelled by the input/output CARIMA form:

$$A(q^{-1})y(t) = B(q^{-1})u(t-1) + C(q^{-1})\xi(t) \Big/ \Delta(q^{-1}) \quad (1)$$

$\xi(t)$ is a zero mean non-correlated white noise, and as previously mentioned, $C(q^{-1})$ models the noise influence (Clarke and Mohtadi, 1989). The introduction of difference operator $\Delta(q^{-1}) = 1 - q^{-1}$ in the disturbance model helps to find an integral action in the controller and so eliminate the static errors. The control signal is obtained by minimisation of a quadratic cost function:

$$J = \sum_{j=N_1}^{N_2} \left[w(t+j) - \hat{y}(t+j)\right]^2 + \lambda \sum_{j=1}^{N_u} \Delta u(t+j-1)^2 \quad (2)$$

Where N_1 and N_2 define the output prediction horizons, and N_u the control horizon. λ is the control weighting factor, w is the reference value, $\hat{y}$ is the predicted output value, obtained solving diophantine equations, and u the control signal.
The receding horizon principle assumes that only the first value of the optimal control sequence is applied, so that at the next sampling period the same procedure is repeated. This control strategy leads to a two-degrees of freedom RST controller:

$$S(q^{-1})\Delta(q^{-1})u(t) = -R(q^{-1})y(t) + T(q)w(t) \quad (3)$$

At this stage, it is assumed that the design has been performed with $C(q^{-1}) = 1$ and N_1, N_2, N_u, λ adjusted to obtain the required input/output behaviour. The resulting two-degrees of freedom RST controller will be denoted R_0, S_0, T_0 in the following sections (Figure 1). It is assumed that the plant model is perfectly identified.

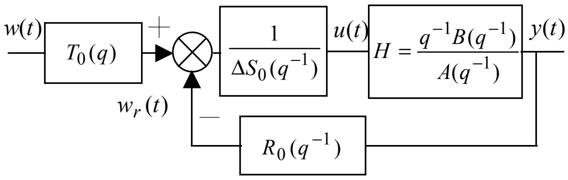

Fig. 1. Two-degrees of freedom GPC controller

3. TWO-DEGREES OF FREEDOM CONTROLLER AS A 2-INPUTS/2-OUTPUTS SYSTEM

A SISO system with a two-degrees of freedom controller can be represented as stated on Figure 2.

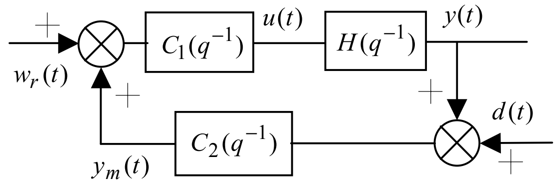

Fig. 2. Two-degrees of freedom controller.

Theorem 1. A two-degrees of freedom controller can be expressed as a one-degree of freedom controller for a 2-inputs/2-outputs plant with a structured 2-inputs/2-outputs controller.

Proof: Figure 2 can be modified to obtain the following scheme of Figure 3.

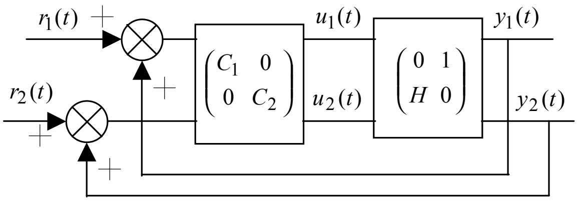

Fig. 3. Two-degrees of freedom controller modelled as a 2-inputs/2-outputs system.

With the following equivalence between transfers:

$$\frac{y_1}{r_1} = \frac{y_m}{w_r};\ \frac{y_1}{r_2} = \frac{y_m}{d};\ \frac{y_2}{r_1} = \frac{y}{w_r};\ \frac{y_2}{r_2} = \frac{y}{d} \quad (4)$$

4. PARAMETRIZATION OF ALL STABILIZING CONTROLLERS

4.1 General consideration.

Figure 4 considers a general feedback loop, with the classical notations used in our following developments.

