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ADAPTIVE FUZZY CONTROLLER FOR HANDLING IMROVEMENT OF FOUR-WHEEL-STEERING SYSTEMS

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Abstract: A fuzzy control design method for four-wheel-steering vehicles, using fuzzy models, is presented. The design model is obtained from a vehicle model using fuzzy modeling approach. In the first step of the controller design, an optimal steering controller is proposed for each local model using LQR method. Then, the local controllers are combined using fuzzy rules to form a fuzzy controller. In the next step, an adaptive fuzzy controller is obtained using adaptive control law. Simulation results show the good performance of the proposed controller as compared to the nonadaptive fuzzy controllers.

Keywords: fuzzy modeling, four-wheel steering, adaptive control, linear quadratic regulator

1. INTRODUCTION

The ever-growing demand for better handling of two-wheel-steering systems has attracted much attention of automobile industries to four-wheel-steering systems (4WS), which has gained popularity among consumers. The aim of 4WS is to reduce the body sideslip angle at vehicle mass center. In more technical terms, the phase difference between the yaw rate response and the lateral velocity has to be minimized (Shibahata, 1986). But, there is controversy among researchers of how the rear wheels should be steered in order to optimize the handling performance and stability of the vehicle. There are two distinctive characteristics, which define the approach to this problem: the control law used to steer the rear wheels and the model used to assess the impact of the control aw on the dynamics of the vehicle (Sanchez, 1994). In addition, the vehicle models are highly nonlinear, which makes it too complicated for use in the controller design procedure. Therefore, there is need to develop a simplified version of the vehicle model that can be used to design a controller. In this paper, using TSK fuzzy modeling method (Takagi and Sugeno, 1985; Sugeno and Kang, 1988), which is based on simple linear models using fuzzy logic, the difficulty in modeling has been solved.

In the next section, the fuzzy model of vehicle will be obtained. In section III, the design procedure for 4WS fuzzy controller is explained. Section IV shows the simulation results of the proposed control scheme. The conclusion is given in section V followed by nonlinear model of vehicle in appendix.

2. VEHICLE FUZZY MODEL

In order to design a controller with satisfactory performance, the model should capture vital characteristics of the process. In many instances, human experts may provide a linguistic description of the process in terms of IF-THEN rules. Combining this linguistic description with the mathematical description of the process results in a fuzzy model (Wang, 1997). In this paper, first, two local linear models of a vehicle are obtained. Then, combining these models with the linguistic description of the vehicle, a fuzzy model will be derived.

2.1 Vehicle Local Models

A linear model with two degrees of freedom for a vehicle with 4WS is shown in Fig. 1. This model contains two dominant state variables, namely the yaw rate *r* and the lateral velocity of the vehicle V_y. The state equations of the vehicle steering dynamics can be shown as follows, assuming that there are no roll steer effect and no lateral weight transfer:

$$\begin{bmatrix} \dot{V}_y \\ \dot{r} \end{bmatrix} = \begin{bmatrix} -2\dfrac{c_f + c_r}{m_{\text{tot}}\, u} & -2\dfrac{a\, c_f - b\, c_r}{m_{\text{tot}}\, u} \\ -2\dfrac{a\, c_f + b\, c_r}{I_{zz}\, u} & -2\dfrac{a^2\, c_f + b^2\, c_r}{I_{zz}\, u} \end{bmatrix} \begin{bmatrix} V_y \\ r \end{bmatrix} + \begin{bmatrix} 2\dfrac{c_f}{m_{\text{tot}}} \\ 2a\dfrac{c_f}{I_{zz}} \end{bmatrix} \delta_f + \begin{bmatrix} 2\dfrac{c_r}{m_{\text{tot}}} \\ -2\dfrac{b\, c_r}{I_{zz}} \end{bmatrix} \delta_r \quad (1)$$

