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H^∞ FAULT DETECTION FILTER FOR A CLASS OF DISCRETE-TIME SYSTEMS WITH MULTIPLE TIME DELAYS

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Abstract: This paper deals with the problem of H^∞ detection filter design for discrete-time systems with multiple time delays, where methods for decoupling the fault effects from unknown inputs including model uncertainties and external plant disturbances are not available. Through the appropriate choice of the filter gain, the filter is convergent if there is no fault in the system, and the effect of disturbances on residual is minimized in the sense of H^∞ norm. The problem of achieving satisfactory sensitivity of the residual to faults and its solution are formulated. The detection threshold of the filter is also discussed. Finally, simulation of a numerical example is made to illustrate the efficiency of the proposed method.

Keywords: H^∞ filter, robust fault detection, discrete-time systems, time delay, unknown inputs.

1. INTRODUCTION

A system designed to provide both sensitivity to faults and robustness to modelling error or disturbances is called a robust FDI scheme. One of the most successful robust fault diagnosis approaches is the use of disturbance decoupling principle. This can be done by using unknown input observer (Seliger and Frank, 1991), by using structured parity equations (Staroswiecki and Comtet-Varga, 2001; Gertler and Kunwer, 1993), or by eigenstructure assignment (Patton and Chen, 1991). But in most cases the effects of faults and unknown inputs can not be completely decoupled from each other. (see (Jiang *et al.*, 1999; Jiang *et al.*, 2001), for example).

In recent years much attention has been focused on the FDI problem by using H^∞ theory in order to improve unknown input attenuation capability. Edelmayer *et al* (Edelmayer *et al.*, 1994) described an H^∞ filtering approach that minimizes the l_2-norm of the disturbance residual component. Qiu and Gertler (Qiu and Gertler, 1993) considered robust FDI based on H^∞ optimization. Chung and Speyer (Chung and Speyer, 1998) developed in detail a related game-theoretic filter. Frank and Ding (Frank and Ding, 1994) applied H^∞ theory to FDI in the frequency domain. Most recently, Chen and Patton (Chen and Patton, 2000) considered standard H^∞ filtering formulation of robust fault detection. However, these papers mentioned above only consider the systems without time delays. Time delay is commonly encountered in various engineering systems, such as chemical processes, long transmission lines in pneumatic, hydraulic and rolling mill systems. More recently, Liu and Frank (Liu and Frank, 1999) studied the problem of H^∞ detection filter design for state delayed linear continuous-time systems.

In this paper, we extend the work in (Liu and Frank, 1999) to discrete-time systems with multiple time delays, in which the effects of faults and unknown inputs (including model uncertainties and external disturbance) cannot be decoupled from each other. The detection filter gains are designed so that if there is no fault in the system, the filter is convergent. The transfer function from the unknown input to the residual satisfies a prescribed H^∞ norm upper bound. The design freedom in filter gain can be used to assure satisfactory sensitivity of the residual to faults. The detection threshold of the filter is also discussed and simulation of a numerical example is made to illustrate the efficiency.

This paper is organized as follows. Section 2 includes system description, definition of the design problem and useful lemma. A sufficient condition under which the H^∞ detection filter exists is given in section 3. Section 4 characterizes the set of the desired filtering gains. Simulation result of a numerical example is included in Section 5, followed by some concluding remarks in Section 6.

