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SEQUENTIAL APPROACH TO PRODUCTION PLANNING IN MULTISITE ENVIRONMENTS

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Abstract: A sequential planning approach is proposed for make-to-order situations that tries to emulate the practitioners approach. It deals sequentially with assignment of sites and transportation, completion time, exact timing and detailed sites planning, allowing negotiation among sites in terms of runtime required for each site. *Copyright 2002IFAC*

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1. INTRODUCTION

The problem considered is production planning in make-to-order situations when several sites are involved requiring transportation tasks. The objective of proposing a sequential approach is to try to represent the decision process of management people in those situations. The practical way to deal with these problems appears to be a sequential procedure with backtrack where production models are sequentially refined in order to represent more and more production details. This procedure starts with an aggregate model that nevertheless captures essential aspects, allowing to limit analysis to a reduced quantity of possible scenarios.

2. PLANNING AND SCHEDULING

Production planning in a multisite environment extends the scope of the only one site problem at least in two aspects: i) different forms (recipes) for the production of intermediates, final products and raw materials procurement are possible and ii) transportation tasks must be considered. The first aspect, alternatives recipes, can be present in a single plant but it is not frequent. Obviously multisite production planning involves sites' planning but certainly a first step is only to consider some aggregate view of each plant. This is the case, for example, of representing the plant through production rates without considering how production is performed and which resources are utilized. This macroscopic point of view can make the problem easier, at this aggregate level, since it eliminates the constraints in equipment' units sharing, which complicate planning and scheduling. These constraints will only appear at the macroscopic multisite level if plants are allowed to switch among production recipes during the planning period.

It is worth to consider the impact of those finite capacity constraints. In a single plant, production planning is only finished when a convenient scheduling solution has been obtained. For example when a detailed shop floor acceptable Gantt chart is obtained, defining which tasks have to be executed, where and when. The scheduling problem can be huge and it is tackled through decomposition approaches, mainly beginning with a planning problem. This planning problem can have many different characteristics depending on the application but always involves time aggregation. This leads to the critical problem of abstracting and representing scheduling constraints, mainly resources capacity constraints, in this aggregated time frame in order to obtain feasible planning solutions. In other words to take into account resources finite capacity. This is the main problem in MRP (Manufacturing Resources Planning) approach as well as multilevel optimization approaches (Subrahmanyam, *et al.,* 1996). This situation has lead to heuristic approaches where planning and scheduling are treated in a sequential way, allowing backtrack to generate different scenarios (Mockus and Reklaitis,1999). These approaches in a certain way emulate the practitioner approach, that generates acceptable solutions, treating those problems in some ordered way with some degree of backtracking.

In a multisite environment a decomposition approach is likely to deal first with: i) the selection of possible scenarios in terms of plants, recipes and respective runtimes, and transport tasks to fulfill a specific demand for end products, that is selection among alternative recipes, and ii) synchronization of plants' production and transport in order to meet demand due dates. These two problems do not involve resources sharing if certain conditions hold. Mainly each plant must utilize a single recipe so that the problem of recipes sequencing in a plant is avoided, and transport equipment must not be shared among different transport tasks. Planning and scheduling of each plant would be done in a later step knowing the time interval and runtime necessary at each plant.

3. SEQUENTIAL APPROACH

The sequential approach proposed for multisite planning deals with the planning problem considering increasing levels of detail. However, since the present version is a one pass procedure no claim can be made that the optimal solution is obtained. We think that production constraints at the different sites must be considered as soon as possible in order to establish some sort of negotiation among sites and between sites and transportation facilities. In this work it is proposed that this negotiation can be done negotiating the production time intervals planned for each site and the needs for transportation tasks. The sequential approach works in two phases.

First phase utilizes a rough model of the whole multisite problem. Sites production capability is given by a production rate (mass units per time unit) for each output product at the site, input materials consumption is given in the same way. Given a demand of end products this phase determines; i) in a first level a scenario of producer sites and transport needs and a production time interval for each site and, ii) in a second level the production start and end times. At the first level production, stock and transportation costs are minimized, at the second level total completion time is minimized. Site production rates are input data that have to be given by site managers. Initial values estimates surely would be lower than full capacity since it is not known what production interval will be proposed for the site. In this way plants' runtimes and transport tasks incorporate a slack to accommodate detailed scheduling. The planning solution of this phase can be unacceptable in the sense that end products supply is late with respect to due dates. In this case other scenarios are analyzed or a negotiation has to start in order to increase, if possible, production rates.