$$\beta = \tan^{-1}\left(\frac{V_y}{u}\right) \quad (2)$$

where a and b are distances from the front and the rear axle to the center of gravity of the vehicle, respectively, c_f and c_r are the front and the rear tire cornering stiffness coefficients, respectively, m_{tot} is the vehicle total mass, u is the longitudinal velocity, I_{zz} is the yaw inertia, and δ_f and δ_r are the angles of the front and the rear tires (Fig. 1), respectively. Also, β is the body sideslip angle of the vehicle. Now, the derivation of two local models will be shown. Since the main contribution to the nonlinearity of the model is from the tires, the stiffness coefficients of the tires will play the main role in the model employed in this paper. According to Fig. 2, these coefficients are large for the tire slip angles less than 0.05 rad. and are small for slip angles more than 0.05 rad. Hence, Two linear models can be constructed, one for the dynamic model with large slip angles and one for the small slip angles. These two models are built according to Eq. (1) but with different values for tire stiffness coefficients c_f and c_r. For the small slip local model $c_f = c_r = 50000\ \text{N/rad}$, and for large slip local model $c_f = c_r = 50000\ \text{N/rad}$.

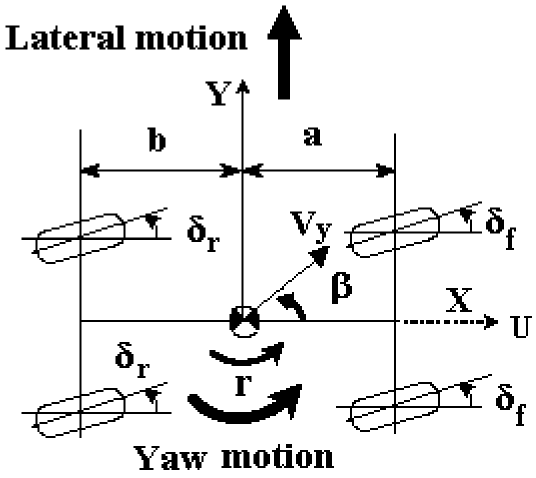

Fig. 1. Plane view of vehicle model.

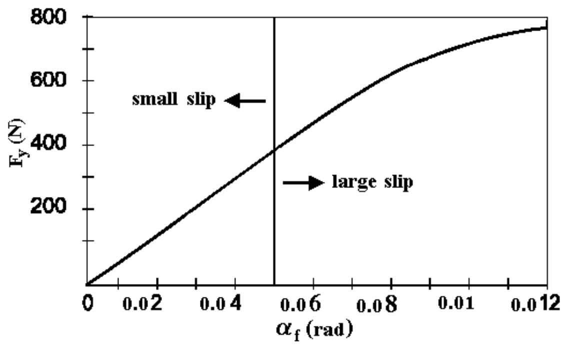

Fig. 2. Typical curve of tire lateral force versus tire slip angles.

Other parameters of the vehicle are as follows:

$u = 33.33\ \text{m}$ $\quad m_{\text{tot}} = 1298.84\ \text{kg}$

$a = 1\ \text{m}$ $\quad I_{zz} = 1627\ \text{kg m}^2$

$b = 1.45\ \text{m}$ $\quad g = 9.81\ \text{m/s}$

Now, the small slip local model can be defined as

$$\dot{\mathbf{x}} = \mathbf{A}_1\, \mathbf{x} + \mathbf{B}_{1f}\ \delta_f + \mathbf{B}_{1r}\ \delta_r \quad (3)$$

$$\mathbf{A}_1 = \begin{bmatrix} -4.6195 & -32.2939 \\ 0.8297 & -5.7207 \end{bmatrix}$$
$$\mathbf{B}_{1f} = \begin{bmatrix} 76.9918 \\ 61.4628 \end{bmatrix} \quad \mathbf{B}_{1r} = \begin{bmatrix} 76.9918 \\ -89.1211 \end{bmatrix} \quad (4)$$

and the large slip local model as

$$\dot{\mathbf{x}} = \mathbf{A}_2\, \mathbf{x} + \mathbf{B}_{2f}\ \delta_f + \mathbf{B}_{2r}\ \delta_r \quad (٥)$$

$$\mathbf{A}_2 = \begin{bmatrix} -2.7717 & -32.7097 \\ 0.4978 & -3.4324 \end{bmatrix}$$
$$\mathbf{B}_{2f} = \begin{bmatrix} 46.1951 \\ 36.8777 \end{bmatrix} \quad \mathbf{B}_{2r} = \begin{bmatrix} 46.1951 \\ -53.4726 \end{bmatrix} \quad (6)$$

where $\mathbf{x} = \begin{bmatrix} V_y & r \end{bmatrix}^T$.