2. SYSTEM DESCRIPTION AND PROBLEM FORMULATION

Consider the following linear discrete-time system with multiple time delays described by

$$\begin{aligned} x(k+1) = & Ax(k) + \sum_{i=1}^{N} A_i x(k-\tau_i) + Bu(k) \\ & + F_1 f(k) + D\omega(k) \qquad (1) \\ y(k) = & Cx(k) + F_2 f(k) \qquad (2) \end{aligned}$$

where $x(k) \in R^n$ is the system state vector, $u(k) \in R^m$ is the control input, $y(k) \in R^r$ is the measurement vector, $k = 0, 1, 2, \cdots$, and τ_i $(i = 1, 2, 3, \cdots)$ are the time-delays. Since this paper deals with fault detection filter design which is not affected by control input delays, only state delays are considered in the model. The matrices A, B, C, D, F_1, F_2 and A_i $(i = 1, \cdots, N)$ are real matrices of appropriate dimensions, and C is of full row rank. The unknown input vector $\omega(k) \in R^d$ including model uncertainties and external plant disturbances is a l_2 sequence each of which components has norm less than one. The unknown fault vector is $f(k) \in R^q$. Many discrete-time systems can be described by the above model (Shen and Hsu, 1998).

The filter considered in this paper is written as

$$\begin{aligned} \hat{x}(k+1) = & \tilde{A}\hat{x}(k) + \sum_{i=1}^{N} A_i \hat{x}(k-\tau_i) \\ & + Bu(k) + Ky(k) \qquad (3) \end{aligned}$$

where $\hat{x}(k) \in R^n$ is the state estimation vector, $\tilde{A} \triangleq A - KC$ and K is the filter's gain matrix to be designed.

Let the filter error be $e(k) = x(k) - \hat{x}(k)$, then the error dynamics is described by

$$\begin{aligned} e(k+1) = & \tilde{A}e(k) + \sum_{i=1}^{N} A_i e(k-\tau_i) \\ & + (F_1 - KF_2)f(k) + D\omega(k) \qquad (4) \end{aligned}$$

The residual vector $\varepsilon(k)$ is defined as

$$\varepsilon(k) = Se(k) \qquad (5)$$

where S is a suitable weighting matrix designed to assure isolability properties. The disturbance transfer matrix $H_\omega(z)$ from $\omega(k)$ to $\varepsilon(k)$ and fault transfer matrix $H_f(z)$ from $f(k)$ to $\varepsilon(k)$ can be described as

$$H_\omega(z) \triangleq S[zI_n - \tilde{A} - \sum_{i=1}^{N} z^{-\tau_i} A_i]^{-1} D \qquad (6)$$

$$\begin{aligned} H_f(z) \triangleq & S[zI_n - \tilde{A} - \sum_{i=1}^{N} z^{-\tau_i} A_i]^{-1} F, \\ F \triangleq & F_1 - KF_2 \qquad (7) \end{aligned}$$

Now, we describe the design problem as follows:

H^∞ **detection filter problem**: Given the state delayed discrete-time system (1) and (2) with the detection filter (3), determine the filter gain matrix K such that
(i) The filter (3) is convergent for all τ_i $(i = 1, \cdots, N)$ and $f(k) = 0$; i.e. the filter error dynamics is asymptotically stable.
(ii) The H^∞ norm of the disturbance transfer matrix $H_\omega(z)$ satisfies the constraint

$$\| H_\omega(z) \|_\infty \leq \gamma \qquad (8)$$

where γ is a given positive scalar, and

$$\| H_\omega(z) \|_\infty = \sup_{\omega \in l_2} \frac{\| \varepsilon \|_2}{\| \omega \|_2} = \sup_{\theta \in [0, 2\pi]} \sigma_{max}[H_\omega(e^{j\theta})].$$

(iii) The satisfactory sensitivity of the residuals to the faults can be achieved using the design freedom of the gains.

Lemma 1: Suppose that x_i $(i = 1, \cdots, n)$ are vectors with appropriate dimensions, then for any symmetric positive definite matrix $P > 0$, the following inequality holds

$$(\sum_{i=1}^{n} x_i)^T P (\sum_{i=1}^{n} x_i) \leq n(\sum_{i=1}^{n} x_i^T P x_i).$$

Proof : It is a direct consequence of the fact that function $f(x) = x^T P x$ is convex. □

3. SUFFICIENT CONDITION FOR H^∞ DETECTION FILTER

In this section, we develop a Riccati equation framework that guarantees the filter (3) to be convergent for all τ_i $(i = 1, \cdots, N)$ when $f(k) = 0$, and satisfies the disturbance attenuation constraint (8).

