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MODEL-BASED NONLINEAR CONTROL OF A LOW-POWER GAS TURBINE

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Abstract:
A simple nonlinear controller for a low-power gas turbine based on direct passivation is proposed in this paper. It uses a nonlinear state space model of the gas turbine in input-affine form based on first engineering principles.
It is found by standard nonlinear analysis that the developed model is reachable and observable in the whole application domain and stable in a small neighborhood of an operating point. Accordingly, the control aim is to guarantee the asymptotic stability of every operating points in the whole application domain and to provide suitable disturbance rejection.
The proposed nonlinear controller is based on a prescribed quadratic Lyapunov-function and it is able to solve the protection of the gas turbine. The robustness of the closed-loop system with respect to the time-varying parameters is also investigated.

Keywords: Nonlinear models, Nonlinear control systems, Transportation systems, Vehicles, Gas turbines

1. INTRODUCTION

Gas turbines are important and widely used prime movers in transportation systems, such as aircraft, cars. Besides of this area gas turbines are found in power systems where they are the main power generators (Evans (1998)). At the same time gas turbines are known to be nonlinear systems which present a challenge for their control.

The nonlinear dynamic model of gas turbines is derived from first engineering principles which consists of differential conservation balances completed by algebraic constitutive relations (Hangos and Cameron (2001)). These constitutive equations describe the steady-state behavior of the gas turbine and they are based upon the characteristics of the component parts of the engine Akhmedzianov (1997)). This nonlinear dynamic model can then be effectively used to develop linear and nonlinear controllers to improve the dynamic response of aircraft engines (Qi et al. (1992)).

Generally linear controllers are developed for the gas turbine, as proposed by Qi et al. (1992), Athans et al. (1986) and Ariffin and Munro (1997), for example. These controllers are based on the LQ-technique, for example on LQ-servo or LQG/LTR methods. Lately robust control system design has also been performed

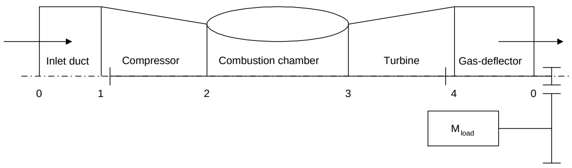

Fig. 1. The main parts of the gas turbine

which can assist in improving the robust stability and performance of a special aero-engine.

Motivation
Linear controllers work well on gas turbines because of two reasons. The first reason is the fact that a gas turbine is stable in a small neighborhood of any operating point in the whole application domain. The second reason is that the nonlinearities present are usually mild, i.e. the nonlinear dynamic model of the turbine is usually well approximated by a linear model.

Therefore, the motivation for applying nonlinear controller design techniques can be to guarantee the global asymptotic stability in all operating points in the application domain and disturbance-rejection, where the disturbances are ambient circumstances and in some cases the load.

System description
The main parts of a gas turbine include the inlet duct, the compressor, the combustion chamber, the turbine and the nozzle or the gas-deflector. The interactions between these components are fixed by the physical structure of the engine.

The operation of gas turbines is basically the same. The air is drawn into the engine through the inlet duct by the compressor, which compresses it and then delivers it to the combustion chamber. Within the combustion chamber the air is mixed with fuel and the mixture is ignited, producing a rise in temperature and hence an expansion of the gases. These gases are exhausted through the engine nozzle or the engine gas-deflector, but first pass through the turbine, which is designed to extract sufficient energy from them to keep the compressor rotating, so that the engine is self sustaining. The main parts of a gas turbine are shown schematically in Fig. 1. A concrete low-power gas turbine is used for our studies. The equipment is installed in the Budapest University of Technology and Economics, Department of Aircraft and Ships on a test-stand.

2. NONLINEAR STATE SPACE MODEL

The nonlinear state equations are derived from first engineering principles. Dynamic conservation balance equations are constructed for the overall mass m, the internal energy U (Hangos and Cameron (2001)) and the mechanical energy E_{shaft}. The notation list is given separately in the Appendix.

These dynamic equations have to be transformed to intensive variable form to contain the measurable quantities. Therefore the set of transformed differential balances include the dynamic mass balance for the combustion chamber, the pressure form of the state equation derived from the internal energy balance for the combustion chamber and the intensive form of the overall mechanical energy balance expressed for the number of revolutions n. Thus 3 independent balance equations can be constructed, therefore the gas turbine can be described by only 3 state variables.