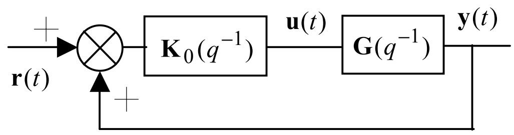

Fig. 4. General feedback loop

Theorem 2. Let:

$$\mathbf{G} = \mathbf{N}\,\mathbf{M}^{-1} = \widetilde{\mathbf{M}}^{-1}\,\widetilde{\mathbf{N}} \tag{5}$$

and:
$$\mathbf{K}_0 = \mathbf{U}_0\,\mathbf{V}_0^{-1} = \widetilde{\mathbf{V}}_0^{-1}\,\widetilde{\mathbf{U}}_0 \tag{6}$$

be the fractional representations of $\mathbf{G}$ and $\mathbf{K}_0$ respectively. If $\mathbf{K}_0$ is a stabilizing controller, then $\mathbf{N},\mathbf{M}$, $\mathbf{U}_0,\mathbf{V}_0,\widetilde{\mathbf{N}},\widetilde{\mathbf{M}},\widetilde{\mathbf{U}}_0$ and $\widetilde{\mathbf{V}}_0$ can be chosen such that:

$$\begin{bmatrix} \widetilde{\mathbf{V}}_0 & -\widetilde{\mathbf{U}}_0 \\ -\widetilde{\mathbf{N}} & \widetilde{\mathbf{M}} \end{bmatrix} \begin{bmatrix} \mathbf{M} & \mathbf{U}_0 \\ \mathbf{N} & \mathbf{V}_0 \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \tag{7}$$

Proof: See (Maciejowski, 1989)

Theorem 3. Let Eqs. 5 and 6 verified, such that Eq. 7 holds. For any compatible dimensions stable transfer matrix $\mathbf{Q}$, define:

$$\mathbf{U} = \mathbf{U}_0 + \mathbf{M}\,\mathbf{Q} \qquad \mathbf{V} = \mathbf{V}_0 + \mathbf{N}\,\mathbf{Q} \tag{8}$$
$$\widetilde{\mathbf{U}} = \widetilde{\mathbf{U}}_0 + \mathbf{Q}\,\widetilde{\mathbf{M}} \qquad \widetilde{\mathbf{V}} = \widetilde{\mathbf{V}}_0 + \mathbf{Q}\,\widetilde{\mathbf{N}} \tag{9}$$

1) Then $\mathbf{U}\,\mathbf{V}^{-1} = \widetilde{\mathbf{V}}^{-1}\widetilde{\mathbf{U}}$ and $\mathbf{K} = \mathbf{U}\,\mathbf{V}^{-1} = \widetilde{\mathbf{V}}^{-1}\widetilde{\mathbf{U}}$ is a stabilizing controller for $\mathbf{G}$ given by Eq. 5.
2) Furthermore, any stabilizing controller has fractional representation Eq. 8 and Eq. 9.

Proof: See (Maciejowski, 1989)

4.2 Parametrization of all stabilizing controllers for a two-degrees of freedom controller represented as a 2-inputs/2-outputs system.

According to the previous notations, the system represented Figure 3 can be modelled with the following matrices:

$$\mathbf{K}_0 = \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix} \quad \mathbf{G} = \begin{bmatrix} 0 & 1 \\ H = \dfrac{H_n}{H_d} & 0 \end{bmatrix} \tag{10}$$

The aim is thus to find a fractional representation that holds Eqs. 5, 6 and 7 in order to parametrize all stabilizing controllers as stated by Eqs. 8 or 9. In order to achieve that, C_1 and C_2 may be decomposed into a numerator and denominator part:

$$C_1 = C_{1n}/C_{1d} \qquad C_2 = C_{2n}/C_{2d}$$

With this notation, the closed loop of Figure 2 is:

$$\begin{aligned} \frac{y}{w_r} &= \frac{C_{2d}\,C_{1n}\,H_n}{H_d\,C_{1d}\,C_{2d} - C_{1n}\,C_{2n}\,H_n} \\ &= \frac{C_{2d}\,C_{1n}\,H_n}{A_o\;A_c} \end{aligned} \tag{11}$$

assuming that the characteristic polynomial of this closed loop can be separated in a control polynomial A_c and an observer polynomial A_o, both stable, as in pole placement, see (Åström and Wittenmark, 1997).

