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ROBUST OUTPUT FEEDBACK CONTROL OF NONLINEAR SYSTEMS WITH RANDOM JUMPS

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Abstract: A class of control systems, modeled as a finite set of differential equations with parameters uncertainty and with sector bounded nonlinearities is considered. Each model of this family describes the individual mode (or regime) of the system. The transitions between these modes are described by a homogeneous Markov chain. At the moment of discontinuous mode change the state vector can be changed by jump with uncertain parameters. The static output feedback control law is obtained, which guarantees exponential stability in the mean square of closed-loop system for all plant parameters uncertainty, all jump parameters uncertainty, and for all transition probabilities matrix uncertainty from the given domains. *Copyright ©2002 IFAC*

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1. INTRODUCTION

In real life we can find a lot of dynamical systems with random jumping changes of their structure or parameters, such as aerospace systems, manufacturing systems, economic systems, etc., see, for example, the books by Kats (1998), Kazakov and Artem'ev (1980), Mariton (1990) and the references therein. Systems with random jumps are *hybrid* ones with many operating modes. Every mode corresponds to an individual deterministic or stochastic dynamics. The system mode switching is governed by a Markov process with a finite set of states $\mathbb{N} = \{1, 2, \dots, \nu\}$ (Markov chain). When the mode $i \in \mathbb{N}$ is fixed, the plant state evolves according to the corresponding individual dynamic. So the state space of these systems is naturally hybrid: to the usual plant state in $\mathbb{R}^n$ we append a discrete variable taking values in the set $\mathbb{N}$.

The stability and control theory for the systems with random jumps began to develop since the pioneering works of Kats and Krasovskii (1960), Krasovskii and Lidskii (1961) correspondingly. The stochastic moment approach to the stability problem was introduced by Mil'stein (1972). The linear quadratic (LQ) control problem was solved by Sworder (1969) using stochastic maximum principle for state feedback in finite horizon case. Wonham (1970) obtained the same results using dynamic programming for both finite and infinite horizon cases. He also obtained a set of sufficient conditions for the existence of a finite solution. Kazakov and Artem'ev (1980) have developed a general theory of random structure systems based on Fokker-Planck-Kolmogorov type equation approach. Now, due the large number of applications several results for this class of systems can be found in the current literature, regarding stability, optimal control, stabilization, controllability and observability problems, see for instance (Ji and Chizeck, 1990; Kats, 1998; Mariton, 1990) and the references therein. For the latest papers in this field the reader is addressed to *Proceedings of the European Control Conference* (Porto, Portugal, 4-7 September 2001).

Robust control offers the advantage to design a controller which enables us to cope with the uncertainties which appear in the more realistic models. Few papers dealing with the robustness of the class of systems with random jumps have been published. The stability, stabilization, H_2, H_∞, mixed H_2/H_∞ problems and their robustness have been investigated.

Boukas (1995) has considered the robustness of the class of linear piecewise deterministic systems whose uncertainties are upper bounded. A sufficient condition for stochastic stabilizability of this class of systems has been given under a state feedback control law. Boukas and Yang (1997) have dealt with the uncertain nonlinear piecewise deterministic systems. Under some special matching conditions, they have established sufficient conditions, which guarantee the stochastic stability robustness of this class of systems. Without any intention of being exhaustive here, we quote the papers by Shi (1996), Benjelloun, *et al.* (1998) and the references therein.

De Souza and Fragoso (1993) was considered the H_∞ problem for both finite and infinite horizon cases. It is shown that the finite-horizon problem can be tackled via a certain set of interconnected Riccati differential equation, while the solution for the infinite horizon case is based upon a set of interconnected algebraic Riccati equation. Aliyu and Boukas (1999) considered mixed H_2/H_∞ control problem and studied its robustness. The linear matrix inequality (LMI) optimization approach, see (Boyd, *et al.*, 1990) is effectively used in cited papers.

Usually it is supposed for considered class of systems that only the plant has parameters uncertainty. In this paper we study the robust static output feedback control problem in the case, when the plant contains sector bounded (Lur'e type) nonlinearities and both the plant and the regime change process (matrix of transition probabilities) have uncertainty in their parameters. Moreover we suppose that at the moment of mode (regime) change the plant state vector can be changed by jump with uncertain parameters. We develop here some ideas by Pakshin (1997), Pakshin and Retinskii (2001), and use the game theoretic approach (Basar and Bernhard, 1995; Kogan, 1999) for solution of the robust control problems under study.

The paper is organized as follows. In Section 2 we give the mathematical description of the considered nonlinear system. In section 3 we formulate and solve robust control problem against both plant parameters uncertainty and jump parameters uncertainty, formalized in the form of quadratic inequalities. We obtain this control in the explicit form of linear switching (regime dependent) plant state feedback, which guarantees exponential stability in the mean square of the considered system. In section 4 we obtain the perfect robust control i.e. the robust control against plant parameters uncertainty, jump parameters uncertainty and regime change parameters uncertainty. Some short concluding remarks ends the paper.