2.2 Fuzzy Rule Base

The front tires slip angle α_f is considered as linguistic variable with two fuzzy sets (large and small), whose membership functions are shown in Fig. 3. Then, using these fuzzy sets, the fuzzy IF-TEHN rules can be defines as follows:

Rule1:

$$\text{IF} \left| \alpha_f \right| \text{ is small, THEN } \dot{\mathbf{x}} = \mathbf{A}_1 \mathbf{x} + \mathbf{B}_{1f}\ \delta_f + \mathbf{B}_{1r} \delta_r$$

Rule2:

$$\text{IF} \left| \alpha_f \right| \text{ is big, THEN} \dot{\mathbf{x}} = \mathbf{A}_2 \mathbf{x} + \mathbf{B}_{2f}\ \delta_f + \boldsymbol{B}_{2\boldsymbol{r}} \delta_r$$

Now, combining these fuzzy rules with the local linear models in Eqs. (3) to (6), the fuzzy model of the vehicle will be obtained. The effective weight of each rule is defines as

$$w_j = w_j(\alpha_f) = \frac{\mu_j(\alpha_f)}{\sum_{i=1}^{2} \mu_i(\alpha_f)} \tag{7}$$

where $\mu_j(\alpha_f)\ (j = 1,2)$ are the membership functions as shown in Fig. 3. Therefore, the following equation can represent the fuzzy model of the vehicle

$$\dot{\mathbf{x}} = \sum_{i=1}^{2} w_i (\mathbf{A}_i \mathbf{x} + \mathbf{B}_{if} \delta_f + \mathbf{B}_{ir} \delta_r) \tag{8}$$

In the next section the 4WS fuzzy controller will be developed.

3. 4WS CONTROLLER DESIGN

3.1 Fuzzy Controller

In this section the 4WS control system will be designed based on the fuzzy model in Eq. (8). Reminding the goal of the 4WS, which is a better handling of vehicle using rear wheel steering based on the front wheels, the 4WS control system will be designed using fuzzy model and the optimal control theory. Employing LQR method, it is desired to minimize the body sideslip angle of the vehicle β with 4WS. In the design procedure of the rear wheel steering the input to the front wheels δ_f is set to zero, which results in minimization of the performance function

$$j = \int_0^{\infty} (\mathbf{x}^T Q\, \mathbf{x} + R \delta_r^2) dt$$

with boundary conditions

$$\dot{\mathbf{x}} = \mathbf{A}_i \mathbf{x} + \mathbf{B}_{if} \delta_f + \mathbf{B}_{ir} \delta_r \quad (i = 1,2)$$

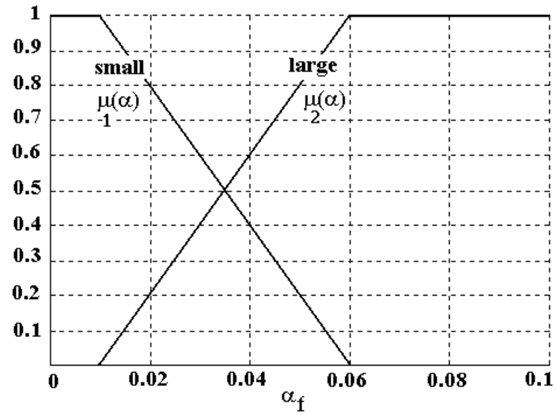

Fig. 3. Membership functions for front-wheel slip.