The fractional representation must hold Eqs. 5, 6 and 7. These expressions include eight matrices equations with eight unknown parameters. In these equations, it is straightforward to show that:

$$\begin{aligned} \widetilde{\mathbf{V}}_0\,\mathbf{U}_0 - \widetilde{\mathbf{U}}_0\,\mathbf{V}_0 &= \mathbf{0} \\ -\widetilde{\mathbf{N}}\,\mathbf{M} + \widetilde{\mathbf{M}}\,\mathbf{N} &= \mathbf{0} \end{aligned}$$

are redundant in Eqs. 6 and 7, and in Eqs. 5 and 7. Only six equations remain with eight unknown parameters, as follows:

$$\begin{cases} \mathbf{K}_0 = \mathbf{U}_0\,\mathbf{V}_0^{-1} \\ \mathbf{K}_0 = \widetilde{\mathbf{V}}_0^{-1}\,\widetilde{\mathbf{U}}_0 \\ \mathbf{G} = \mathbf{N}\,\mathbf{M}^{-1} \\ \mathbf{G} = \widetilde{\mathbf{M}}^{-1}\,\widetilde{\mathbf{N}} \\ \widetilde{\mathbf{V}}_0\,\mathbf{M} - \widetilde{\mathbf{U}}_0\,\mathbf{N} = \mathbf{I} \\ -\widetilde{\mathbf{N}}\,\mathbf{U}_0 + \widetilde{\mathbf{M}}\,\mathbf{V}_0 = \mathbf{I} \end{cases} \tag{12}$$

However, if $\mathbf{M}$ and $\widetilde{\mathbf{M}}$ are fixed, the other unknown parameters can be derived solving previous relations:

$$\begin{cases} \mathbf{U}_0 = (\mathbf{I} - \mathbf{K}_0\,\mathbf{G})^{-1}\mathbf{K}_0\,\widetilde{\mathbf{M}}^{-1} \\ \mathbf{V}_0 = \mathbf{K}_0^{-1}\,\mathbf{U}_0 \\ \widetilde{\mathbf{U}}_0 = \mathbf{M}^{-1}\,\mathbf{K}_0(\mathbf{I} - \mathbf{G}\,\mathbf{K}_0)^{-1} \\ \widetilde{\mathbf{V}}_0 = \widetilde{\mathbf{U}}_0\,\mathbf{K}_0^{-1} \\ \mathbf{N} = \mathbf{G}\,\mathbf{M} \\ \widetilde{\mathbf{N}} = \widetilde{\mathbf{M}}\,\mathbf{G} \end{cases} \tag{13}$$

The fractional representation found in this way is valid if all transfers of $\mathbf{N},\mathbf{M},\mathbf{U}_0,\mathbf{V}_0,\widetilde{\mathbf{N}},\widetilde{\mathbf{M}},\widetilde{\mathbf{U}}_0$ and $\widetilde{\mathbf{V}}_0$ are stable.

Theorem 4. For a system with structure of Figure 3, a Youla parametrization exists such that the four transfers of the system can be expressed as:

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} A_{cl\,11} & A_{cl\,12} \\ A_{cl\,21} & A_{cl\,22} \end{pmatrix} \begin{pmatrix} r_1 \\ r_2 \end{pmatrix} \tag{14}$$

with:
$$\left| \begin{aligned} A_{cl\,11} &= \frac{H_n\,C_{1n}\,C_{2n} + A_o\,A_c\,Q_{21}}{A_o\,A_c} \\ A_{cl\,12} &= \frac{H_d\,(C_{1d}\,C_{2n} + A_c\,Q_{22})}{A_o\,A_c} \\ A_{cl\,21} &= \frac{H_n\,(C_{1n}\,C_{2d} + A_o\,Q_{11})}{A_o\,A_c} \\ A_{cl\,22} &= \frac{H_n\,(C_{1n}\,C_{2n} + H_d\,Q_{12})}{A_o\,A_c} \end{aligned} \right.$$