Second phase considers the planning problem at each site. It utilizes the planning system developed for single sites (Rodrigues *et al.*, 2000) which analyzes planning and scheduling constraints using tasks processing time windows. Those time windows, defined through earliest beginning times and latest finishing times, in the multisite situation come from phase one.

A two level procedure is proposed for phase one where objectives at each level are the following. Level 1a only considers mass production and transportation with no timing. An economical cost function is minimized representing costs due to i) production inventories of final products and intermediates and ii) transportation costs. Mathematical formulation, as described below leads to a Mixed Integer Linear Problem (MILP) with a reduced number of binary variables. This level gives the plants selected for production and the production time necessary at each plant to fulfill demands on final products. If there are not alternative plants this first level does not apply.

Level 1a allows a rough estimation of the global production time necessary, that is the production completion time. If the completion time is unacceptable at the planning phase other solutions with lower completion times may be interesting in spite of a cost degradation. A Level 1b is used to achieve this objective where completion time is minimized allowing an increase in Level 1a cost. Level 1b comprises all the equations utilized at Level 1a plus a formulation representing completion time. It is a MILP, nevertheless the number of binary variables is maintained low since they are only defined for pairs of plants.

As far as Level 1b comprises all the equations of Level 1a it could replace Level 1a. A problem arises because cost function should balance mass production and transportation cost terms with completion time. It seems better to solve Level 1b only minimizing completion time subject to a constraint that merely states that some degradation on the value of the cost function at Level 1a is accepted by the user.

Once an acceptable solution is obtained, it remains the time production allocation at the different plants. This is done at Level 2 through a MILP formulation where binary variables represent plants' production start times. Makespan is minimized subject to the constraints represented by mass balances among plants. Time discretization is necessary as far as plants production start time has to be modeled, nevertheless time discretization interval is only constrained by user time scale and transport times.

- Level 1a

Sets and parameters:

s	*states*
p	*plants*
$Out_{p,s}$	*states s output of plant p*
$In_{p,s}$	*states s input of plant p*

$Product_s$	*= true if state s is an end product*
D_s	*external demand of state s*
$Link_{p1,p2}$	*= true if plants p_1 and p_2 are linked*
$Ratep_{p,s}$	*rate of production of state s by plant p*
$Ratec_{p,s}$	*rate of consumption of state s by plant p*
$Hd_{p,s}$	*time interval between plant p start and state s production/consumption (Head)*
$Tl_{p,s}$	*time between latest state s consumption or production and plant p stop (Tail)*
$CTT_{p1,p2,s}$	*unitary transport cost of state s between plants p_1 and p_2*

Variables (positive):

WP(p)	*binary variable = 1 if plant p is used*
T(p)	*run time for plant p (integer)*
$QT(p_1,p_2,s)$	*quantity of s transferred from p_1 to p_2*
S(s)	*final stock of state s*

- Additional variables for Level 1b (positive)

$WT(p_1,p_2)$	*binary variable = 1 if some state is transported between p_1,p_2*
CT(p)	*estimation of completion time of plant p*
$Delta(p_1,p_2)$	*contribution of plant p_2 to completion time of plant p_1*

- Level 1b (equations 1 – 5)