where $\mathbf{x} = [V_y \quad r]^T$ and R and Q are weighting parameters. The closed loop response δ_r of each local model can found as

$$\delta_r = \frac{-1}{R} \mathbf{B}_{ir}^{\mathbf{T}}\ \mathbf{P}_i\ \mathbf{x} \qquad (i = 1,2)$$

where $\mathbf{P}_i$ is a symmetrical positive matrix, which satisfies the following Riccati equation:

$$-\mathbf{P}_i\ \mathbf{A}_i - \mathbf{A}_i^T\ \mathbf{P}_i + \mathbf{P}_i\ \mathbf{B}_{ir}\ \mathbf{R}^{-1}\ \mathbf{B}_{ir}^T\ \mathbf{P}_i - \mathbf{Q} = \mathbf{0}$$

After solving the Riccati equation for linear models, the optimal gain vectors will be found as

For small slip model: $\mathbf{k}_1 = [7.0131 \quad -0.3991]$

For large slip model: $\mathbf{k}_1 = [7.0141 \quad -0.6616]$

Therefore, the controller for rear wheels is as follow:

$$\delta_r = -(w_1 \mathbf{k}_1 + w_2 \mathbf{k}_2) \mathbf{x} \tag{9}$$

In The following section it will be shown how the center of the membership functions will be updated in order to create an adaptive controller.

3.2 Adaptive Fuzzy Controller

Sometimes the controllers are working with some uncertainties or unknown changes in the parameters or in the structure of the system. In general, the main goal of the adaptive controllers is to maintain the performance of the system in the presence of these uncertainties. Hence, an advanced fuzzy controller should be adaptive as well. To make the proposed fuzzy controller, adaptive, let c_s and c_l be the center of the membership functions for the small and the large slip models, respectively. Also, let w_s and w_l be the value of the memberships for the small and the large slip models, respectively. Then, the adaptive law for the center of the membership functions will be

$$c_s(n+1) = k_w \times w_s(n)$$
$$c_l(n+1) = k_w \times w_l(n)$$

where k_w is a positive coefficient.

4. SIMULATION RESULTS

The simulations are performed in three parts. In the first part, the open loop response of the system is being considered. That is, the modeling precision of the proposed fuzzy model will be compared with the modeling accuracy of the linear model of the vehicle. In this part of the simulations, the input to the vehicle and the fuzzy model is a step function with an amplitude of 0.0345 rad. The results have been shown in Figs. 5-7. As these graphs show, the proposed fuzzy model has greater accuracy as compared to the linear model. The nonlinear dynamic equations of vehicles are given in the appendix. In the second part of the simulations (Figs. 8-10), the results of the proposed controller are compared to the results obtained by Taxeria, et al. (1997). The input function has been shown in Fig. 4. Since there is direct correlation between lateral velocity and the body sideslip angle of the vehicle, the body sideslip angle of the adaptive controller has been considerably reduced as compared to the fixed fuzzy controller. In the third part of the simulations (Figs. 11-13), the proposed adaptive controller has been tested against the change of the parameters in the vehicle (Yeh, at al., 1989), considering the lateral velocity, the body sideslip and the yaw rate. The equations for changes are as follows:

$$a' = a + \frac{p \cdot b}{1+p}, \quad b' = b - \frac{p \cdot b}{1+p}$$

$$m'_{\text{tot}} = (1+p)\, m_{\text{tot}}, \quad I'_{zz} = I_{zz} + p\, m\, b^2$$

where the change factor $p = 0.05$. As these graphs in Figs. 11-13 show, V_y has been decreased, which results in less body sideslip angles β.

5. CONCLUSIONS

In this paper, first a TSK fuzzy model was proposed to model the 4WS. This model consists of two local linear models, one for large body sideslip angles and one for small ones. Then, combining this fuzzy model and the LQR control method, an optimal fuzzy controller was constructed for the 4WS vehicles. Next, employing adaptive control methods, an adaptive fuzzy controller was proposed. Simulation results show the good performance of the proposed method. That is, less body sideslip and less yaw rate for vehicles.