Theorem 1: If there exist symmetric positive definite matrices P, Q and scalar $\epsilon > 0$ such that

$$G_i \triangleq Q - N A_i^T P A_i > 0, \quad i = 1, \cdots, N. \tag{9}$$

$$H \triangleq \gamma^2 I_d - D^T P D > 0 \tag{10}$$

$$\begin{aligned} P = {}& \tilde{A}^T P \tilde{A} + \sum_{i=1}^{N} \tilde{A}^T P A_i G_i^{-1} A_i^T P \tilde{A} \\ & + \sum_{i=1}^{N} P A_i G_i^{-1} A_i^T P + \tilde{A}^T P D H^{-1} D^T P \tilde{A} \\ & + S^T S + NQ + \epsilon I_n \end{aligned} \tag{11}$$

hold, then
(a) Filter error system (4) is asymptotically stable for all τ_i $(i = 1, \cdots, N)$ and $f(k) = 0$.
(b) The H^∞ norm of the disturbance transfer matrix $H_\omega(z)$ satisfies constraint (8).

Proof: The Lyapunov function is chosen as

$$\begin{aligned} V(e(k)) = {}& e^T(k) P e(k) \\ & + \sum_{i=1}^{N} \sum_{l=k-\tau_i}^{k-1} e^T(l) Q e(l) \end{aligned} \tag{12}$$

The corresponding Lyapunov difference along the trajectories $e(k)$ of the error system (4) with $\omega(k) = 0$ and $f(k) = 0$ is given by

$$\begin{aligned} \triangle V(k) = {}& V(e(k+1)) - V(e(k)) \\ = {}& e^T(k+1) P e(k+1) - e^T(k) P e(k) \\ & + \sum_{i=1}^{N} [e^T(k) Q e(k) - e^T(k-\tau_i) Q e(k-\tau_i)] \\ = {}& e^T(k) [\tilde{A}^T P \tilde{A}] e(k) \\ & + [\sum_{i=1}^{N} A_i e(k-\tau_i)]^T P [\sum_{i=1}^{N} A_i e(k-\tau_i)] \\ & + 2 e^T(k) \tilde{A}^T P \sum_{i=1}^{N} A_i e(k-\tau_i) - e^T(k) P e(k) \\ & + \sum_{i=1}^{N} [e^T(k) Q e(k) - e^T(k-\tau_i) Q e(k-\tau_i)] \\ = {}& e^T(k) [\tilde{A}^T P \tilde{A} - P + NQ] e(k) \\ & + 2 e^T(k) \tilde{A}^T P \sum_{i=1}^{N} A_i e(k-\tau_i) \\ & + [\sum_{i=1}^{N} A_i e(k-\tau_i)]^T P [\sum_{i=1}^{N} A_i e(k-\tau_i)] \\ & - \sum_{i=1}^{N} e^T(k-\tau_i) Q e(k-\tau_i) \end{aligned} \tag{13}$$

According to Lemma 1 and (9), equation (13) can be rewritten as

$$\begin{aligned} \triangle V(k) \le {}& e^T(k) [\tilde{A}^T P \tilde{A} - P + NQ] e(k) \\ & + 2 e^T(k) \tilde{A}^T P \sum_{i=1}^{N} A_i e(k-\tau_i) \\ & - \sum_{i=1}^{N} e(k-\tau_i) G_i e(k-\tau_i) \end{aligned} \tag{14}$$