In order to complete the model, constitutive algebraic equations are also needed. These equations describe the static behavior of the gas turbine in various operating points, and all of them can be substituted into the dynamic equations (Ailer et al. (2001)).

The final form of the nonlinear dynamic model equations is the following:

$$\frac{dm_{Comb}}{dt} = v_C + v_{fuel} - v_T \tag{1}$$

$$\begin{aligned}\frac{dp_3^*}{dt} &= \frac{p_3^*}{m_{Comb}}\left(v_C + v_{fuel} - v_T\right) + \frac{R_{med}}{V_{Comb}c_{vmed}} \\ &\left(v_C c_{pair} T_1^* \left(1 + \frac{1}{\eta_C}\left(\left(\frac{p_3^*}{p_1^*\sigma_{Comb}}\right)^{\frac{\kappa_{air}-1}{\kappa_{air}}} - 1\right)\right) - \right. \\ &v_T c_{pgas}\frac{p_3^* V_{Comb}}{m_{Comb}R_{med}} + Q_f \eta_{comb} v_{fuel} - \\ &\left. c_{vmed}\frac{p_3^* V_{Comb}}{m_{Comb}R_{med}}\left(v_C + v_{fuel} - v_T\right)\right)\end{aligned} \tag{2}$$

$$\begin{aligned}\frac{dn}{dt} &= \frac{1}{4\Pi^2\Theta n}\left(v_T c_{pgas}\frac{p_3^* V_{Comb}}{m_{Comb}R_{med}}\eta_T \eta_{mech}\right. \\ &\left(1 - \left(\frac{p_1^*}{p_3^*\sigma_I\sigma_N}\right)^{\frac{\kappa_{gas}-1}{\kappa_{gas}}}\right) - v_C c_{pair}\frac{T_1^*}{\eta_C} \\ &\left.\left(\left(\frac{p_3^*}{p_1^*\sigma_{Comb}}\right)^{\frac{\kappa_{air}-1}{\kappa_{air}}} - 1\right) - 2\Pi\frac{3}{50}nM_{load}\right)\end{aligned} \tag{3}$$

where:

$$v_C = \beta_{air}\, A_1\, q(\lambda_1)\frac{p_1^*}{\sqrt{T_1^*}} \tag{4}$$

$$v_T = \beta_{gas}\, A_3\, q(\lambda_3)\frac{p_3^*}{\sqrt{\frac{p_3^* V_{Comb}}{m_{Comb}R_{med}}}} \tag{5}$$

$$\eta_C = a_1\frac{n}{\sqrt{\frac{T_1^*}{288.15}}}\, q(\lambda_1) +$$

$$a_2 \frac{n}{\sqrt{\frac{T_1^*}{288.15}}} + a_3\, q(\lambda_1) + a_4 \tag{6}$$

$$\eta_T = b_1(const \frac{n}{\sqrt{\frac{p_3^* V_{Comb}}{m_{Comb} R_{med}}}}) \frac{p_1^*}{p_3^* \sigma_I \sigma_N} +$$

$$b_2(const \frac{n}{\sqrt{\frac{p_3^* V_{Comb}}{m_{Comb} R_{med}}}}) + b_3 \frac{p_1^*}{p_3^* \sigma_I \sigma_N} + b_4 \tag{7}$$

and:

$$q(\lambda_1) = a_1 \frac{n}{\sqrt{\frac{T_1^*}{288.15}}} \frac{p_3^*}{p_1^* \sigma_{Comb}} +$$

$$a_2 \frac{n}{\sqrt{\frac{T_1^*}{288.15}}} + a_3 \frac{p_3^*}{p_1^* \sigma_{Comb}} + a_4 \tag{8}$$

$$q(\lambda_3) = b_1(const \frac{n}{\sqrt{\frac{p_3^* V_{Comb}}{m_{Comb} R_{med}}}}) \frac{p_1^*}{p_3^* \sigma_I \sigma_N} +$$

$$b_2(const \frac{n}{\sqrt{\frac{p_3^* V_{Comb}}{m_{Comb} R_{med}}}}) + b_3 \frac{p_1^*}{p_3^* \sigma_I \sigma_N} + b_4) \tag{9}$$

The parameters, constants of the nonlinear dynamic model are known or have been determined from measurements. We have determined the static characteristics of the compressor and the turbine and applied nonlinear parameter-estimation for the rest (Ailer et al. (2001)).