Where each transfer is parametrized independently from the others, and with Q_{ij} free stable transfers. Taking into account the equivalences in Eq. 4, the transfer modified by each parameter Q_{ij} is:

Q_{11} modifies $\frac{y_2}{r_1} = \frac{y}{w_r}$ $\quad$ Q_{12} modifies $\frac{y_2}{r_2} = \frac{y}{d}$

Q_{21} modifies $\frac{y_1}{r_1} = \frac{y_m}{w_r}$ $\quad$ Q_{22} modifies $\frac{y_1}{r_2} = \frac{y_m}{d}$

Proof: Choosing:

$$\mathbf{M} = \begin{pmatrix} \frac{H_d}{A_c} & 0 \\ 0 & 1 \end{pmatrix} \qquad \tilde{\mathbf{M}} = \begin{pmatrix} 1 & 0 \\ 0 & \frac{H_d}{A_o} \end{pmatrix} \tag{15}$$

the following fractional representation is obtained according to Eq. 13:

$$\mathbf{N} = \begin{pmatrix} 0 & 1 \\ \frac{H_n}{A_c} & 0 \end{pmatrix} \qquad \tilde{\mathbf{N}} = \begin{pmatrix} 0 & 1 \\ \frac{H_n}{A_o} & 0 \end{pmatrix} \tag{16a}$$

$$\begin{aligned} \mathbf{U}_0 &= \begin{pmatrix} \frac{H_d\, C_{1n}\, C_{2d}}{A_o\, A_c} & \frac{C_{1n}\, C_{2n}}{A_c} \\ \frac{H_n\, C_{1n}\, C_{2n}}{A_o\, A_c} & \frac{C_{1d}\, C_{2n}}{A_c} \end{pmatrix} \\ \mathbf{V}_0 &= \begin{pmatrix} \frac{H_d\, C_{1d}\, C_{2d}}{A_o\, A_c} & \frac{C_{1d}\, C_{2n}}{A_c} \\ \frac{H_n\, C_{1n}\, C_{2d}}{A_o\, A_c} & \frac{C_{1d}\, C_{2d}}{A_c} \end{pmatrix} \end{aligned} \tag{16b}$$

$$\begin{aligned} \tilde{\mathbf{U}}_0 &= \begin{pmatrix} \frac{C_{1n}\, C_{2d}}{A_o} & \frac{C_{1n}\, C_{2n}}{A_o} \\ \frac{H_n\, C_{1n}\, C_{2n}}{A_o\, A_c} & \frac{H_d\, C_{1d}\, C_{2n}}{A_o\, A_c} \end{pmatrix} \\ \tilde{\mathbf{V}}_0 &= \begin{pmatrix} \frac{C_{1d}\, C_{2d}}{A_o} & \frac{C_{1n}\, C_{2d}}{A_o} \\ \frac{H_n\, C_{1d}\, C_{2n}}{A_o\, A_c} & \frac{H_d\, C_{1d}\, C_{2d}}{A_o\, A_c} \end{pmatrix} \end{aligned} \tag{16c}$$

Each transfer is stable because $\mathbf{K}_0$ is considered as a stabilizing controller and A_o and A_c are both stable.

With this fractional representation and applying Eq. 8 or Eq. 9, all stabilizing controllers can be deduced. Applying Eq. 8, e.g., the following controller is found:

$$\mathbf{K} = \mathbf{U}\,\mathbf{V}^{-1} = \begin{pmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{pmatrix} \tag{17}$$

with: $\mathbf{U} = \{U_{ij}\}\big|_{i=1,2;j=1,2}$, $\mathbf{V} = \{V_{ij}\}\big|_{i=1,2;j=1,2}$, and:

$$\begin{aligned} U_{11} &= \frac{H_d\,(C_{1n}\, C_{2d} + A_o\, Q_{11})}{A_o\, A_c} \\ U_{12} &= \frac{C_{1n}\, C_{2n} + H_d\, Q_{12}}{A_c} \\ U_{21} &= \frac{H_n\, C_{1n}\, C_{2n} + A_o\, A_c\, Q_{21}}{A_o\, A_c} \\ U_{22} &= \frac{C_{1d}\, C_{2n} + A_c\, Q_{22}}{A_c} \\ V_{11} &= \frac{H_d\, C_{1d}\, C_{2d} + A_o\, A_c\, Q_{21}}{A_o\, A_c} \\ V_{12} &= \frac{C_{1d} C_{2n} + A_c\, Q_{22}}{A_c} \\ V_{21} &= \frac{H_n\,(C_{1n}\, C_{2d} + A_o\, Q_{11})}{A_o\, A_c} \\ V_{22} &= \frac{C_{1d}\, C_{2d} + H_n\, Q_{12}}{A_c} \end{aligned} \tag{18}$$