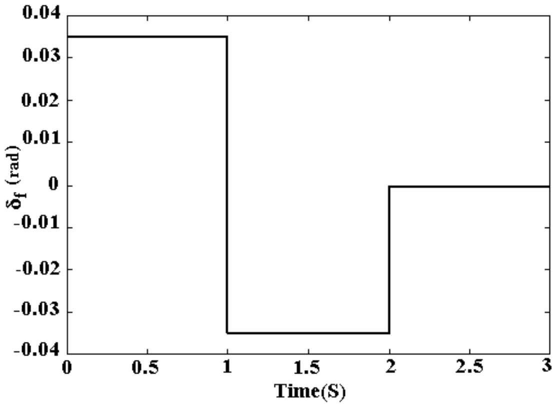

Fig. 4. The front wheels angle as the input to the system.

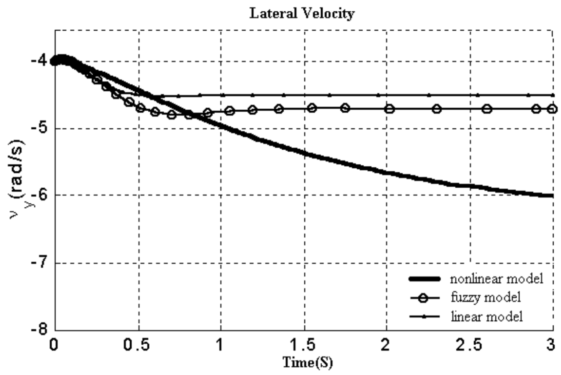

Fig. 5. Lateral velocity of the vehicle, the fuzzy models and the linear model.

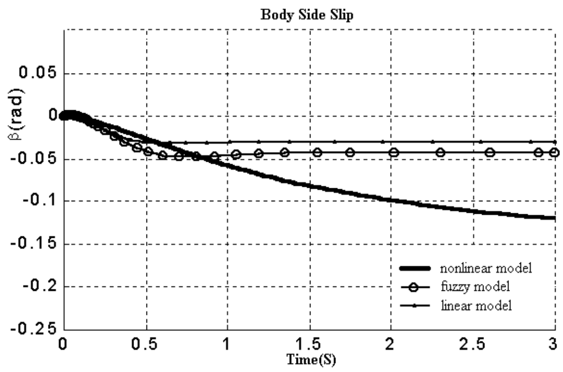

Fig. 6. Body sideslip angle of the vehicle, the fuzzy model and the linear model.

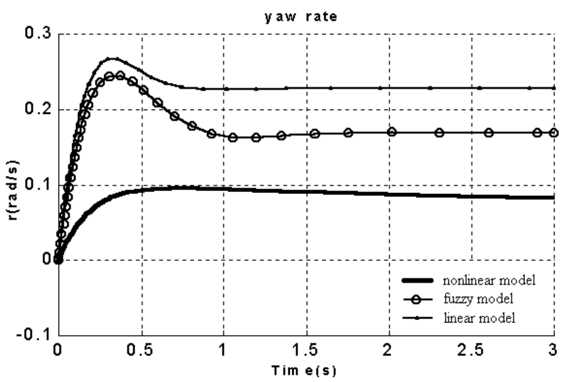

Fig. 7. The yaw rate of the vehicle, the fuzzy model and the linear model.

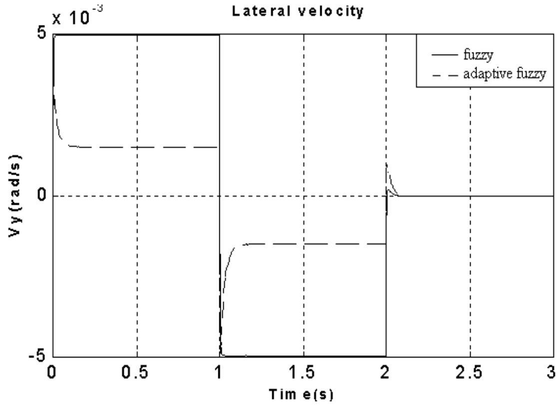

Fig. 8. Lateral velocity of the controlled 4WS vehicle.