1. External demand: External demand (final products) D_s satisfaction

$$\sum_p Ratep_{p,s}[T(p)-WP(p)(Hd_{p,s}+Tl_{p,s})] \geq D_s$$

$$\forall s,\ p \mid Product_s,\ Out_{p,s} = True$$

2. Transportation: $\sum_{p_2} QT(p_1,p_2,s)$

$$\leq Ratep_{p_1,s}[T(p_1)-WP(p_1)(Hd_{p_1,s}+Tl_{p_1,s})]$$

$$\forall s,\ p_1,\ p_2 \mid Out_{p1,s},\ In_{p2,s},\ Link_{p1,p2} = True$$

3. Internal demand: $\sum_{p_1} QT(p_1,p_2,s)$

$$\leq Ratec_{p_2,s}[T(p_2)-WP(p_2)(Hd_{p_2,s}+Tl_{p_2,s})]$$

$$\forall s,\ p_1,\ p_2 \mid In_{p2,s},\ Out_{p1,s},\ Link_{p1,p2} = True$$

4. Intermediates and final products stocks

$$S(s) = \sum_{p_1}\{-\sum_{p_2} QT(p_1,p_2,s) + Ratep_{p_1,s}([T(p_1)-WP(p_1)(Hd_{p_1,s}+Tl_{p_1,s})]\}$$

$$\forall s,\ p_1,\ p_2 \mid Out_{p1,s},\ In_{p2,s},\ Link_{p1,p2} = True$$

5. Cost function: minimization of costs of: i) final products excess production, ii) intermediaries stock at planning period end and iii) transportation .

$$\sum_{s_1}\ [\sum_{p_1} Ratep_{p_1,s_1}(T(p_1)-WP(p_1)(Hd_{p_1,s_1}+Tl_{p_1,s_1}))] - D_{s_1} + \alpha\sum_{s_2}[S(s_2)] + \beta\sum_{p_1,p_2,s_3} QT(p_1,p_2,s_3)CTT_{p_1,p_2,s_3}$$

$$\forall s_1, s_2, s_3, p,\ p_1,\ p_2 \mid Product_{s1},\ Out_{p1,s},\ In_{p2,s},\ Out_{p2,s},\ Link_{p1,p2} = True;\ Product_{s2} = False$$

- Level 1b (equations 1 – 4 and 6 – 8 plus equation 5 as a constraint allowing a greater value than that obtained at Level 1a)

6. Binary variables definition

$$M * WT(p_1,p_2) \geq QT(p_1,p_2,s)$$

$$WT(p_1,p_2) \leq QT(p_1,p_2,s)$$

$$\forall s,\ p_1,\ p_2 \mid In_{p2,s},\ Out_{p1,s},\ Link_{p1,p2} = True$$

$$WT(p_1,p_3) \geq 0.5(WT(p_1,p_2)+WT(p_2,p_3))$$

$$\forall p_1,\ p_2,\ p_3 \mid Link_{p1,p2},\ Link_{p2,p3} = True$$

7. Completion time estimation

$$CT(p) \geq T(p)$$

$$CT(p_1) \geq T(p_1) + Delta(p_1,p_2)$$

$$Delta(p_1,p_2) - CT(p_2) \geq -M*(1-WT(p_2,p_1))$$

$$Delta(p_1,p_2) - CT(p_2) \leq M*(1-WT(p_2,p_1))$$

M: big positive constant

8. Cost function: $\min\{\max_p CT(p)\}$

- Level 2.

Parameters

RT_p *runtime (integer) of plant p from Level1 (T(p))*

Variables (positive):

Cumpro(s,p,t)	*accumulated production of state s in plant p until slot t*
Cumdel(s,p,t)	*accumulated delivery of state s*
Spro(s,p,t)	*stock of s at producer plant p in t*
Shi(s,p,q,t)	*quantity of state s shipped from plant p to q at slot t*
Ava(s,p,q,t)	*quantity of state s available at plant q, coming from plant p at slot t*
Cumcon(s,q,t)	*accumulated consumption of state s in plant q until slot t*
Cumrec(s,q,t)	*accumulated reception of state s in plant q until slot t*
Scon(s,q,t)	*stock of s at consuming plant q in t*
W(p,t)	*binary; =1 if plant p starts at slot t*