2. SYSTEM DESCRIPTION

Consider a control system described by the family of differential equations

$$\begin{aligned}\dot{\mathbf{x}}(t) = [\mathbf{A}(r_t) + \mathbf{F}(r_t)\boldsymbol{\Omega}(t,r_t)\mathbf{E}(r_t)]\mathbf{x}(t) + \\ \mathbf{B}(r_t)\mathbf{u}(t) + \mathbf{D}(r_t)\varphi(t,\mathbf{z}_t), \\ \mathbf{y}(t) = \mathbf{C}(r_t)\mathbf{x}(t), \ \ \mathbf{z}(t) = \mathbf{L}(r_t)\mathbf{x}(t), \quad (1)\end{aligned}$$

where $\mathbf{x}(t)$ is the n-dimensional plant state vector; $\mathbf{u}(t)$ is the k-dimensional control vector; $\mathbf{y}(t),\ \mathbf{z}(t)$ are s-dimensional and m- dimensional output vectors; r_t is homogeneous discrete state Markov process (Markov chain) representing a mode (or regime) of system and taking values in a finite set $\mathbb{N} = \{1,\dots,\nu\}$ with a matrix of transition probabilities $\mathbf{P}(\tau) = [p_{ij}(\tau)]_1^\nu$, from mode i to mode j during the time interval $[t, t+\tau]$ given by $\mathbf{P}(\tau) = \exp(\mathbf{Q}\tau)$, $p_{ij}(\tau) = \mathrm{P}\{r(t+\tau) = j \mid r(t) = i\}\ (i,j \in \mathbb{N})$, $\mathbf{Q} = [q_{ij}]_1^\nu$, $q_{ij} \geq 0\ (i \neq j)$, $q_{ii} = -\sum_{j\neq i}^{\nu} q_{ij}$; $\boldsymbol{\Omega}(t,r_t)$ is a matrix of uncertain parameters, satisfying for every t and r_t the following inequality

$$\mathbf{I} - \boldsymbol{\Omega}^T(t,r_t)\boldsymbol{\Omega}(t,r_t) \geq 0; \qquad (2)$$

$\varphi(t,\mathbf{z})$ is a nonlinear m-dimensional vector function, whose components have form

$$\begin{aligned}\varphi_l(t,\mathbf{z}) = \varphi_l(t,z_l), \\ \varphi_l(t,0) = 0 \ \ (l = 1,...,m) \qquad (3)\end{aligned}$$

and satisfy restrictions

$$\begin{aligned}0 \leq \varphi_l(t,z_l)z_l \leq \kappa_l(i)z_l^2, \ \text{ if } \ r_t = i \\ (l = 1,\dots,m,\ i \in \mathbb{N}); \qquad (4)\end{aligned}$$

$\mathbf{A}(i), \mathbf{B}(i), \mathbf{L}(i), \mathbf{C}(i), \mathbf{D}(i), \mathbf{E}(i), \mathbf{F}(i)\ (i \in \mathbb{N})$ are known matrices of appropriate dimensions. For simplicity, but without the loss of generality we assume that $\kappa_l(i) = 1\ (l = 1,\dots,m,\ i \in \mathbb{N})$. Then we can write

$$\varphi(t,\mathbf{z})\boldsymbol{\Gamma}[\varphi(t,\mathbf{z}) - \mathbf{z}] \leq 0, \qquad (5)$$

where $\boldsymbol{\Gamma} = \mathrm{diag}[\gamma_l]_1^m\ \ (\gamma_l > 0,\ l = 1,\dots,m)$.

Let $\tau > t_0$ be the moment of discontinuous mode change, i.e. the moment of transition from $r(\tau - 0) = i$ to $r(\tau) = j \neq i$. It is supposed that at the moment τ the plant state vector $\mathbf{x}$ can changed discontinuously too and its value after jump linearly dependent on the same value before the jump:

$$\mathbf{x}(\tau) = [\Phi_{ij} + \mathbf{F}_{ij}\boldsymbol{\Omega}_{ij}(\tau - 0)\mathbf{E}_{ij}]\mathbf{x}(\tau - 0), \quad (6)$$

where $\Phi_{ij},\ \mathbf{F}_{ij},\ \mathbf{E}_{ij}\ (i,j \in \mathbb{N}),\ i \neq j$ are $n \times n$ constant matrices, $\boldsymbol{\Omega}_{ij}(t),\ (i,j \in \mathbb{N})$ are matrices of uncertain parameters, satisfying for every t the following inequalities

$$\mathbf{I} - \boldsymbol{\Omega}_{ij}^T(t)\boldsymbol{\Omega}_{ij}(t) \geq 0\ (i,j \in \mathbb{N}). \qquad (7)$$

Note that as a rule the case of continuous change of the plant state vector is considered ($\Phi_{ij} = \mathbf{I}$), but in many real systems the situation, when some plant state variables are changed by jump is more typical. This situation is natural for mechanical systems with sudden change of mass or moment of inertia; in this case the linear or angular velocity will be changed by jump, see (Kats, 1998) for more details.