Adding and subtracting

$$e^T(k) [\sum_{i=1}^{N} \tilde{A}^T P A_i G_i^{-1} A_i^T P \tilde{A}] e(k)$$

to equation (14) yields

$$\begin{aligned} \triangle V(k) \le {}& e^T(k) [\tilde{A}^T P \tilde{A} - P + NQ \\ & + \sum_{i=1}^{N} \tilde{A}^T P A_i G_i^{-1} A_i^T P \tilde{A}] e(k) \\ & - \sum_{i=1}^{N} [G_i^{-1/2} A_i^T P \tilde{A} e(k) - G_i^{1/2} e(k-\tau_i)]^T \\ & \times [G_i^{-1/2} A_i^T P \tilde{A} e(k) - G_i^{1/2} e(k-\tau_i)] \end{aligned} \tag{15}$$

At last, considering (11), one can get

$$\begin{aligned} \triangle V(k) \le {}& - e^T(k) W e(k) \\ & - \sum_{i=1}^{N} [G_i^{-1/2} A_i^T P \tilde{A} e(k) - G_i^{1/2} e(k-\tau_i)]^T \\ & \times [G_i^{-1/2} A_i^T P \tilde{A} e(k) - G_i^{1/2} e(k-\tau_i)] \end{aligned} \tag{16}$$

where

$$\begin{aligned} W \triangleq {}& \sum_{i=1}^{N} P A_i G_i^{-1} A_i^T P + \tilde{A}^T P D H^{-1} D^T P \tilde{A} \\ & + S^T S + \epsilon I_n \end{aligned} \tag{17}$$

Since $W > 0$, it follows that $\triangle V(k) < 0$ for $x(k) \neq 0$ and hence the filter error dynamics described by (4) is asymptotically stable.

Next, to prove condition (8), one can rewrite equation (11) as

$$\begin{aligned} S^T S = {}& e^{j\theta} P e^{-j\theta} - \tilde{A}^T P \tilde{A} \\ & - \sum_{i=1}^{N} \tilde{A}^T P A_i G_i^{-1} A_i^T P \tilde{A} - \sum_{i=1}^{N} P A_i G_i^{-1} A_i^T P \end{aligned}$$

$$-\tilde{A}^T PDH^{-1}D^T P\tilde{A} - NQ - \epsilon I_n \quad (18)$$

Define $z \triangleq e^{j\theta}, \bar{z} \triangleq e^{-j\theta}$, equation (18) becomes

$$\begin{aligned}
S^T S = &[\bar{z}I_n - \tilde{A} - \sum_{i=1}^{N} \bar{z}^{-\tau_i} A_i]^T \\
&\times P[zI_n - \tilde{A} - \sum_{i=1}^{N} z^{-\tau_i} A_i] \\
&-\tilde{A}^T P\tilde{A} - NQ - \sum_{i=1}^{N} \tilde{A}^T PA_i G_i^{-1} A_i^T P\tilde{A} \\
&-\sum_{i=1}^{N} PA_i G_i^{-1} A_i^T P - \tilde{A}^T PDH^{-1}D^T P\tilde{A} \\
&-\epsilon I_n - \tilde{A}^T P\tilde{A} + \bar{z}P\tilde{A} + z\tilde{A}^T P \\
&+\sum_{i=1}^{N}[\bar{z}PA_i z^{-\tau_i} + \bar{z}^{-\tau_i} A_i^T Pz \\
&-z^{-\tau_i}\tilde{A}^T PA_i - \bar{z}^{-\tau_i} A_i^T P\tilde{A}] \\
&-(\sum_{i=1}^{N} \bar{z}^{-\tau_i} A_i^T)P(\sum_{i=1}^{N} A_i z^{-\tau_i}) \quad (19)
\end{aligned}$$