In the other hand, the system of Figure 3, with the controller Eq. 17, has the input/output transfers:

$$\begin{aligned} \frac{y1}{r1} &= \frac{K_{21}\, H_d - H_n\, K_{12} K_{21} + H_n\, K_{11} K_{22}}{(1-K_{21})(H_d - H_n\, K_{12}) - K_{11} K_{22} H_n} \\ \frac{y1}{r2} &= \frac{A K_{22}}{(1-K_{21})(H_d - H_n\, K_{12}) - K_{11} K_{22} H_n} \\ \frac{y2}{r1} &= \frac{H_n\, K_{11}}{(1-K_{21})(H_d - H_n\, K_{12}) - K_{11} K_{22} H_n} \\ \frac{y2}{r2} &= \frac{H_n\, K_{11} K_{22} + (1-K_{21})\, H_n\, K_{12}}{(1-K_{21})(H_d - H_n\, K_{12}) - K_{11} K_{22} H_n} \end{aligned} \tag{19}$$

Replacing in these transfers the controller found with Eqs. 17 and 18 leads after calculation to the transfers of Theorem 4, Eq. 14, which achieved the proof.

4.3 Back to the diagonal controller.

In order to find the class of stabilizing controllers that can be implemented as two-degrees of freedom controllers, only the diagonal controllers in the previous parametrization of all stabilizing controllers are considered. As a consequence, $K_{12} = K_{21} = 0$ is imposed, leading to the following controller after calculations:

$$\begin{aligned} K_{11} &= \frac{C_{1n}\, C_{2d} + A_o\, Q_{11}}{C_{1d}\, C_{2d} + H_n\, Q_{12}} \\ K_{22} &= \frac{C_{1n}\, C_{2n} + H_d\, Q_{12}}{C_{1n}\, C_{2d} + A_o\, Q_{11}} \end{aligned} \tag{20}$$

with:

$$Q_{21} = \frac{H_d\, H_n\, Q_{12}}{A_o\, A_c}$$

$$Q_{22} = \frac{(C_{1n}\, C_{2n} + H_d\, Q_{12})(C_{1d}\, C_{2d} + H_n\, Q_{12}) - C_{1d}\, C_{2n}\,(C_{1n}\, C_{2d} + A_o\, Q_{11})}{A_c\,(C_{1n}\, C_{2d} + A_o\, Q_{11})}$$

Consequently, among the four parameters Q_{11}, Q_{12}, Q_{21} and Q_{22}, only two of them remain free and only two transfers can be independently modified.

4.4 Application to a two-degrees of freedom GPC-controller initially designed with $C(q^{-1}) = 1$.

According to Figure 1 and previous notations, the plant and the controller are respectively given by the following relations:

$$H = \frac{q^{-1}B}{\Delta A} \quad C_1 = \frac{1}{S_0} \quad C_2 = -R_0 = \frac{R_0}{-1} \quad (21)$$

Δ is introduced in the denominator of H in order to preserve the integral action of the controller even after parametrization. Applying Eq. 20, all stabilizing diagonal controllers are:

$$\begin{aligned} K_{11} &= \frac{1 - A_o Q_{11}}{S_0 - q^{-1}B\,Q_{12}} \\ K_{22} &= -\frac{R_0 + \Delta A\,Q_{12}}{1 - A_o Q_{11}} \end{aligned} \quad (22)$$

N_1, N_2, N_u and λ have been adjusted to provide the desired behaviour of the *y*/*w* transfer. To maintain this transfer unchanged, Q_{11} must be zero, and the final controller becomes (Figure 5):

$$\begin{aligned} K_{11} &= \frac{1}{S_0 - q^{-1}B\,Q_{12}} \\ K_{22} &= -(R_0 + \Delta A\,Q_{12}) \end{aligned} \quad (23)$$