3. ROBUST CONTROL AGAINST THE PLANT PARAMETERS UNCERTAINTY

Suppose that both output vector $\mathbf{y}(t)$ and mode change process $r(t)$ are available for controller. Let the control law has the form of static linear output feedback

$$\mathbf{u}(t) = -\mathbf{K}(i)\mathbf{y}(t), \;\text{ if }\; r(t) = i, \tag{8}$$

such that for every fixed $i \in \mathbb{N}$ it is stabilizing control for deterministic system

$$\dot{\mathbf{x}}(t) = \mathbf{A}(i)\mathbf{x}(t) + \mathbf{B}(i)\mathbf{u}(t),$$
$$\mathbf{y}(t) = \mathbf{C}(i)\mathbf{x}(t),$$

or in other words such that the matrices

$$\mathbf{A}_c(i) = \mathbf{A}(i) - \mathbf{B}(i)\mathbf{K}(i)\mathbf{C}(i) \; (i \in \mathbb{N})$$

are Hurwitz. The matrix $\mathbf{K}(i)$ can be obtained by *known methods of solving of the deterministic static output feedback control problem*, see (Syrmos *et al*, 1998). In this section we obtain an additional conditions for this matrix which guarantees that the control law (8) stabilizes the original system (1) in the sense of exponential stability in the mean square for all plant parameters uncertainty, satisfying inequality (2), all nonlinearities, satisfying (3), (4) and for all jump parameters uncertainty, satisfying (7). We say that such a control is robust stabilizing control. For this purpose we introduce an auxiliary system

$$\dot{\mathbf{x}}(t) = \mathbf{A}(r_t)\mathbf{x}(t) + \mathbf{B}(r_t)\mathbf{u}(t) + \mathbf{F}(r_t)\mathbf{v}(t) + \mathbf{D}(r_t)\mathbf{v}_1(t), \tag{9}$$
$$\mathbf{y}(t) = \mathbf{C}(r_t)\mathbf{x}(t), \;\; \mathbf{z}(t) = \mathbf{L}(r_t)\mathbf{x}(t), \tag{10}$$
$$\mathbf{x}(\tau) = \Phi_{ij}\mathbf{x}(\tau - 0) + \mathbf{F}_{ij}\mathbf{v}_{ij}(\tau - 0), \tag{11}$$

where $\mathbf{v}(t), \;\; \mathbf{v}_1(t), \;\; \mathbf{v}_{ij}(t)(i, j \in \mathbb{N})$ are random disturbance vectors. If

$$\mathbf{v}(t) = \mathbf{\Omega}(t, r_t)\mathbf{E}(r_t)\mathbf{x}(t), \tag{12}$$
$$\mathbf{v}_1(t) = \varphi(t, \mathbf{z}_t), \tag{13}$$
$$\mathbf{v}_{ij}(t) = \mathbf{\Omega}_{ij}(t)\mathbf{E}_{ij}\mathbf{x}(t), \tag{14}$$

then the system (9) coincides with original system (1).

Let us define the vector of all disturbances as $\mathbf{w} = [\mathbf{v}, \mathbf{v}_1, \mathbf{v}_{ij}]^T \; (i, j \in \mathbb{N})$. We say that the disturbances are admissible if

$$\mathcal{E}\{\int_0^\infty \mathbf{w}^T(t)\mathbf{w}(t)dt\} < \infty,$$

where $\mathcal{E}$ is the expectation operator.

Consider the cost functional of the form

$$J(\mathbf{u}, \mathbf{w}) = \mathcal{E}_x\{\int_0^\infty [\mathbf{x}^T(t)[\bar{\mathbf{M}}(r_t)\mathbf{x}(t) + \mathbf{u}^T(t)\mathbf{R}(r_t)\mathbf{u}(t) - \mathbf{v}_1^T(t)\mathbf{\Gamma}(\mathbf{v}_1(t) - \mathbf{z}(t)) - \gamma\mathbf{v}^T(t)\mathbf{v}(t) - \sum_{j\neq r}^{\nu}\gamma_{rj}\mathbf{v}_{rj}^T(t)\mathbf{v}_{rj}(t)q_{rj}]dt\} \tag{15}$$

along the trajectories of the system (9), where $\mathcal{E}_x$ is the expectation operator at $\mathbf{x}_0 = \mathbf{x}$; $\gamma, \;\; \gamma_{ij}(i, j \in \mathbb{N})$ are positive numbers; $\mathbf{\Gamma} = \mathrm{diag}[\gamma_l]_1^m$ is positive definite matrix, $\bar{\mathbf{M}}(i) = \mathbf{S}(i) + \mathbf{M}_+(i), \;\; \mathbf{S}(i) = \mathbf{M}(i) + \gamma\mathbf{E}^T(i)\mathbf{E}(i) + \sum_{j\neq i}^{\nu}\gamma_{ij}\mathbf{E}_{ij}^T\mathbf{E}_{ij}q_{ij}; \; \mathbf{M}(i) = \mathbf{M}^T(i) \;\; (i \in \mathbb{N})$ is positive semidefinite matrix. $\mathbf{M}_+(i) = \mathbf{M}_+^T(i) \;\; (i \in \mathbb{N})$ is positive definite matrix. The disturbances will attempt to maximize this functional, while the control will to keep it to a minimum.