The following operation domain can be investigated experimentally on the gas turbine test-stand expressed in terms of the *intensive set of state variables*:

$$x = [\ m_{Comb} \ \ p_3^* \ \ n\ ]^T \tag{10}$$

$$0.002100317923 \le m_{Comb} \le 0.01092691283\ [kg]$$
$$101333.9168 \le p_3^* \le 357894.2736\ [Pa]$$
$$650 \le n \le 833.33\ [1/sec]$$

The value of the only *input variable* v_{fuel} is also constrained by:

$$0.003669480223 \le v_{fuel} \le 0.02701065149\ [kg/sec]$$

The set of possible *disturbances* includes:

$$d = [\ p_1^* \ \ T_1^* \ \ M_{load}\ ]^T \tag{11}$$

$$60000 \le p_1^* \le 110000\ [K]$$
$$243.15 \le T_1^* \le 308.15\ [Pa]$$
$$0 \le M_{load} \le 363\ [Nm]$$

Finally we construct the *set of measurable output variables*:

$$y = [\ T_4^* \ \ p_3^* \ \ n\ ]^T \tag{12}$$

The dynamic equations with these vectors can be transformed into input-affine form (Hangos et al. (2000)):

$$\frac{dx}{dt} = f(x) + g(x)u \tag{13}$$

$$y = h(x) \tag{14}$$

with the following f, g and h functions:

$$f(x) = \begin{bmatrix} f_1(x_1,x_2,x_3,d_1,d_2) \\ f_2(x_1,x_2,x_3,d_1,d_2) \\ f_3(x_1,x_2,x_3,d_1,d_2,d_3) \end{bmatrix} \tag{15}$$

$$g(x) = \begin{bmatrix} constant_1 \\ constant_2 \\ 0 \end{bmatrix} \tag{16}$$

$$h(x) = \begin{bmatrix} h_1(x_1,x_2,x_3,d_1) \\ x_2 \\ x_3 \end{bmatrix} \tag{17}$$

It is important to note that these functions do not depend on only the state variables, but also the disturbance vector. Further observe, that $g(x)$ does not depend on the state vector x. This means, that the effect of the input is linear to the time derivative of the state vector.

3. CONTROLLER DESIGN

3.1 Control aims

The following control aims is set for our low-power gas turbine:

- The number of revolutions has to follow the position of the throttle and should not be effected by the load and the ambient circumstances (the disturbance vector).
- The temperatures (basically the total temperature after the turbine) and the number of revolutions has to be limited, their values are constrained from above by their maximum values.

3.2 Open loop properties of the system

It is found by standard nonlinear analysis (Isidori (1995)) that the developed model is reachable and observable in the whole application domain and stable in a small neighborhood of any admissible operating point (Ailer et al. (2001)). Therefore the asymptotic stability of all operating points can be guaranteed in the whole application domain.

The measurable time-varying parameters of the system are collected into the disturbance vector. All elements of this vector are correctly measurable and significantly change their values during the operation of the gas turbine.

There are also some unmeasurable time-varying parameters in the nonlinear model. The value of the parameters σ_I, σ_{Comb}, σ_N and η_{Comb} are not constant, but they change their values only about ± 2 percent.

Because of these properties we have to achieve disturbance-rejection and robustness of the closed-loop system as stated in the control aims before.

3.3 Static nonlinear stabilizing controller

In the first step, a static nonlinear full state feedback is designed to stabilize the system in the whole operating region. For this, let us assume that the point x_0 is an equilibrium point for the system (i.e. $f(x_0) = 0$) and define a positive definite storage function

$$V(x) = (x - x_0)^T M(x - x_0) \tag{18}$$

where M is a $n \times n$ positive definite symmetric matrix. If we set the input u as

$$u = v_p + v + w \tag{19}$$

where

$$v_p = -\frac{L_f V(x)}{L_g V(x)} \tag{20}$$

and

$$v = -k \cdot L_g V(x), \;\; k > 0 \tag{21}$$

then the closed loop system will be passive with respect to the supply rate $s = w^T y$ with storage function V where $y = L_g V(x)$ and w is the new reference input (Byrnes et al. (1991)). Note that the feedback law in Eq. (20) has singular points where $L_g V(x) = 0$, therefore x_0 has to be selected carefully to be outside the real operating region. The new reference input w is calculated from the linear shift between x_0 and the set-point.

3.4 Protection of the gas turbine

The aim of the protection of the gas turbine is twofolds: to avoid too high temperatures and too high number of revolutions.