1. Accumulated production: $Cumpro(s,p,t) =$

$$Ratep_{p,s}\sum_{t-(RT_p-Tl_{p,s})+2}^{t-Hd_{p,s}} W(p,t')[t-t'-Hd_{p,s}+1] + Ratep_{p,s}[RT_p-Hd_{p,s}-Tl_{p,s}]\sum_{1}^{t-(RT_p-Tl_{p,s})+1} W(p,t')$$

$$\forall s,\ p,\ t;\ Out_{p,s}=True$$

2. Accumulated delivery: $Cumdel(s,p,t) =$

$$Cumdel(s,p,t-1) + \sum_{q, In_{q,s}=True} Shi(s,p,q,t)$$

$$\forall s,\ p,\ t;\ Out_{p,s}=True$$

3. Material balance at producer plant

$$Spro(s,p,t) = Cumpro(s,p,t) - Cumdel(s,p,t)$$

$$\forall s, p,\ t;\ Out_{p,s}=True$$

4. Availability at a consumer plant

$$Ava(s,p,q,t) = Shi(s,p,q,t - Transp_{pq})$$
$$\forall s,p,q,\ t;\ Out_{p,s},\ In_{q,s}=True$$

5. Accumulated consumption: $Cumcon(s,q,t) =$

$$Ratec_{p,s} \sum_{t-(RT_q - Tl_{q,s})+2}^{t-Hd_{q,s}} W(q,t')[t - t' - Hd_{q,s} + 1]$$
$$+Ratec_{q,s}[RT_q - Hd_{q,s} - Tl_{q,s}] \sum_{1}^{t-(RT_q - Tl_{q,s})+1} W(q,t')$$
$$\forall s,q,\ t;\ In_{q,s}=True$$

6. Accumulated reception

$$Cumrec(s,q,t) = Cumreq(s,q,t-1) + \sum_{p,Out_{p,s}=True} Ava(s,p,q,t)$$
$$\forall s,q,t;\ In_{q,s}=True$$

7.Material balance at consumer plant

$$Scon(s,q,t) = Cumreq(s,q,t) - Cumcon(s,q,t)$$
$$\forall s,q,\ t;\ In_{q,s}=True$$

8. Allocation

$$\sum_t W(p,t) = 1 \qquad \forall p \,/\, RT_p \,\#\, 0$$

9.Cost function (makespan)

$$min\{ \max_p [\sum_t W(p,t)t + RT(p)]\}$$

4. EXAMPLE

The example considered is shown in Figure 1. Each site has a production recipe represented through its State Task Network (Kondili *et al.*,1993) as shown in Figure 2 for site 5.

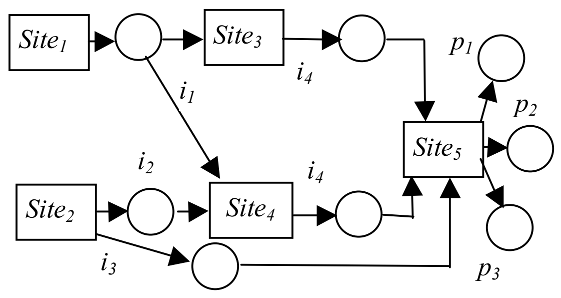

Figure 1. Sites and products

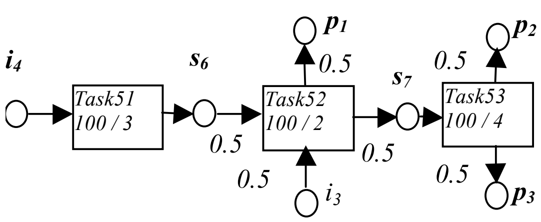

Figure 2. State Task Network for site 5. Tasks' batchsize and processing time and mass factors.

Rectangles represent tasks and circles stand for input and output states. Each task is characterized by its batch size, processing time and input/output states mass factors (as batch size percentages).

Demands for end products p_1, p_2 and p_3 are 400, 200 and 200 respectively. All transport costs are unitary except transport of intermediate i_4 between site3 and site5 which has a value of 10. All transport times have a value of 2 time units. A discretization interval (time slot) of 2 time units has been chosen for phase 2 according to transport times, so input data are scaled accordingly. Scaled production/consumption rates, and heads and tails in sites cycling operation, are given in Table 1.