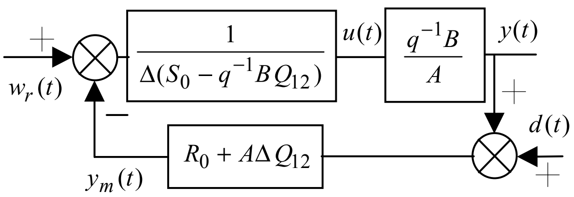

Fig. 5. Parametrization of the RST controller keeping the *y*/*w* transfer unchanged.

The only transfer to be modified thus remains *y*/*d* via the Q_{12} parameter. This particular result, deduced from the general "full" Youla parametrization presented above, is similar to the parametrization proposed by (Kouvaritakis, *et al.*, 1992; Yoon and Clarke, 1995; Ansay, *et al.*, 1998).
This transfer *y*/*d* is expressed as:

$$\begin{aligned} \frac{y}{d} &= \frac{q^{-1}\,B(C_{1n}C_{2n} + \Delta A\,Q_{12})}{A_o\,A_c} \\ &= \frac{q^{-1}B\,R_0}{A_o\,A_c} + \frac{q^{-1}B\,\Delta A}{A_o\,A_c}Q_{12} \end{aligned}$$

with $A_o\,A_c = \Delta A S_0 + q^{-1}BR_0$.

5. APPLICATION TO AN INDUCTION MACHINE

An interesting application of the previous result is connected to rejection of measurement noise in high frequencies, and Q_{12} can be chosen for that purpose to induce a 'low pass' behaviour to the *y*/*d* transfer and enhance robustness of the initial GPC controller against high frequencies uncertainties. Consider the speed control of an induction motor, for which measurement noise rejection must be achieved. An identified transfer function of this induction motor, between torque and velocity, (Dumur and Boucher, 1998) is, for a 5ms sampling period:

$$H(q^{-1}) = \frac{q^{-1}B(q^{-1})}{A(q^{-1})} = \frac{1.344q^{-1} + 3.024q^{-2}}{1 - 0.98q^{-1} - 0.02q^{-2}}$$

An initial GPC controller is designed with $C(q^{-1}) = 1$ with the following tuning parameters selected according to rules given in (Dumur and Boucher, 1998): $N_1 = 1, N_2 = 8, N_u = 1, \lambda = 200$.
The velocity and torque signals corresponding to a step setpoint are given respectively Figures 6 and 7 upper part. For this simulation, a zero mean random measurement noise of 0.25^2 variance is added as shown on Figure 2. The influence of this noise on the output and on the control signal clearly appears.

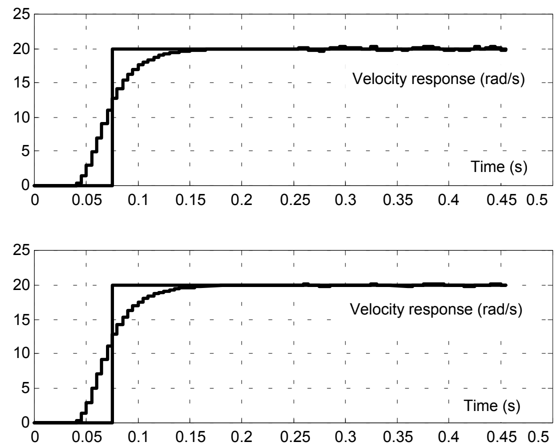

Fig. 6. Velocity output (upper part: initial GPC, lower part: robustified GPC)

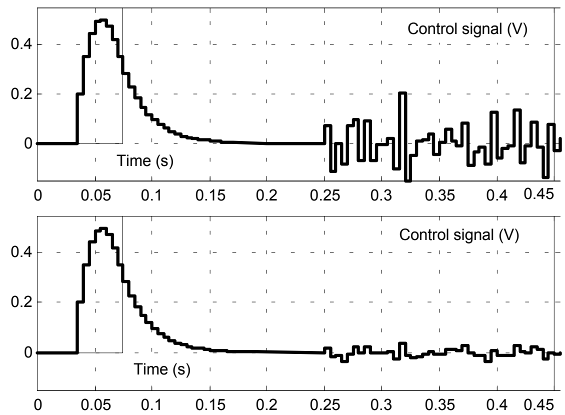

Fig. 7. Torque signal (upper part: initial GPC, lower part: robustified GPC)