Define the worst-case disturbance as

$$\mathbf{w}^* = \arg\max_w J(\mathbf{u}, \mathbf{w})$$

This disturbance satisfy the Hamilton-Jacobi equation (Basar and Bernhard, 1995)

$$\max_w\{\mathcal{L}V(\mathbf{x}, i) + \Theta_i(\mathbf{x}, \mathbf{u}, \mathbf{w}\} = 0, \tag{16}$$

where $\mathcal{L}$ is the differential generator of the Markov process $\{\mathbf{x}_t, \; r_t\}_{t\geq 0}$ along the trajectories of the system (9) with jump conditions (11), and $\Theta_i(\mathbf{x}, \mathbf{u}, \mathbf{w}) = \mathbf{x}^T\bar{\mathbf{M}}(i)\mathbf{x} + \mathbf{u}^T\mathbf{R}(i)\mathbf{u} - \mathbf{v}_1^T\mathbf{\Gamma}(\mathbf{v}_1 - \mathbf{z}) - \gamma\mathbf{v}^T\mathbf{v} - \sum_{j\neq i}^{\nu}\gamma_{ij}\mathbf{v}_{ij}^T\mathbf{v}_{ij}(t)q_{ij}(i \in \mathbb{N})$. Taking into account that

$$\mathcal{L}V(\mathbf{x}, i) = \left[\frac{\partial V(\mathbf{x}, i)}{\partial \mathbf{x}}\right]^T [\mathbf{A}(i)\mathbf{x} + \mathbf{B}(i)\mathbf{u} + \mathbf{F}(i)\mathbf{v} + \mathbf{D}(i)\mathbf{v}_1] + \sum_{j\neq i}^{\nu}[V(\mathbf{\Phi}_{ij}\mathbf{x} + \mathbf{F}_{ij}\mathbf{v}_{ij}, j) - V(\mathbf{x}, i)]q_{ij},$$

see (Kats, 1998) and choosing the Bellman function in the quadratic form $V(\mathbf{x}, i) = \mathbf{x}^T\mathbf{H}(i)\mathbf{x}$ we can rewrite (16) as

$$\max_w\{2\mathbf{x}^T\mathbf{H}(i)[\mathbf{A}(i)\mathbf{x} + \mathbf{B}(i)\mathbf{u} + \mathbf{F}(i)\mathbf{v} + \mathbf{D}(i)\mathbf{v}_1] + \mathbf{x}^T\bar{\mathbf{M}}(i)\mathbf{x} + \mathbf{u}^T\mathbf{R}(i)\mathbf{u} - \gamma\mathbf{v}^T\mathbf{v} - \mathbf{v}_1^T\mathbf{\Gamma}(\mathbf{v}_1 - \mathbf{L}\mathbf{x}) + \sum_{j\neq i}^{\nu}[\mathbf{x}^T(\mathbf{\Phi}_{ij}^T\mathbf{H}(j)\mathbf{\Phi}_{ij} - \mathbf{H}(i))\mathbf{x} - \gamma_{ij}\mathbf{v}_{ij}^T\mathbf{v}_{ij} + 2\mathbf{x}^T\mathbf{\Phi}_{ij}^T\mathbf{H}(j)\mathbf{F}_{ij}\mathbf{v}_{ij} +$$

$$\mathbf{v}_{ij}^T\mathbf{F}_{ij}^T\mathbf{H}(j)\mathbf{F}_{ij}\mathbf{v}_{ij}]q_{ij}\} = 0. \quad (17)$$

By standard calculations from (17) we obtain that the worst case disturbances for the system (9) with jump conditions (11) and with $r(t) = i$ are given by

$$\mathbf{v}^* = \mathbf{G}(i)\mathbf{x},\ \mathbf{v}_1^* = \mathbf{G}_1(i)\mathbf{x},\ \text{if } r(t) = i, \quad (18)$$
$$\mathbf{v}_{ij}^* = \mathbf{G}_{ij}\mathbf{x},\ i,j \in \mathbb{N}, \quad (19)$$

where

$$\mathbf{G}(i) = \gamma^{-1}\mathbf{F}^T(i)\mathbf{H}(i), \quad (20)$$
$$\mathbf{G}_1(i) = \mathbf{\Gamma}^{-1}[\mathbf{D}^T(i)\mathbf{H}(i) + \frac{1}{2}\mathbf{\Gamma}\mathbf{L}(i)], \quad (21)$$
$$\mathbf{G}_{ij} = [\gamma_{ij}\mathbf{I} - \mathbf{F}_{ij}^T\mathbf{H}(j)\mathbf{F}_{ij}]^{-1}\mathbf{F}_{ij}^T\mathbf{H}(j)\Phi_{ij}, \quad (22)$$

and the matrix $\mathbf{H}(i)$ satisfies the system of coupled matrix quadratic equations:

$$\begin{aligned}&\mathbf{H}(i)\mathbf{A}_c(i) + \mathbf{A}_c^T(i)\mathbf{H}(i) - \\ &\mathbf{H}(i)\mathbf{B}(i)\mathbf{R}^{-1}(i)\mathbf{B}^T(i)\mathbf{H}(i) + \\ &\gamma^{-1}\mathbf{H}(i)\mathbf{F}(i)\mathbf{F}^T(i)\mathbf{H}(i) + \bar{\mathbf{M}}(i) + \\ &(\mathbf{H}(i)\mathbf{D}(i) + \frac{1}{2}\mathbf{L}^T(i)\mathbf{\Gamma})\mathbf{\Gamma}^{-1}(\mathbf{H}(i)\mathbf{D}(i) + \\ &\frac{1}{2}\mathbf{L}^T(i)\mathbf{\Gamma})^T + \sum_{j\neq i}^{\nu}[\mathbf{\Phi}_{ij}^T\mathbf{H}(j)\mathbf{\Phi}_{ij} - \\ &\mathbf{H}(i) + \mathbf{\Phi}_{ij}^T\mathbf{H}(j)\mathbf{F}_{ij}(\gamma_{ij}\mathbf{I} - \\ &\mathbf{F}_{ij}^T\mathbf{H}(j)\mathbf{F}_{ij})^{-1}\mathbf{F}_{ij}^T\mathbf{H}(j)\mathbf{\Phi}_{ij}]q_{ij} = 0. \quad (23)\end{aligned}$$

If there exists a positive definite solution $\mathbf{H}(i)$ $i \in \mathbb{N}$ of the equation (23) then for all admissible disturbances we have

$$\begin{aligned}0 > {} & 2\mathbf{x}^T\mathbf{H}(i)[\mathbf{A}(i)\mathbf{x} + \mathbf{B}(i)\mathbf{u} + \mathbf{F}(i)\mathbf{v}^* + \\ &\mathbf{D}(i)\mathbf{v}_1^*] + \mathbf{x}^T\mathbf{S}(i)\mathbf{x} + \mathbf{u}^T\mathbf{R}(i)\mathbf{u} - \\ &\gamma\mathbf{v}^{*T}\mathbf{v}^* - \mathbf{v}_1^{*T}\Gamma(\mathbf{v}_1^* - \mathbf{L}(i)\mathbf{x}) + \\ &\sum_{j\neq i}^{\nu}[\mathbf{x}^T(\Phi_{ij}^T\mathbf{H}(j)\Phi_{ij} - \mathbf{H}(i))\mathbf{x} - \\ &\gamma_{ij}\mathbf{v}_{ij}^{*T}\mathbf{v}_{ij}^* + 2\mathbf{x}^T\Phi_{ij}^T\mathbf{H}(j)\mathbf{F}_{ij}\mathbf{v}_{ij}^* + \\ &\mathbf{v}_{ij}^{*T}\mathbf{F}_{ij}^T\mathbf{H}(j)\mathbf{F}_{ij}\mathbf{v}_{ij}^*]q_{ij} > \\ &2\mathbf{x}^T\mathbf{H}(i)[\mathbf{A}(i)\mathbf{x} + \mathbf{B}(i)\mathbf{u} + \mathbf{F}(i)\mathbf{v} + \\ &\mathbf{D}(i)\mathbf{v}_1] + \mathbf{x}^T\mathbf{M}(i)\mathbf{x} + \mathbf{u}^T\mathbf{R}(i)\mathbf{u} - \\ &\gamma[\mathbf{v}^T\mathbf{v} - \mathbf{x}^T\mathbf{E}^T(i)\mathbf{E}(i)\mathbf{x}] - \mathbf{v}_1^T\Gamma[\mathbf{v}_1 - \\ &\mathbf{L}(i)\mathbf{x}] + \sum_{j\neq i}^{\nu}[\mathbf{x}^T(\Phi_{ij}^T\mathbf{H}(j)\Phi_{ij} - \mathbf{H}(i))\mathbf{x} - \\ &\gamma_{ij}[\mathbf{v}_{ij}^T\mathbf{v}_{ij} - \mathbf{x}^T\mathbf{E}_{ij}^T\mathbf{E}_{ij}\mathbf{x}] + \\ &2\mathbf{x}^T\Phi_{ij}^T\mathbf{H}(j)\mathbf{F}_{ij}\mathbf{v}_{ij} + \mathbf{v}_{ij}^T\mathbf{F}_{ij}^T\mathbf{H}(j)\mathbf{F}_{ij}\mathbf{v}_{ij}]q_{ij}.\end{aligned}$$

It follows from this inequality that if the system of matrix equations (23) has positive definite solution $\mathbf{H}(i)$ $i \in \mathbb{N}$ then along the trajectories of the the system (9) with $\mathbf{u}$ given by (8) we have

$$\begin{aligned}&\mathcal{L}V(\mathbf{x}, i) + \mathbf{x}^T\mathbf{M}(i)\mathbf{x} + \mathbf{u}^T\mathbf{R}(i)\mathbf{u} - \\ &\gamma[\mathbf{v}^T\mathbf{v} - \mathbf{x}^T\mathbf{E}^T(i)\mathbf{E}(i)\mathbf{x}] - \\ &\mathbf{v}_1^T\Gamma[\mathbf{v}_1 - \mathbf{L}(i)\mathbf{x}] - \\ &\sum_{j\neq i}^{\nu}\gamma_{ij}[\mathbf{v}_{ij}^T\mathbf{v}_{ij} - \mathbf{x}^T\mathbf{E}_{ij}^T\mathbf{E}_{ij}\mathbf{x}]q_{ij} < 0. \quad (24)\end{aligned}$$

Substituting in (24) the disturbance given by (12), (13), (14) we obtain according to Kats (1998) and taking into account the S-procedure, see (Boyd *et al.*, 1994), that the system (1) is robustly stable. The robust stabilizing control law is given by (8). It is easy to see that in this case the disturbances (13)-(14) are admissible. So we have proved the following theorem.