(1) The turbine outlet temperature y_1 has a maximum value of $938.15\,K$. If the set-point for y_1 is higher then its maximum value then we can increase the number of revolutions (the temperature will then decrease), but the number of revolutions has also a maximum value: $833.33\ 1/sec$.

(2) If the position of the throttle specify higher value for the number of revolutions x_3 than its maximum, then we do not allow it; the value will be its maximum.

3.5 Achieving robustness with a PI controller

In order to remove the steady state error in the number of revolutions caused by the change of the unmeasurable time-varying parameters, a PI controller (which is itself again a passive system) is applied in the outer loop i.e.

$$w(t) = k_p(y_{3s} - y_3(t)) + k_i \cdot \int_{t_0}^{t} (y_{3s} - y_3(\tau))d\tau \tag{22}$$

where y_{3s} is the constant set-point for the number of revolutions, $y_3(t)$ is the number of revolutions as a function of time, k_p and k_i are the parameters of the PI controller.

4. SIMULATION RESULTS

For the simulations a typical operating point has been selected:

$$x^* = [\ 0.00580436\ 197250.6068\ 700\ ]^T \tag{23}$$

$$d^* = [\ 101325\ 288.15\ 10\ ]^T \tag{24}$$

$$u^* = 0.009252624089 \tag{25}$$

4.1 Simulation of the open-loop system

The properties of the open-loop system are well demonstrated by the simulation results. Fig. 2 shows a case when the number of revolutions has been changed to 400 and the system is not able to reach the set-point indicating that the open-loop system is not globally asymptotic stable. Fig. 3 shows that the open-loop system is sensitive with respect to the change of the disturbances: if we change the M_{load} from 10 to 30 then the engine shuts down.

4.2 Simulation of the closed-loop system

In order to achieve appropriate dynamics of the closed-loop system, we have to tune M being positive definite and symmetric; the positive constant k and the parameters of the PI controller k_i and k_p. The tuning of these

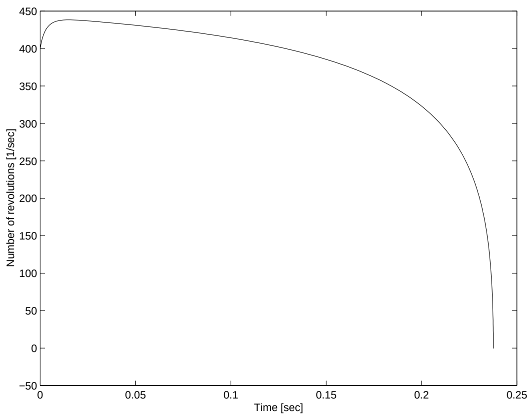

Fig. 2. Response of the number of revolutions of the open-loop system (unstable)

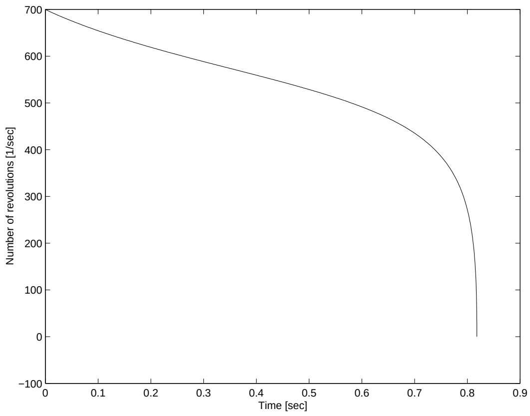

Fig. 3. Response of the number of revolutions of the open-loop system (sensitive)

parameters was carried out by trial and error method and the following values obtained:

$$M = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 1000 & 0 \\ 0 & 0 & 0.001 \end{bmatrix} \tag{26}$$

and $k = 0.0005$, $k_i = 0.7$, $k_p = 1.1$.

Fig. 4 shows the same case as in Fig. 2 with our nonlinear controller. It indicates that the controller indeed stabilizes the system globally. Fig. 5 shows that the closed-loop system is not sensitive to the elements of the disturbance-vector. If we increase the load, the number of revolutions is decreasing, but the nonlinear controller is able to set the original operating point.

Fig. 6 shows a set-point change and the robustness of the controller. The position of the throttle has been changed from 700 $[1/sec]$ to 750 $[1/sec]$ and simultaneously one time-varying parameter has been raised with 2 percent.