Table1. Production and consumption rates

Site	State	Prod. rate	Cons. rate	Head	Tail
Site1	i_1	*40*		*7*	*0*
Site2	i_2	*25*		*0*	*0*
Site2	i_3	*25*		*0*	*0*
Site3	i_1		*33.3*	*0*	*1*
Site3	i_4	*33.3*		*2*	*0*
Site4	i_4	*33.3*		*3*	*0*
Site4	i_1		*25*	*0*	*5*
Site4	i_2		*25*	*2*	*3*
Site5	p_1	*50*		*2*	*2*
Site5	p_2	*25*		*4*	*0*
Site5	p_3	*25*		*4*	*0*
Site5	i_3		*50*	*2*	*2*
Site5	i_4		*66.6*	*0*	*5*

Mixed Linear Problems (MILP) have been solved using CPLEX6.6 through GAMS language (PentiumII/600). Weighing in equation 5 of Level1 is $\alpha = 5$, $\beta = 1$. MILP for this level has 39 equations, 14 continuous variables and 10 binary variables and the solution is obtained after 7 iterations (0.1 s.) giving a minimum cost of 1587 that corresponds entirely to transportation costs. Site3 is not utilized given its higher transportation cost and run time for the other sites, expressed in time slots, are 15 (site1),16 (site2), 17 (site4) and 12 (site5). If Level 1b is bypassed, Level 2 leads to the situation shown in Figure 3: Run times lead to a makespan of 31 slots. Accumulated intermediates production is shown at the top; below with enlarged scales it is represented, for each intermediate, its accumulated production and consumption. A dotted intermediate pattern shows the accumulated deliveries obtained by the solver. MILP for Level 2 has 1641 continuous variables and 200 binary variables when time horizon is 40 time slots. The optimal solution is obtained after 162 iterations (0.4 s.)

The results presented in figure 3 use the nominal production rates given in Table1; these nominal values are obtained from the recipe utilized at each plant, as shown in Figure 2 for site5, and do not include any slack to allow some flexibility in plant scheduling.