Theorem 1. Let for some positive scalars $\gamma, \gamma_l (l = 1, \ldots, m), \gamma_{ij}$ $(i,j \in \mathbb{N})$ and matrices $\mathbf{M}(i) \geq 0$, $\mathbf{R}(i) > 0$ there exists a positive definite solution $\mathbf{H}(i)$ $(i \in \mathbb{N})$, of the system of coupled matrix quadratic inequalities

$$\begin{aligned}&\mathbf{H}(i)\mathbf{A}_c(i) + \mathbf{A}_c^T(i)\mathbf{H}(i) - \\ &\mathbf{H}(i)\mathbf{B}(i)\mathbf{R}^{-1}(i)\mathbf{B}^T(i)\mathbf{H}(i) + \\ &\gamma^{-1}\mathbf{H}(i)\mathbf{F}(i)\mathbf{F}^T(i)\mathbf{H}(i) + \mathbf{S}(i) + \\ &(\mathbf{H}(i)\mathbf{D}(i) + \frac{1}{2}\mathbf{L}^T(i)\mathbf{\Gamma})\mathbf{\Gamma}^{-1}(\mathbf{H}(i)\mathbf{D}(i) + \\ &\frac{1}{2}\mathbf{L}^T(i)\mathbf{\Gamma})^T + \sum_{j\neq i}^{\nu}[\mathbf{\Phi}_{ij}^T\mathbf{H}(j)\mathbf{\Phi}_{ij} - \\ &\mathbf{H}(i) + \mathbf{\Phi}_{ij}^T\mathbf{H}(j)\mathbf{F}_{ij}(\gamma_{ij}\mathbf{I} - \\ &\mathbf{F}_{ij}^T\mathbf{H}(j)\mathbf{F}_{ij})^{-1}\mathbf{F}_{ij}^T\mathbf{H}(j)\mathbf{\Phi}_{ij}]q_{ij} < 0. \quad (25)\end{aligned}$$

Then the output feedback control (8) is the robust stabilizing control. The function $V(\mathbf{x}, i) = \mathbf{x}^T\mathbf{H}(i)\mathbf{x}$ $(i \in \mathbb{N})$ is stochastic Lyapunov function which guarantees robust stability of the system (1).

4. PERFECT ROBUST STABILIZING CONTROL

As a rule the jumping variable $r(t)$ has uncertainty in its parameters too. We suppose that the matrix $\mathbf{Q}$ of transition intensities of Markov chain $r(t)$ is not exactly known and we have only some bounds for its elements (maximal "switching frequency"):

$$q_i = -q_{ii} \leq \bar{q}_i. \quad (26)$$

In this section we obtain a state feedback control in the same form as in previous section such that the closed loop system (1) is exponentially stable in the mean square for all transition probabilities, satisfying (26), for all plant parameters uncertainty, satisfying inequality (2), for all nonlinearities, satisfying (3) (4) and for all jump parameters uncertainty, satisfying (7) We say that such a system is perfectly robustly stable and appropriate control is perfect robust stabilizing control. To solve the stated robust control problem consider the set of the following auxiliary deterministic systems

$$\dot{\mathbf{x}}(t) = \mathbf{A}(\rho_i, i)\mathbf{x}(t) + \mathbf{B}(i)\mathbf{u}(t)dt + \mathbf{F}(i)\mathbf{v}(t) + \mathbf{D}(i)\mathbf{v}_1(t), \quad (27)$$

with the corresponding cost functionals

$$J_a(\mathbf{u}, \mathbf{v}, \mathbf{v}_1) = \int_0^\infty [\mathbf{x}^T(t)[\tilde{\mathbf{M}}(i)\mathbf{x}(t) + \mathbf{u}^T(t)\mathbf{R}(i)\mathbf{u}(t) - \mathbf{v}_1^T(t)\mathbf{\Gamma}(\mathbf{v}_1(t) - \mathbf{L}(i)\mathbf{x}) - \gamma\mathbf{v}^T(t)\mathbf{v}(t)]dt, \quad (28)$$

where $\mathbf{A}(\rho_i, i) = \mathbf{A}(i) + \frac{1}{2}\rho_i\mathbf{I},\ \rho_i$ is a positive scalar, $\tilde{\mathbf{M}}(i) = \mathbf{M}(i) + \mathbf{M}_+(i) + \gamma\mathbf{E}^T(i)\mathbf{E}(i)\ (i \in \mathbb{N})$; $\mathbf{u}(t)$ is the control vector and $\mathbf{v}(t) \in L_2[0,\infty),\ \mathbf{v}_1(t) \in L_2[0,\infty)$ play the role of disturbances.