Let $L(z) \triangleq (zI_n - \tilde{A} - \sum_{i=1}^{N} z^{-\tau_i} A_i)$, then equation (19) can be rewritten as

$$\begin{aligned}
S^T S = &L^*(z)PL(z) + L^*(z)P\tilde{A} + \tilde{A}^T PL(z) \\
&-\tilde{A}^T PDH^{-1}D^T P\tilde{A} \\
&-M - U^*(z)U(z) - \epsilon I_n \quad (20)
\end{aligned}$$

where

$$\begin{aligned}
U(z) &\triangleq \sum_{i=1}^{N}(zG_i^{-1/2} A_i^T P - z^{-\tau_i} G_i^{1/2}) \\
M &\triangleq \sum_{i=1}^{N}(2NA_i^T PA_i + \tilde{A}^T PA_i G_i^{-1} A_i^T P\tilde{A})
\end{aligned}$$

and $(\cdot)^*$ stands for complex conjugate transpose. Substituting (20) into (6) yields

$$\begin{aligned}
&H_\omega^*(z)H_\omega(z) = D^T(L^{-1})^*(z)S^T SL^{-1}(z)D \\
&= D^T PD + D^T P\tilde{A}L^{-1}(z)D + D^T(L^{-1})^*(z)\tilde{A}^T PD \\
&\quad -D^T(L^{-1})^*(z)[M + U^*(z)U(z)]L^{-1}(z)D \\
&\quad -D^T(L^{-1})^*(z)\tilde{A}^T PDH^{-1}D^T P\tilde{A}L^{-1}(z)D \\
&\quad -\epsilon D^T(L^{-1})^*(z)L^{-1}(z)D \quad (21)
\end{aligned}$$

From (10) and (21), one can obtain

$$\begin{aligned}
&H_\omega^*(z)H_\omega(z) - \gamma^2 I_d \\
&= -Y^*(z)Y(z) \\
&\quad -D^T(L^{-1})^*(z)[U^*(z)U(z) + M]L^{-1}(z)D \\
&\quad -\epsilon D^T(L^{-1})^*(z)L^{-1}(z)D \le 0 \quad (22)
\end{aligned}$$

where

$$Y(z) \triangleq H^{-1/2}D^T P\tilde{A}L^{-1}(z)D - H^{1/2} \quad (23)$$

Therefore (8) holds. This completes the proof of Theorem 1. □

Remark 1: Although the structure of the proposed filter in equation (3) is standard, the conditions (9)-(11) under which the filter is convergent are novel. As it can be seen in the proof of Theorem 1, extension of detection filter design for continuous-time systems to the case of discrete-time systems is not trivial.

4. CHARACTERIZATION OF THE FILTER GAIN

First, we derive the sufficient condition for the existence of an achievable (P, Q, ϵ), that is, the set of filter gains K can be found such that the matrix inequalities (9), (10) and (11) hold.

Theorem 2: For a given H^∞ attenuation constraint γ, (P, Q, ϵ) is achievable if (P, Q, ϵ) satisfy (9), (10) and the following algebraic matrix inequality

$$\begin{aligned}
R \triangleq &P + A^T \triangle A - NQ - S^T S - \epsilon I_n \\
&-\sum_{i=1}^{N} PA_i G_i^{-1} A_i^T P \ge 0 \quad (24)
\end{aligned}$$

where

$$\begin{aligned}
\triangle = &P + \sum_{i=1}^{N} PA_i G_i^{-1} A_i^T P \\
&+PDH^{-1}D^T P \quad (25)
\end{aligned}$$

Proof: Substituting $\tilde{A} = A - KC$ into (11) yields

$$\begin{aligned}
P = &C^T K^T \triangle KC - A^T \triangle KC - C^T K^T \triangle A \\
&+NQ + S^T S + \epsilon I_n \\
&+\sum_{i=1}^{N} PA_i G_i^{-1} A_i^T P \quad (26)
\end{aligned}$$

where $\triangle$ is given by (25). Since $\triangle > 0$, there exists $\triangle_1$ that is full column rank and $\triangle = \triangle_1^T \triangle_1$. Equation (26) can be rewritten as

$$\begin{aligned}
&(\triangle_1 KC - \triangle_1 A)^T(\triangle_1 KC - \triangle_1 A) \\
&= P + A^T \triangle A - NQ \\
&\quad -S^T S - \epsilon I_n - \sum_{i=1}^{N} PA_i G_i^{-1} A_i^T P \quad (27)
\end{aligned}$$