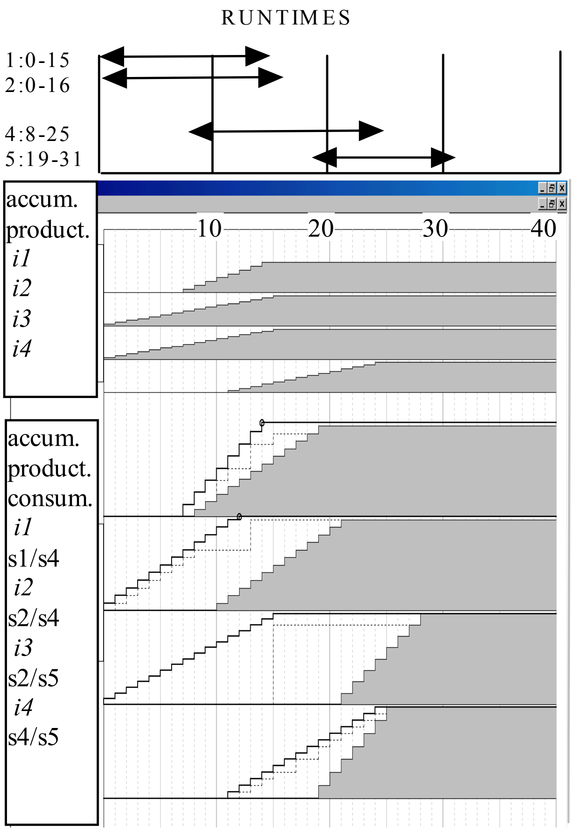

Figure 3. Level 2 results after Level 1a

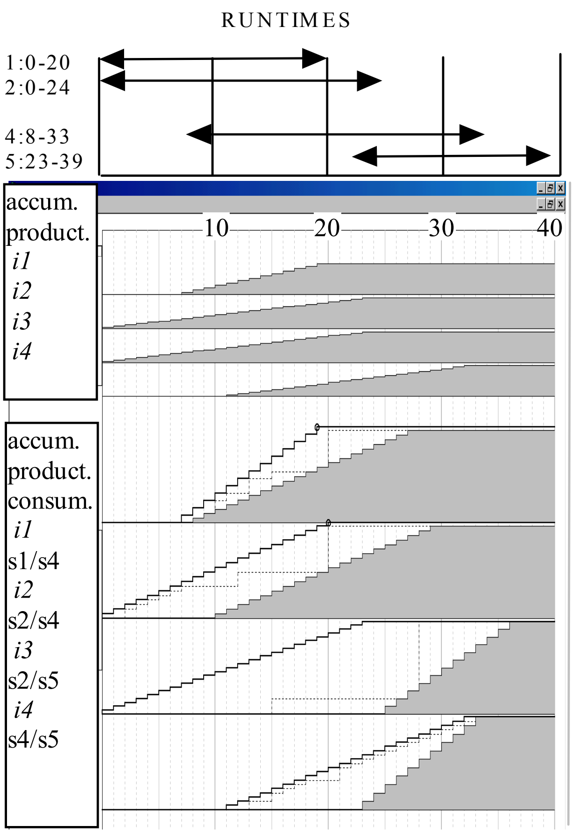

Figure 4. Level 2 results after Level 1a with production rates at 70%

Production rates must be smaller to allow some flexibility in plants scheduling. Moreover lower values are likely to be used at the beginning of the negotiation. Figure 4 shows the results obtained when production rates are taken as 70% of the nominal values: run times are now 20 (site1), 24 (site2), 25 (site4) and 16 (site5), leading to a makespan of 39.

Let's suppose this is the initial situation and that a makespan of 39 is not acceptable because it does not allow to fulfill final products due dates. It would be possible to negotiate shorter run times with increased production rates but another possibility is to accept a higher cost with a different multisite configuration. In this case Level 1b is executed allowing an increase in the cost obtained at Level 1a and minimizing an estimation of makespan. A makespan of 32 is obtained with an increase in cost from 1757 to 4361. Now site3 is used leading to the increased cost due to its higher transportation cost. Run times obtained are: 24 (site1), 24 (site2), 14 (site3), 13 (site4) and 16 (site5). The result obtained at Level 2 is shown in Figure 5.

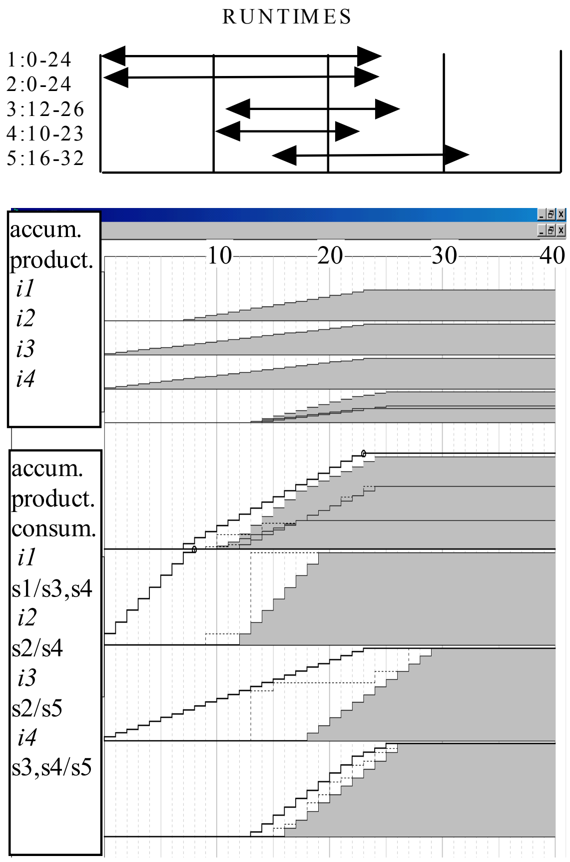

Figure 5.Level 2 results after Level 1b (70%)

Detailed planning after phase one is done at each site using the single plant planning system - PCPIP (Rodrigues *et al.*,2000). It generates a processing time window for each batch using final products due dates and input materials availability as inputs. These time windows allow to estimate equipment units loading and cumulative resources utilization, so that they can be the basis for a negotiation among sites to determine intermediates transportation and final run times.