Theorem 2. Let for some positive scalars $\gamma,\ \gamma_l\ (l = 1, \ldots, m),\ \alpha_i,\ \rho_i,\ \gamma_{ij}\ (i, j \in \mathbb{N})$ and matrices $\mathbf{M}(i) \geq 0,\ \mathbf{R}(i) > 0$ the system of inequalities

$$\mathbf{H}(i)\mathbf{A}_c(\rho_i, i) + \mathbf{A}_c^T(\rho_i, i)\mathbf{H}(i) - \mathbf{H}(i)\mathbf{B}(i)\mathbf{R}^{-1}(i)\mathbf{B}^T(i)\mathbf{H}(i) + \mathbf{M}(i) + \gamma\mathbf{E}^T(i)\mathbf{E}(i) + \gamma^{-1}\mathbf{H}(i)\mathbf{F}(i)\mathbf{F}^T(i)\mathbf{H}(i) + (\mathbf{H}(i)\mathbf{D}(i) + \frac{1}{2}\mathbf{L}^T(i)\mathbf{\Gamma})\mathbf{\Gamma}^{-1}(\mathbf{H}(i)\mathbf{D}(i) + \frac{1}{2}\mathbf{L}^T(i)\mathbf{\Gamma})^T < 0, \quad (29)$$

$$\sum_{j\neq i}^{\nu} \Phi_{ij}^T\mathbf{H}_j\Phi_{ij} - \Phi_{ij}^T\mathbf{H}_j\mathbf{F}_{ij}[\mathbf{F}_{ij}^T\mathbf{H}(j)\mathbf{F}_{ij} - \gamma_{ij}\mathbf{I}]^{-1}\mathbf{F}_{ij}^T\mathbf{H}(j)\Phi_{ij} + \gamma_{ij}\mathbf{E}_{ij}^T\mathbf{E}_{ij} \leq \alpha_i\mathbf{H}_i, \quad (30)$$

$$\frac{\rho_i}{\alpha_i} \geq \bar{q}_i, \quad (31)$$

$$\Phi_{ij}^T\mathbf{H}_j\Phi_{ij} - \Phi_{ij}^T\mathbf{H}_j\mathbf{F}_{ij}[\mathbf{F}_{ij}^T\mathbf{H}(j)\mathbf{F}_{ij} - \gamma_{ij}\mathbf{I}]^{-1}\mathbf{F}_{ij}^T\mathbf{H}(j)\Phi_{ij} + \gamma_{ij}\mathbf{E}_{ij}^T\mathbf{E}_{ij} \geq 0, \quad (32)$$

where

$$\mathbf{A}_c(\rho_i, i) = \mathbf{A}(\rho_i, i) - \mathbf{B}(i)\mathbf{K}(i)\mathbf{C}(i)\ (i \in \mathbb{N}),$$

has positive definite solution $\mathbf{H}(i)\ (i \in \mathbb{N})$. Then the control (8) is perfect robust stabilizing control for the system (1).

PROOF. Define the worst-case disturbances as

$$[\mathbf{v}^*\ \mathbf{v}_1^*]^T = \arg\max_{v,v_1} J_a(\mathbf{u}, \mathbf{v}, \mathbf{v}_1)$$

and suppose that the control law is given by (8). It follows from Theorem 1, see also (Kogan, 1999), that these disturbances are given by (18), where matrices $\mathbf{G}(i),\ \mathbf{G}_1(i)$ are defined by the formulae (20), (21) and matrix $\mathbf{H}(i)\ (i \in \mathbb{N})$ is positive definite solution of the following matrix equation.

$$\mathbf{H}(i)\mathbf{A}_c(\rho_i, i) + \mathbf{A}_c^T(\rho_i, i)\mathbf{H}(i) - \mathbf{H}(i)\mathbf{B}(i)\mathbf{R}^{-1}(i)\mathbf{B}^T(i)\mathbf{H}(i) + \gamma^{-1}\mathbf{H}(i)\mathbf{F}(i)\mathbf{F}^T(i)\mathbf{H}(i) + \tilde{\mathbf{M}}(i) + (\mathbf{H}(i)\mathbf{D}(i) + \frac{1}{2}\mathbf{L}^T(i)\mathbf{\Gamma})\mathbf{\Gamma}^{-1}(\mathbf{H}(i)\mathbf{D}(i) + \frac{1}{2}\mathbf{L}^T(i)\mathbf{\Gamma})^T = 0. \quad (33)$$

This solution exists for some positive definite matrix $\mathbf{M}_+$ in virtue of the inequality (29). From (29), taking into account that $\mathbf{v}^*, \mathbf{v}_1^*$ maximize the left hand side of (33) and (31) we have for all $\mathbf{x} \in \mathbb{R}^n$