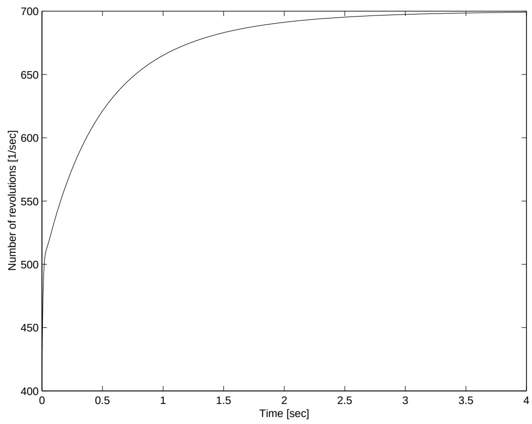

Fig. 4. Response of the number of revolutions of the closed-loop system (stable)

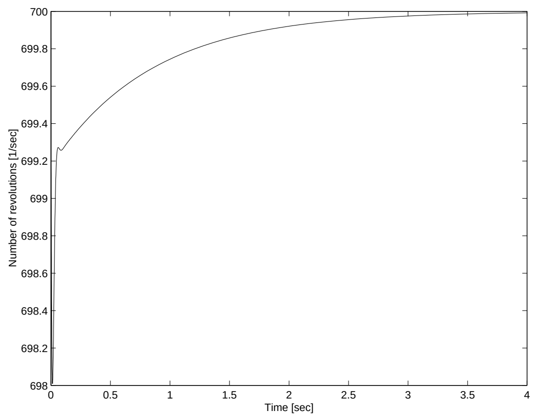

Fig. 5. Response of the number of revolutions of the closed-loop system (not sensitive)

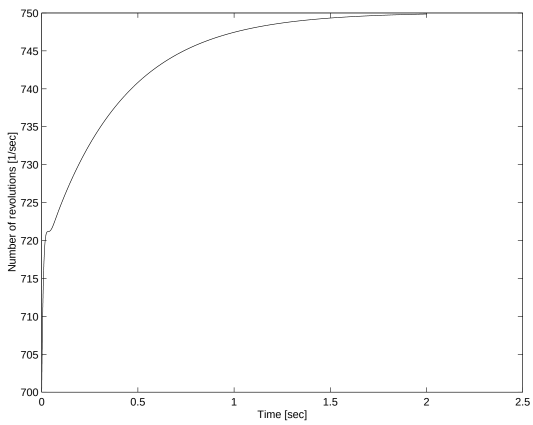

Fig. 6. Response of the number of revolutions of the closed-loop system (set-point change)

5. CONCLUSION

A nonlinear controller is proposed in this paper for a low-power gas turbine which is able to guarantee the asymptotic stability of the closed-loop system in every operating of the application domain. This controller keeps the number of revolutions in accordance with the position of the throttle while the number of revolutions is not affected by the load and ambient circumstances. At the same time the controller can protect the gas turbine against the too high temperature and the too high number of revolutions in a set-point and the closed-loop system is robust with respect to the time-varying parameters.

6. APPENDIX: NOTATIONS

Variables

c	specific heat $[J/kg\ K]$
i	specific enthalpy $[J/kg]$
m	mass $[kg]$
n	number of revolutions $[1/sec]$
p	pressure $[Pa]$
q	dimensionless mass flow rate $[\frac{\sqrt{K}sec}{m}]$
t	time $[sec]$
Q_f	lower thermal value of fuel $[J/kg]$
M	moment $[Nm]$
Q	heat $[J]$
R	specific gas constant $[J/kg\ K]$
T	temperature $[K]$
U	internal energy $[J]$
V	volume $[m^3]$
W	work $[J]$
η	efficiency $[-]$
κ	adiabatic exponent $[-]$
λ	dimensionless speed $[\sqrt{\frac{kg}{J}}\frac{m}{sec}]$
v	mass flow rate $[kg/sec]$
σ	pressure loss coefficient $[-]$
Θ	inertial moment $[kg\ m^2]$

Indices

$*$	total
0	inlet duct inlet
1	compressor inlet
2	compressor outlet
3	turbine inlet
4	turbine outlet
I	refers to inlet duct
C	refers to compressor
$Comb$	refers to combustion chamber
T	refers to turbine
N	refers to gas-deflector
air	refers to air
gas	refers to gas
med	refers to medium parameters
$comb$	refers to combustion
$mech$	mechanical
$load$	loading
$fuel$	refers to fuel
in	inlet
out	outlet
p	refers to constant pressure
v	refers to constant volume

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