To enhance the behaviour of the system with regard to the measurement noise, the Q_{12} parameter is chosen in order to confer to the *y*/*d* transfer a low pass behaviour. For that purpose, this choice may be performed through an optimisation procedure, or for example here following (Ansay, *et al.*, 1998). The *A* polynomial is first factorised in stable and unstable parts, $A = A^s A^u$, then Q_{12} is chosen as:

$$Q_{12} = \frac{R_0 M^*}{A^s C^*}$$

The transfer *y*/*d* is thus:

$$\frac{y}{d} = \frac{q^{-1} B\, R_0}{A_o\, A_c}\left[1 + \frac{\Delta A^u\, M^*}{C*}\right]$$

M^* and C^* can be chosen as:

$$M^* = -\mu_1 \mu_2$$
$$C^* = (1 - \mu_1\, q^{-1})(1 - \mu_2\, q^{-1})$$

Figures 6 and 7 (lower part) show the response obtained for $\mu_1 = 0.9$ and $\mu_2 = 0.8$. The influence of the measurement noise is obviously smaller, both on the velocity output and the torque signal, compared to results with the previous controller.

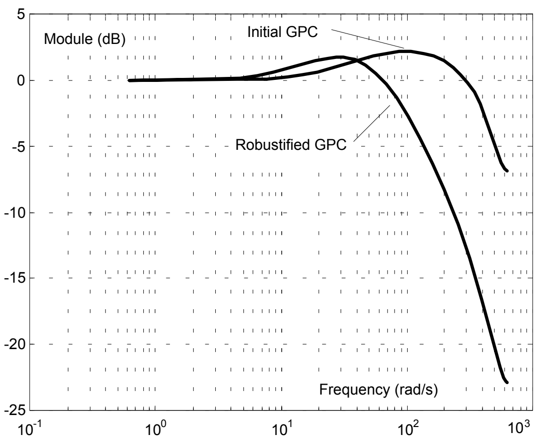

Fig. 8. Frequency response of the *y*/*d* transfer

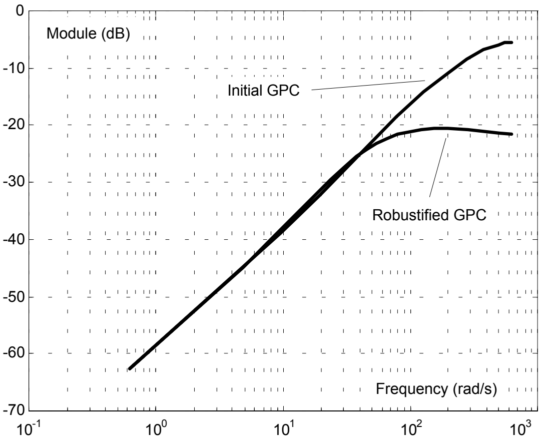

Fig. 9. Frequency response of the *u*/*d* transfer

Figures 8 and 9 give the frequency response of the *y*/*d* and *u*/*d* transfers, showing that the low-pass behaviour of both transfers has been increased due to the parametrization.

6. CONCLUSIONS

A general method which parametrizes all two-degrees of freedom stabilizing controllers has been presented through the definition of the Youla parameters. This theory has been applied to the GPC structure as a particular case of two-degrees of freedom controllers, in order to robustify its performance.
Moreover, a restricted family of stabilizing controllers keeping particular transfers unchanged has been examined. It has been shown that, for the case where the input/output transfer remains invariant, the presented theory leads to classical results found in the literature.
This strategy has been applied to a GPC-controlled induction motor, to improve measurement noise rejection. A good behaviour is obtained, mainly on the control signal, compared to GPC without extra degree of freedom.
With the general results presented here, other transfers than those classically considered may be robustified, using an optimisation procedure for the design of the Youla parameters.

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