$$0 > 2\mathbf{x}^T\mathbf{H}(i)[\mathbf{A}(i)\mathbf{x} + \mathbf{B}(i)\mathbf{u} + \mathbf{F}(i)\mathbf{v}^* + \mathbf{D}(i)\mathbf{v}_1^*]\mathbf{x} + \mathbf{x}^T(\mathbf{M}(i) + \gamma\mathbf{E}^T(i)\mathbf{E}(i))\mathbf{x} + \mathbf{u}^T\mathbf{R}(i)\mathbf{u} - \gamma\mathbf{v}^{*T}\mathbf{v}^* - \mathbf{v}_1^{*T}\Gamma(\mathbf{v}_1^* - \mathbf{L}(i)\mathbf{x}) + \rho_i\mathbf{x}^T\mathbf{H}(i)\mathbf{x} \geq 2\mathbf{x}^T\mathbf{H}(i)[\mathbf{A}(i)\mathbf{x} + \mathbf{B}(i)\mathbf{u} + \mathbf{F}(i)\mathbf{v} + \mathbf{D}(i)\mathbf{v}_1]\mathbf{x} + \mathbf{x}^T\mathbf{M}(i)\mathbf{x} + \mathbf{u}^T\mathbf{R}(i)\mathbf{u} - \gamma[\mathbf{v}^T\mathbf{v} - \mathbf{x}^T\mathbf{E}^T(i)\mathbf{E}(i)\mathbf{x}] - \mathbf{v}_1^T\Gamma[\mathbf{v}_1 - \mathbf{L}(i)\mathbf{x}] + \mathbf{x}^T\alpha_i\bar{q}_i\mathbf{H}(i)\mathbf{x}. \quad (34)$$

For the last term in (34) taking into account the property of the matrix $\mathbf{Q}$ and (30), (32) we obtain

$$\mathbf{x}^T\alpha_i\bar{q}_i\mathbf{H}(i)\mathbf{x} \geq \mathbf{x}^T\sum_{j\neq i}^{\nu} q_{ij}[\Phi_{ij}^T\mathbf{H}(i)\Phi_{ij} - \mathbf{H}(i) - \Phi_{ij}^T\mathbf{H}_j\mathbf{F}_{ij}[\mathbf{F}_{ij}^T\mathbf{H}(j)\mathbf{F}_{ij} - \gamma_{ij}\mathbf{I}]^{-1}\mathbf{F}_{ij}^T\mathbf{H}(j)\Phi_{ij} + \gamma_{ij}\mathbf{E}_{ij}^T\mathbf{E}_{ij}]\mathbf{x} = \sum_{j\neq i}^{\nu} q_{ij}\mathbf{x}^T[[\Phi_{ij}^T\mathbf{H}(i)\Phi_{ij} - \mathbf{H}(i) + \gamma_{ij}\mathbf{E}_{ij}^T\mathbf{E}_{ij}]\mathbf{x} + 2\mathbf{x}^T\Phi_{ij}\mathbf{H}(j)\mathbf{F}_{ij}\mathbf{v}_{ij}^* + \mathbf{v}_{ij}^{*T}(\mathbf{F}_{ij}^T\mathbf{H}(j)\mathbf{F}_{ij} - \gamma_{ij}\mathbf{I})\mathbf{v}_{ij}^*], \quad (35)$$

where $\mathbf{v}_{ij}^*$ is given by (19), (22). It is easily to see that such a $\mathbf{v}_{ij}^*$ maximizes the right hand side of (35). Then for all admissible disturbances the following inequality is true

$$\sum_{j\neq i}^{\nu} q_{ij}\mathbf{x}^T[[\Phi_{ij}^T\mathbf{H}(i)\Phi_{ij} - \mathbf{H}(i) + \gamma_{ij}\mathbf{E}_{ij}^T\mathbf{E}_{ij}]\mathbf{x} + 2\mathbf{x}^T\Phi_{ij}\mathbf{H}(j)\mathbf{F}_{ij}\mathbf{v}_{ij}^* + \mathbf{v}_{ij}^{*T}(\mathbf{F}_{ij}^T\mathbf{H}(j)\mathbf{F}_{ij} - \gamma_{ij}\mathbf{I})\mathbf{v}_{ij}^* \geq \sum_{j\neq i}^{\nu} q_{ij}\mathbf{x}^T[\Phi_{ij}^T\mathbf{H}(i)\Phi_{ij} - \mathbf{H}(i) + \gamma_{ij}\mathbf{E}_{ij}^T\mathbf{E}_{ij}]\mathbf{x} + 2\mathbf{x}^T\Phi_{ij}\mathbf{H}(j)\mathbf{F}_{ij}\mathbf{v}_{ij} + \mathbf{v}_{ij}^T(\mathbf{F}_{ij}^T\mathbf{H}(j)\mathbf{F}_{ij} - \gamma_{ij}\mathbf{I})\mathbf{v}_{ij}]. \quad (36)$$

It follows from (34), (36) that along the trajectories of the the system (9) with $\mathbf{u}$ given by (8) the inequality (24) holds. Substituting in (24) the disturbance given by (12), (13), (14) we obtain according to (Kats, 1998) and taking into account the S-procedure, see (Boyd *et al.*, 1994), that the system (1) is perfectly robustly stable and the robust stabilizing control law is given by (8). It is easy to see that in this case the disturbances (13)-(14) are admissible. The function $\mathbf{x}^T\mathbf{H}(i)\mathbf{x}$ is stochastic function of Lyapunov which guarantees perfect robust stability of system (1).

5. CONCLUSION

To compute the robust output feedback control law in the form of (8) it is necessary to solve *the standard deterministic output feedback control problem* together with the additional system of standard linear matrix inequalities and nonstandard quadratic matrix Riccati like inequalities. One possible approach is to formulate this problem as a consequent iterative procedure of solution of linear matrix inequalities (LMI) (Boyd *et al.*, 1994), leading to numerical MATLAB based algorithms.

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