Site5 is considered in the scenario represented in Figure 4. Figure 6 shows the type of results obtained: time is represented using actual values instead of time slots and due dates for end products p_1, p_2 and p_3 are fixed at t = 78 (given the makespan of 39 slots found acceptable at phase one). PCPIP determines through an exploding procedure the quantity of batches for tasks *T51*(4)*, T52*(8) and *T53*(4) and each batch latest finishing time (*lft*) in order to satisfy final products due dates. This step also determines due dates for input materials in a batch per batch basis (shown in gray at the bottom of Figure 6). An availability plan for input states (i_3 and i_4) has to be entered by the user, this plan will be surely more aggregated and is negotiated with sites 2 and 4. In Figure 6 an availability plan has been entered as one delivery of i_4 (400 at t = 52) and i_3 (400 at t = 52). From this availability plan PCPIP determines batches earliest beginning times (*ebt*), which together with *lfts* determined before define each batch time window. Batches time windows are shown in the upper part of Figure 6 along with the consumption estimate of a cumulative resource. Black intervals in time windows are obligatory used by batches since time window is smaller than twice the processing time, which means that cumulative resource will also be utilized.

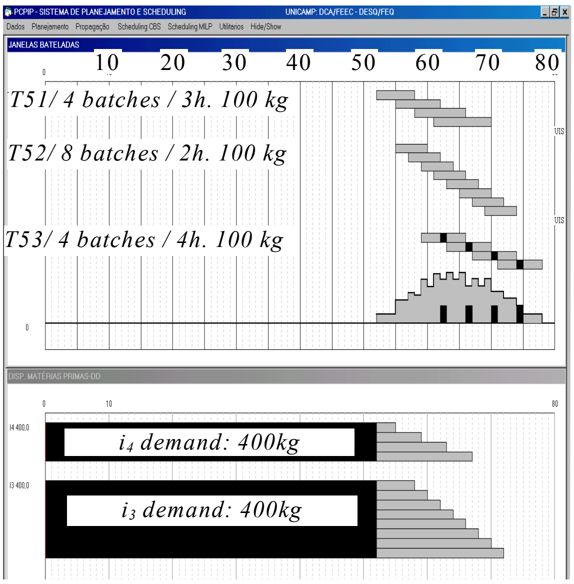

Figure 6. Processing time windows

Figure 7 represents equipment units loading estimates obtained from time windows. Slack times per batch are: 6/4 h. for *T51* batches, 3/8 h. for *T52* batches and 3/4 h for *T53* batches, with processing times 3, 2 and 4 h. respectively (Figure 2). Let's suppose that this situation is acceptable for site5. The consequences on sites 2 and 4 which supply its input states are different. For site2 there is no problem in supplying 400 of i_3 at t = 52 (t = 26 in time slots) as can be seen from Figure 4. In contrast site4 cannot supply this quantity at t = 52 since at 70% of nominal production site4 has not yet produced such quantity; an increase in production rate could be negotiated since at 100% (Figure 3) 400 could be sent at t = 52. Obviously a less aggregated supply plan for i_4 can be chosen so that it is suitable for site5 and less demanding for site4.

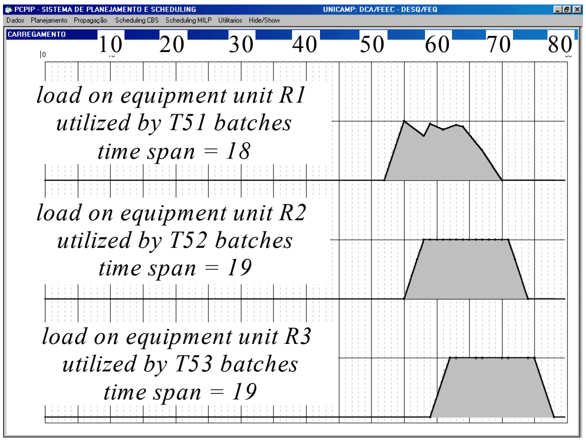

Figure 7. Equipment units loading,

5. SUMMARY

A sequential approach for multisite planning is being developed for make to order situations characterized by specific demands with due dates. The main idea is to treat separately the configuration problem, that selects which sites will be used, and sites' capacity analysis in order to get an acceptable scenario agreed by all the partners. The objective has been to avoid dimension problems with detailed time representation in the first phase, allowing to obtain quickly different possible configuration scenarios. Each solution has different impacts on global criteria such as cost and planning horizon, and sites criteria such as resources utilization. This last point is quickly analyzed through a planning system based on batches processing time windows which allows negotiation among sites based on intermediates deliveries and availability plans.

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