Since matrix C is full row rank, from (27), there exists a solution K to (11) (i.e. the (P, Q, ϵ) is achievable) if (9) and (10) hold and the right-hand side of (27) is semi positive definite, i.e. (24) holds. This complete the proof of Theorem 2. □

Theorem 2 gives the existence condition of an achievable (P, Q, ϵ) in terms of matrix inequality. The following theorem characterizes the algebraic expression of all filter gains K related to the achievable (P, Q, ϵ).

Theorem 3: If (P, Q, ϵ) is achievable, the set of filtering gain can be characterized as follows

$$K = AC^{\dagger} + \triangle_1^{\dagger} X R_1 C^{\dagger} \tag{28}$$

where $(\cdot)^{\dagger}$ stands for the pseudo-inverse of a matrix, R_1 is the square root of R (i.e. $R_1^T R_1 = R$), and X is an arbitrary orthogonal matrix.

Proof: It follows from (27) that

$$\triangle_1 KC - \triangle_1 A = XR_1 \tag{29}$$

where X and R_1 are given by Theorem 3. Since $\triangle_1$ is of full column rank and C is of full row rank, (28) follows immediately from (29). This complete the proof of Theorem 3. □

Remark 2: It can be seen from the above theorem that there exists large freedom in the design process, such as the choice of (P, Q, ϵ) and the arbitrary orthogonal matrix X. This design freedom can be used to reach satisfactory value of $tr[H_f^T(e^{j0})H_f(e^{j0})]$, which is the sensitivity of the residual to faults, as in (Patton and Hou, 1997). i.e.

$$\begin{aligned} &tr\{[S(KC - A - \sum_{i=1}^{N} z^{-\tau_i} A_i)^{-1} F]^T \\ &\times [S(KC - A - \sum_{i=1}^{N} z^{-\tau_i} A_i)^{-1} F]\} \geq \beta^2 \end{aligned} \tag{30}$$

where $\beta > \gamma > 0$ is a prescribed scalar.

Remark 3: By substituting (28) into (30), the H^{∞} fault detection filter design can be reformulated as a constrained optimization problem

$$\begin{aligned} &\min_X tr\{[S(KC - A - \sum_{i=1}^{N} z^{-\tau_i} A_i)^{-1} F]^T \\ &\quad \times [S(KC - A - \sum_{i=1}^{N} z^{-\tau_i} A_i)^{-1} F]\} \\ &s.t. \quad X^T X = I \end{aligned} \tag{31}$$

where K is an explicit function of X, given by (28). This is a special type of constrained nonlinear optimization. Some effective nonlinear programming algorithms can be used to solve this problem.

Remark 4: If $\| \omega \|_2 \leq \omega_0$, and the desired filter sensitivity against worst-case disturbance $\omega(k)$ could be achieved which means that $\| H_\omega(z) \|_\infty << \| H_f(z) \|_\infty$, then the detection threshold of the filter can be given as (Edelmayer *et al.*, 1994)

$$T_r = \gamma \, \omega_0 \tag{32}$$

As a result, fault detection can be carried out as follows:

$$\left. \begin{array}{l} \| \varepsilon(k) \| < T_r, \text{ no fault occurs} \\ \| \varepsilon(k) \| \geq T_r, \text{ fault has occurred} \end{array} \right\} \tag{33}$$

From (32), it can be seen that the threshold T_r is proportional to the performance index γ. Thus γ can be determined (estimated) based on the minimization of criteria such as false alarm rate and missed detection rate.