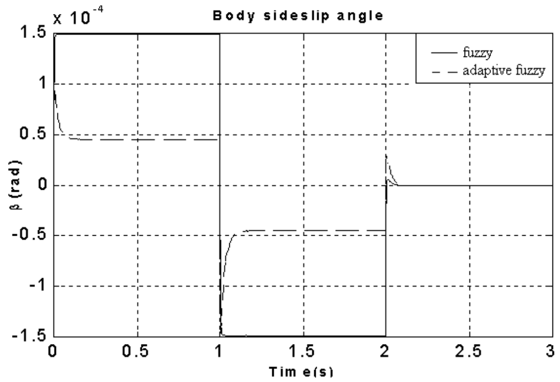

Fig. 9. Body sideslip of the controlled 4WS vehicle.

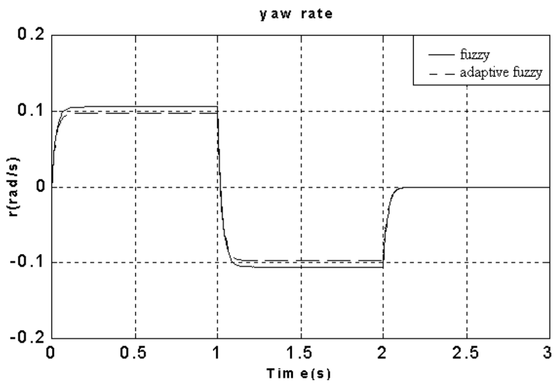

Fig. 10. The yaw rate of the controlled 4WS vehicle.

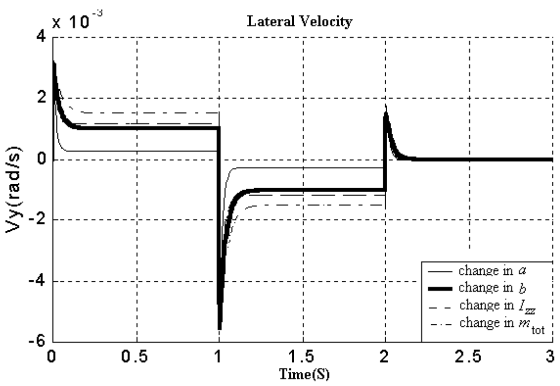

Fig. 11. Lateral velocity of the vehicle, when the parameters are changing.

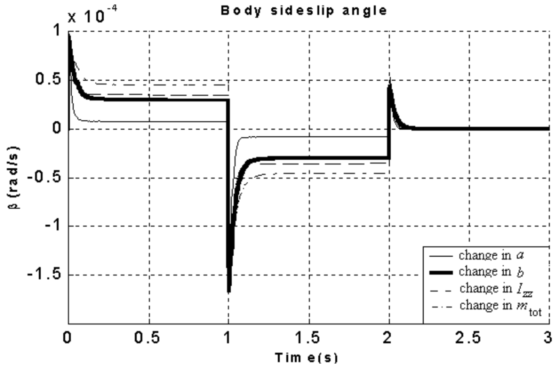

Fig. 12. Body sideslip angle of the vehicle, when the parameters are changing.

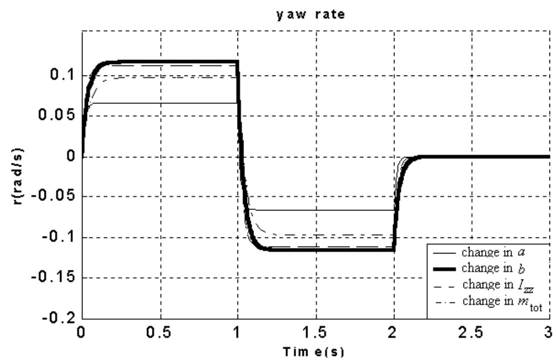

Fig. 13. The yaw rate of the vehicle, when the parameters are changing.