Remark 5: This paper is an extension of the work in (Liu and Frank, 1999). The main differences between the obtained results in this paper and that in (Liu and Frank, 1999) are in three aspects. (i) H^{∞} detection filter design for discrete-time systems with multiple time delays is investigated in this paper, which is more complex than the case of continuous-time systems in (Liu and Frank, 1999). (ii) The detection threshold of the filter is proposed in this paper, which was not discussed at all in (Liu and Frank, 1999), the simulation result of a numerical example will be presented in next section to illustrate the efficiency. (iii) The problem of achieving satisfactory sensitivity of the residual to faults and its solution are explicitly described in Remarks 1-2 of this paper.

5. AN ILLUSTRATIVE EXAMPLE

Consider a perturbed state delayed linear discrete-time system described by (1) and (2), where the parameters are given by

$$N = 2, \quad \tau_1 = 0.1, \; \tau_2 = 0.2;$$

$$A = \begin{bmatrix} -2 & 1 \\ 0 & 0 \end{bmatrix}, \; A_1 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \end{bmatrix}, \; A_2 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix};$$

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \; S = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}, \; D = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix};$$

$$F_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad F_2 = \begin{bmatrix} -1 \\ -2 \end{bmatrix}.$$

In this example, the performance index is:

$$\gamma = 0.95.$$

One may choose Q and ϵ as follows

$$Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \; \epsilon = 0.001.$$

A solution of the positive definite matrix P can be obtained from the matrices inequalities (9), (10) and (24):

$$P = \begin{bmatrix} 2.83 & -0.18 \\ -0.18 & 2.79 \end{bmatrix}$$

By simple calculation, one can get

$$G_1 = G_2 = \begin{bmatrix} 0.7736 & 0.0072 \\ 0.0072 & 0.9434 \end{bmatrix}, \ H = 0.8742;$$

$$\triangle = \begin{bmatrix} 3.6597 & -0.2525 \\ -0.2525 & 3.0952 \end{bmatrix},$$

$$\triangle_1 = \begin{bmatrix} 1.9118 & -0.0688 \\ -0.0688 & 1.7580 \end{bmatrix};$$

$$R = \begin{bmatrix} 14.3884 & -7.4327 \\ -7.4327 & 4.0651 \end{bmatrix},$$

$$R_1 = \begin{bmatrix} 3.4473 & -1.5826 \\ -1.5826 & 1.2492 \end{bmatrix}.$$

Therefore the above (P, Q, ϵ) is achievable according to Theorem 2.

Choose the orthogonal matrix as follows

$$X = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}.$$

Substituting the above matrix into (28) yields the corresponding desired filtering gain matrix

$$K = \begin{bmatrix} 0.4132 \\ 0.8985 \end{bmatrix}.$$

In the simulation, the sampling period is 0.01s, the disturbance $\omega = 0.5\ rand$, the fault considered is created as follows

$$f(t) = \begin{cases} 0 & \text{for} \ \ 0 \leq t \leq 2 \\ 0.5 & \text{for} \ \ 2 < t \leq 6 \end{cases} \tag{34}$$

Figure 1 shows the response of the detection signal $\varepsilon(k)$ when there is a fault as described by (34) in the system. It can be seen that fault detection shows good performance despite the disturbance ω in the system.

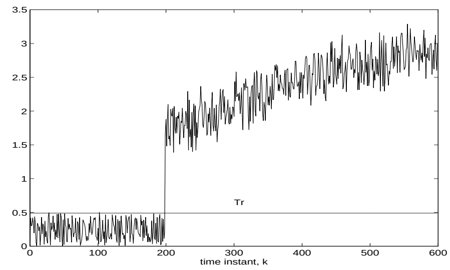

Fig. 1. Detection signal $\mid \varepsilon(k) \mid$

6. CONCLUSION

In this paper, the problem of H^∞ detection filter design for a class of discrete-time systems with multiple time delays is investigated. Simulation of a numerical example is made to show the applicability of the proposed method. Based on linear matrix inequalities approaches, to derive convergent conditions of detection filter which can be solved more efficiently will be investigated in the future.

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