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APPENDIX

Here the nonlinear model of vehicle will be given. A three degree-of-freedom model of 4WS vehicle has been used in simulations. This model contains lateral velocity, yaw and roll degrees of freedom and the nonlinear equations of tires. The equations of motion are as follows (Segal, 1956):

$$m_{\text{tot}}\left(V_y + u\,r\right) + m_s h \ddot{\psi} = F_{yrf} + F_{yrr} + F_{ylf} + F_{ylr} + 2\frac{\partial F}{\partial \dot{\psi}_f}\frac{\partial \dot{\psi}_f}{\partial \psi}\psi + 2\frac{\partial F_r}{\partial \dot{\psi}_r}\frac{\partial \dot{\psi}_r}{\partial \psi}\psi$$

$$I_{zz}\dot{r} - I_{zx}\ddot{\psi} = a\left[F_{yrf} + F_{ylf} + 2\frac{\partial F}{\partial \dot{\psi}_f}\frac{\partial \dot{\psi}_f}{\partial \psi}\psi\right] - \left[F_{yrr} + F_{ylr} + 2\frac{\partial F}{\partial \dot{\psi}_f}\frac{\partial \dot{\psi}_f}{\partial \psi}\psi\right]$$

$$I_{xx}\ddot{\psi} - I_{xz}\dot{r} + m_s h\left(V_y + u\,r\right) = -\left(L_{\psi f} + L_{\psi r}\right)\psi - \left(L_{\dot{\psi} f} + L_{\psi r}\right)\dot{\psi}$$

The tire side forces are functions of the slip angles and tire normal loads

$$F = \mu F_z \frac{\left[f(\sigma)k_s \tan\alpha\right]}{\left[k_s^2 \tan^2\alpha + k_c'^2 s^2\right]^{1/2}}$$

where $f(\sigma)$ is the composite slip function

$$f(\sigma) = \frac{c_1\sigma^3 + c_2\sigma^2 + \frac{4}{\pi}\sigma}{c_1\sigma^3 + c_2\sigma^2 + \frac{4}{\pi}\sigma + 1}$$

σ is the tire saturation function

$$\sigma = \frac{\pi a_p^2}{8\mu_o F_z}\left[k_s^2 \tan^2\alpha + k_c^2\left(\frac{s}{1-s}\right)\right]$$

k_s is the lateral stiffness coefficient

$$k_s = \frac{2\left[A_0 + A_1 F_z - \frac{A_1}{A_2} F_z^2\right]}{a_{p_0}^2}$$

k_c is the longitudinal coefficient

$$k_c = \frac{2F_z\left(\frac{cs}{F_z}\right)}{a_{p_0}^2}$$

μ_o is the peak tire-road coefficient friction

$$\mu_o = 1.176\,\mu_{\text{nom}}\left(B_1 F_z + B_3 + B_4 F_z^2\right)$$

a_p is the tire constant patch length

$$a_p = a_{p_o}\left[1 - k_a\left(\frac{F_x}{F_z}\right)\right]$$

in which

$$a_{p_o} = \frac{0.0768\sqrt{F_z F_{zT}}}{T_w(T_p + 5)}$$

Also, μ is the transition coefficient of friction

$$\mu = \mu_o\sqrt{1 - k_\mu\ (\sin^2\alpha + s^2\cos^2\alpha)}$$

k_c' is the transition stiffness

$$k_c' = k_c + \left(k_s - k_c\right)\sqrt{\sin^2\alpha + s^2\cos^2\alpha}$$

k_μ is the transition constant

$$k_\mu = -\frac{1}{11}v_{wl}^{1/4}$$

F_{zT} is the tire design load at operating pressure, T_w is the tread width, T_p is the tire pressure, k_a is the proportional effect of F_x on patch length, cs/F_z is the coefficient obtained from Calspan test, v_{wl} is the velocity of tire, m_s is the sprung mass of vehicle, h is the height of the center gravity of the sprung mass, $L_{\psi f}$ and $L_{\psi r}$ are the roll stiffness produced by front and rear springs, respectively, $L_{\dot{\psi} f}$ and $L_{\dot{\psi} r}$ are the roll damping produced by the front and rear shock absorbers, respectively, and A_i $\left(i = 1, 2, 3, 4\right)$, B_i and c_i $\left(i = 0, 1, 2, 3, 4\right)$ are standard SAE tire parameters.